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Factors Affecting Students' Change of Learning Behaviour

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Abstract

A model describing factors affecting students' learning will be presented. The model was based on Norman and Prichard's work which identified three major components of the teaching and learning situation, namely Entry Mastery, Student Motivation and Opportunity to Learn. I have focused on two components: **Opportunity to Learn** and **Students' Motivation** in the teaching of Engineering Mathematics at UTM. I changed my teaching practice to support the development of students' independent learning and problem solving skills. Naturalistic data of class interactions and how they influence students' motivation and attitudes towards learning will be presented. From the analysis, additional elements were added to the model.

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Keywords: teaching and learning situation; mathematical thinking; students' learning; motivation and opportunity to learn

1. Introduction

This paper will be discussing some results obtained from a study concerned with changing students' attitudes in Engineering Mathematics through working on mathematical thinking. The research was conducted in three different phases due to changes to the university's entry requirements, the curriculum as well as due to the review of the previous phase. In total, eight groups of students were studied who were from the Faculties of Mechanical, Civil and Electrical Engineering. The research covered a general period from the second semester of 2001/2002 to the first semester of the academic session 2007/2008. An action research methodology was used as the process has several features that were appropriate to the research concerns. Firstly, action research is an intervention in personal practice with a commitment to educational improvement (McNiff, Lomax & Whitehead, 1996). The teacher becomes the subject and object of research but with a greater awareness of the actions that are being carried out. It means that the teacher must investigate her actions and motives systematically, be critical of her interpretations and findings and be more open to alternative viewpoints. There should be a commitment to the actions implemented and the actions must be intentional. The process itself demands that the teacher becomes aware of the cycle of planning, action and review. She must alternate action with critical reflection, evaluating the research situation and back to the planning, modifying or changing if and when required. Secondly, action research will allow authentic description of the classroom environment to be made, the teacher's actions as well as the students' behaviour in the class. Various methods of collecting data was used, namely, observation, students' working, discussion and interviews and various

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questionnaires such as, problem solving questionnaire, course and teacher evaluation questionnaire. Thus the data collection and analysis at every semester and every phase were inputs into the next teaching sessions.

We have seen our students struggling with new concepts in Advanced Calculus, unable to recall the necessary prior knowledge and unsure of how to choose suitable mathematical techniques to solve problems. They worked on many problems to increase their understanding but still found non-routine questions difficult. We have conducted many remedial classes and given extra tuition to 'weak' students and have come to realize that one of the most typical problem was that these students could not 'see' the 'generality' nor 'see' the 'particular' in the general. They could not recognise essential features of a technique especially when presented in different forms.

We strongly felt that a most effective way to help is to empower students with skills to learn and think about the mathematics. Thus, the purpose of the research was to bring about improvements to the teaching of Engineering Mathematics that will develop students facility with mathematical concepts and techniques. The research method was chosen so that the study was in a natural setting. Information from that every encounter with our students, within and beyond the classroom, was noted and used in the reflection about teaching, in the review of the strategies and consequently, in the way the we teach and interact with the students.

Views from our peers were actively sought, within the Department of Mathematics as well as from the various Faculties of Engineering to add variety to perceptions of students learning and development. The findings to be reported here will touch on the various factors that affect students learning. The research covered was carried out in various semesters from the academic sessions 2001/2002 to 2007/2008.

2. Description of the Students

The students were generally made up of two main categories, those who had entered UTM before July 2002 and those who came after. Two main differences between the categories were their entrance qualifications and medium of communication. Pre-July 2002 students had the Sijil Pelajaran Malaysia (SPM, *Malaysian Certificate of Education*) which are equivalent to the GCE O-Level qualifications as an entry qualification. They were also taught in Bahasa Malaysia (*Malay Language*), the official medium of communication of UTM. Meanwhile, post-July students had post-SPM qualifications such as Matriculation qualifications, Sijil Tinggi Pelajaran Malaysia (STPM, *Higher School Certificate of Education*), Diplomas from local colleges or other qualifications recognised as of similar standing. These qualifications are equivalent to the GCE A-Levels. The other significant change was that English Language became the medium of communication. These changes also brought about changes to the curriculum which were modified accordingly.

Ongoing analysis of the data collected had contributed to changes made to teaching materials and tasks during the various phases, however the basic ideas were kept intact but the tasks and delivery was modified and adapted. Some particular examples, was the use of worksheets which then became a workbook. When 'rapport' was identified as a significant factor by students, some personal connection and negotiation helped to support some of the students' efforts at making changes in the way they worked with the mathematics.

A particular mathematical pedagogy was developed and implemented to achieve those objectives with changes in teaching practice to support students' to become more aware of their own ability in mathematical learning (Roselainy, 2009). In order to appreciate the motivation of the change, several issues culled from research about students' learning of mathematics will be presented.

3. Review of Research on Students' Learning

Findings on students learning and difficulties are widely documented. Of significant interest is the finding that students had difficulties in the learning of specific mathematical concepts. Among these were some concepts that were basic to Engineering Mathematics such as functions, the limiting process and the ability to use various representations of functions, numerically, symbolically and graphically (Bennet, 1977; Eisenberg, 1991; Md. Nor Bakar, 1991; Tall, 1996, Artigue, 1991, Maselan Ali, 1996). In terms of students' facility with algebraic manipulation and problem solving, Tall & Razali (1993) have identified possible qualitative differences between the able and less able students in mathematics. The more able students could flexibly manipulated concepts as objects or processes but the less able had a tendency to concentrate on coordinating processes. Thus, they preferred routine processes but were less likely to be able to relate ideas in a meaningful way. They also faced difficulties in organizing known facts and could not master the mathematical language and symbols. Furthermore, their

main strategy in answering questions was to accumulate procedures and they showed a tendency to over generalize the use of certain mathematical procedures. Some students were unable to select and use appropriate mathematical representations, such as, visual, numerical, symbolic, algebraic and analytical. Another finding to be considered was that many students, even the good ones, were unable to solve non-routine problems (Selden, Mason & Selden, 1994).

To further strengthen our understanding of the factors that influences students' behaviour, we also looked at various research that explored the impact and influence of students' beliefs about mathematics and their mathematical performance. In a study by Tall and Razali (1993), it was found that students perceived mathematics as a subject wholly consisting of a collection of facts and procedures. Generally, students expected to master mathematics by memorization and drill exercises as they considered the subject to be an objective and objectively graded discipline. It was also reported (Schoenfeld, 1989) that students believed that most mathematics questions should be immediately solvable and should not take up more 10-15 minutes. Good teaching practice should consist of the ability of teachers to ensure that their students could learn all the necessary rules and procedures. These beliefs have particularly influenced the students' problem solving behaviour. Thus, they would easily give up when faced with difficulties and showed great reluctance in persevering with new ideas and techniques. Students with poor track record in mathematics achievements in the past were over anxious when exposed to new problems and concepts (Tall & Razali, 1993). They showed little cooperation when teaching approaches that required their participation were instituted.

Teaching and learning methods that promoted understanding were effectively undermined by the students' beliefs that mathematics was best learned by rote and exercises. On the other hand, students who felt that mathematics was useful and would bring much benefit were more likely to persevere, put in more effort and were more motivated in their studies (Schoenfeld, 1989; Anthony, 2000). Such positive attitudes would also help to improve their ability to learn (Tall & Razali, 1993, Mohd. Yusof & Tall, 1996). Successful students believed that hard work was the key to success whereas students with poorer achievements believed that it was their inability that contributed to their failure. Schoenfeld (1989) had also identified a strong correlation between students' positive self-concept and success related behaviours, such as persistence and efforts, to their achievements. There was a strong indication that motivation was also correlated to success. The better student was less likely to believe that mathematics was mostly memorizing and problems were always solved in step-by-step procedures. They were also more likely to find mathematics interesting and perceived themselves as working harder in mathematics than most other students. The weaker students believed that success was attributed to luck or inherent ability and that their failure was mainly due to their fault. In addition, conclusions from research (Tall & Razali, 1993; Mohd. Yusof, 1995, Anthony, 2000) indicated that positive attitudes and good study skills could improve students' learning. There were suggestions that changes to the learning environment so as to promote good problem solving behaviour, to increase students' participation and that which provide more challenging ideas could stimulate active learning.

Engineering undergraduates were expected to be able to make connections between relevant mathematical concepts and choose suitable mathematical techniques when solving problems in their engineering fields. It is clear that to be able to do these, they will need a strong understanding of basic concepts and the ability to adapt, modify and extend their mathematical knowledge appropriate to the problem situation.

Schoenfeld (1985 & 1992) presented the view that problem solving can support better understanding of mathematics and required four categories of knowledge and skills. These were: *resources* which were the mathematical knowledge and skills that students had as they tackled the tasks; *heuristic* strategies and techniques that they used in solving problems; *control* and *monitoring* which were decisions that students made about when and what resources and strategies to use during the problem solving process and finally their *beliefs* about mathematics which determined how they approached a problem.

Gray et al (2001) discussed the differences on the construction of mathematical knowledge in elementary and advanced mathematics; the different emphases as well as the different manner in which knowledge was presented. Thus, it has been suggested that the confusion and inconsistency in mathematical learning seen amongst first year students can be attributed to the above reasons and that these could become significant barriers to advanced mathematical thinking. The first encounter in the move from elementary to more advanced mathematical thinking through the different mathematical topics has been described as the transition phase. For example, the move into calculus involves the idea of approximation, which is different from the concept of equality previously learnt. In advanced algebra, concepts such as vector product violates the commutative law of multiplication and the idea of four or more dimensions, break the visual link between equation and geometry. This transition would require students to engage in a cognitive reconstruction. This can be seen from their initial struggle to understand the concepts being taught. Indeed, the move from elementary to advance mathematical thinking as described by Tall (1995), "*involves a significant transition, in particular, from describing to defining, from convincing to proving in a*

logical manner based on definitions. It is the transition from the coherence of elementary mathematics to the consequence of advanced mathematics, based on abstract entities which the students must construct through deductions from formal definitions”.

Was it possible to identify strategies to support students to to adopt more successful ways of learning and thinking about mathematics? Mason, Burton & Stacey (1982) described mathematical thinking as a way to improve understanding and extending control over the study of mathematics. In particular, three aspects, namely, the operations, processes and dynamics of mathematical thinking was discussed. Certain operations were identified as mathematical such as enumeration, iteration, ordering, making correspondence, forming equivalence classes, combining or substituting one from another to transform into a new state. These operations were independent of content area but very necessary for understanding and using mathematical ideas. Four processes were identified as central to mathematical thinking, specialising, conjecturing, generalising and convincing. Specialising is the exploration of meaning by looking at particular cases to make clear some common properties. Conjecturing should naturally follow as a student search for relationships that connects the examples and tries to express and substantiate any underlying patterns. Generalisation was the ability to recognize those patterns or regularity and making an attempt in expressing it mathematically. Convincing oneself and then another about the conjecture of the generalization that has been made encourages students to examine their ideas and explicitly communicate it first to themselves and then to others. In a book about developing strategies and skills to enhance problem-solving powers, Mason (1988 & 1999) showed how learning mathematics involved adopting a particular perspective of maximal involvement which he summarised as “learning is by doing”.

“Learning mathematics is a process of using the work of others to guide and inspire your own reconstruction of these ideas for yourself.”

(Mason, 1999)

In proposing strategies to provoke learners to become aware of mathematical thinking processes, Watson & Mason (1998) described a framework to generate and organized generic questions which can be asked about mathematical topics in various contexts. These questions reflected the internal structures of mathematics and mathematical thinking and thus served the objectives of increasing learners’ awareness of their own powers of thinking. Another useful framework was a structure that allowed a teacher to develop a topic systematically, by looking at three different aspects of the students, namely, behaviour, emotions and awareness. These frameworks were used in developing the implemented teaching strategies, in turning ideas into classroom tasks and activities.

4. A Model of Teaching and Learning

The basic model was first introduced by Cocking and Chipman (1988) who attempted to identify linguistic and cultural variable to explain the poorer performance of language minority students in mathematics when compared to students who spoke English as a primary language. They proposed a model that categorises the factors that influences mathematical learning at school expanded along the lines of Input to the children and Output or Mastery which is child performance. The input consists of (1) Entry Mastery – the cognitive ability patterns in terms of mathematical concepts, language skills, reading and learning ability, (2) Educational Opportunity – the teaching: time on task, quality of instruction, appropriate language and parental or other assistance and (3) Motivation to Engage – cultural/parental values, expectation of awards, and motivational nature of instructional interaction. The output consists of Mastery evaluation looking at students’ achievement and performance. However, the model was also used by Norman and Prichard (1994) in their study of cognitive obstacles in the learning of Calculus. In their project, they were primarily concerned with the Entry Mastery category as they said that this is where many cognitive obstacles originate. The Input part of the model is given below (Figure 1):

We had also used the Input model as a base to describe the learning situation in our class. In particular, the nature of the changes in the teaching was to provide educational opportunities for students to use their mathematical thinking powers and communicate their mathematical knowledge, and anticipated that these activities would support changes in students’ attitudes towards learning Engineering Mathematics. The research results showed that these factors were interconnected whereby students’ motivations were affected by the lecturer’s actions and sometimes vice-versa.

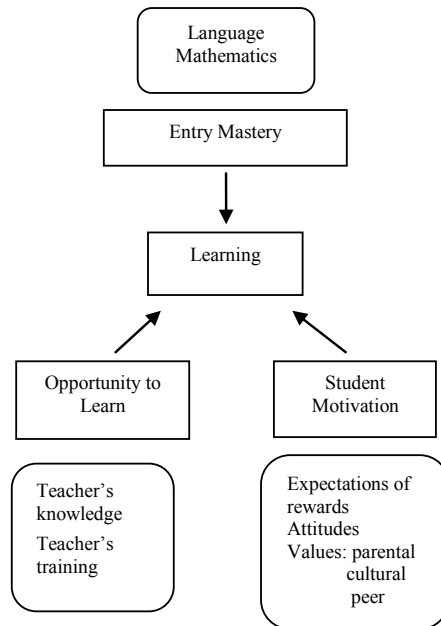


Fig. 1. Input to Learning

5. Factors Affecting Students' Learning Behaviour

The study had provided data (Roselainy, 2009) about students' responses and attitudes towards changes in teaching practice and in this section, we will discuss data that had contributed to a more detailed description of the various factors in the model by Norman & Prichard (1994; Figure 1).

5.1. Prior Knowledge

The majority of the students coming into UTM came through local schools or other educational establishments. Thus, they all had to follow the standard curriculum at their institutions and this included the mathematics curriculum at SPM or post-SPM levels. The level of mastery might be different for each student but in UTM, there were bridging subjects that were designed to help students achieve a more homogenous mathematical background. Some pre-July 2000 students were still facing difficulties after going through our own courses. The bridging mathematics course was more successful for post-July students, recruited under new policies. Thus, generally, our first year mathematics course had tried to create a situation so that students had a strong common background knowledge to prepare them for the Engineering Mathematics courses.

5.2. Motivation

The students identified several factors as important in their learning. Firstly, they wanted to succeed, were willing to work hard and were prepared to participate in the mathematics teaching and assessment processes. There were a small number of students with less positive attitudes but despite that, they still wanted to pass the course. The drive to get a degree was strong. Most other students were very confident in their abilities and their main concern was to get a good 'score' for the subject. Thus, 'rewards' in terms of marks was very important for the students and their participation in class depended upon the prospect of getting marks. Another consequence is the mind-set of working to '*what the lecturer wants*'. These outlook are highlighted in the following excerpt from notes of a class meeting (2nd meeting, semester 1 2002/03, Calculus II):

"Then, I asked the students to discuss the examples using the prompts and questions: "*what is the same?*" and "*what is different?*" and "*what can change?*"

There was a mixed response from the students. Students who knew me from Basic Calculus appeared to know what I meant and started discussing with students sitting next to them. Some of the new students were not sure of how to respond – they asked “*what do you want?*” or “*what do you mean?*”

When they were invited to interpret the questions as they wished – they were not happy and a couple of students immediately inquired if they will be evaluated on this work. One of them said: “*have to work smart*” – which meant (upon enquiry) – “*do just enough of what is required to get the marks*”.

Secondly, they believed that their lecturer could influence the way they studied or learned the subject. The way the lectures were presented, how the lecturer connected or interacted with them was considered important and they claimed that these could increase or decrease their enthusiasm for classroom work. Rapport with their lecturer was declared as an important motivating element. The following excerpts were views given by two students in an interview at the end of the course in semester 1, 2004/2005.

Tikah: “*the lecturer really helps, if her teaching is effective, we want to do more and more (exercises) ... she doesn't need to use...you know sophisticated methods, if she's good in explaining ...that's enough ...*”

Fizah: “*lecturer can really influence a student, I don't like a lecturer who just reads her notes. I like it when she makes us do the work together in the lectures...*”

Thirdly, they had to trust their lecturer as someone who was interested in their success and well-being as students. These notions were strongly held even if not supported by their own behaviour. For instance, in interviews, many students said that they would not seek help from the lecturer if they had any difficulty with the mathematics, or that the lecturer was a last resort resource person. Thus, their beliefs only came from their contact with the lecturer in the scheduled class meetings. They expected these to be conducted in such a way that they need not seek out the lecturer at all after class, a pervasive cultural norm. Clearly, motivation was wholly the students' domain of influence and choice. They can choose to participate and act upon the learning opportunities provided as well as respond to any overtures made by the lecturer for personal connection or support. Or, they can choose to ignore or limit their participation.

5.3. Opportunities to Learn

Providing, designing and planning the learning opportunities were in our domain of influence and opportunities for intervention. In providing the learning experiences, we have chosen to develop teaching strategies intended to increase students' awareness of their own mathematical thinking powers and to support them in using these powers in classroom work. To support the implementation of the teaching strategies, an active learning environment was chosen. The teaching was conducted to opportunities to (i) engage with mathematical thinking processes and structures, (ii) communicate their knowledge, and (iii) develop group work skills.

5.3.1. Working on Mathematical Thinking

The descriptions of the strategies that were used and the design of tasks have been given elsewhere (Roselainy, 2009). The topics were structured so that students could focus on the mathematics, in particular, to be able to identify the structures in mathematics (definitions, facts, theorems, properties, techniques, examples, etc) and made explicit the mathematical powers used (specialising, generalising, conjecturing, characterising, etc). The examples given were also structured in a manner that would lead towards a generality (Roselainy et al, 2002a; 2002b). Students will work on typical examples and generic examples leading towards more general examples. To further strengthen their understanding and knowledge, non-typical examples were also given and they were asked to make up their own examples. Students' own examples were used to assess what they were attending to in the topics taught.

All throughout, the tasks encouraged students' use of their own thinking powers as well as provided opportunities for students to explore linkages and connections between mathematical ideas. The tasks allowed students to experience the mathematical thinking activities of specialising and generalising, imagining and anticipating some mathematical relationship, and then expressing that in words, pictures, and eventually, symbols. Students were given tasks such that they experienced the powers of ordering, organising or characterising mathematical objects according to concepts which are used in the topic. Some opportunities for students to conjecture and convince, first themselves, and then others were included as an important base for advanced mathematical thinking.

5.3.2. Communication and Group work

We provided and created a conducive environment for active learning – student-centred teaching with an emphasis on students' participation and engagement with the mathematics, communication and skills in group work. These was based on the important elements to support effective active learning which were talking, listening, reading, writing and reflecting (Meyers & Jones, 1993).

Students were given many opportunities to communicate, in verbal and written form. To support verbal communication, students were required to express their mathematical knowledge with their group members, the class or to me. Talking about the mathematics was made easy as there were various 'prompts and questions' that were used and students were expected to state their answers to these. They were also asked to write down their answers, ideas or comments. There is a specific section in the workbook where students were required to write down their reflections of their learning of a topic taught. Solving problems was carried out in groups thus there was also opportunities for developing better team work skills, more communication and develop a mutual support system.

Consequently, as they became involved in the tasks and activities, they became more aware of their own powers and how they could use their understanding of the processes to help in problem solving. The students have had years of successful learning using memorization, drill and practice methods. Thus, they do have very strong powers of identifying patterns, similarities and differences but many were not aware of their powers or the way they could be used more effectively. They also have a strong desire to succeed and in coping with the mathematics in the context of their own engineering subjects thus they also wanted methods of learning that could help them achieve this. There was some resistance to changing they way they learn and at first, they did the tasks and other activities because marks were given for their work. However many students realised that studying to understand was beneficial and much more effective in coping with solving non-routine problems.

There were some significant factors that determined students' participation and these factors were interconnected between their own motivation and how my interaction with them could help increase their motivation. Rapport and trust between the students and the lecturer were important in giving some room to negotiate or mediate with them to at least try to participate during the lessons as well as encourage them to share what their problems or difficulties with the mathematics were. Not all the students were like this, some were more responsive and decided to participate in the lessons by themselves and they were a small number that did not appreciate the teaching changes although they went through the lessons without too many complaints.

In terms of measuring change, students who had come to appreciate that they had powers to use were considered as those who had changed their attitudes. The measures that were used to evaluate change was mainly based on students' own reflections, their responses in questionnaires and the way they worked on mathematical tasks. We have not as yet undertaken any study to ascertain whether the changes were permanent or were carried forward to other subjects.

5.4. Obstacles to Students Changing

The main obstacle to change was obviously the nature of motivation itself. As mentioned above, motivation was in the students' domain of influence. They decided what they would like to do, felt or believed in. Much of the data showed that there was a mismatch of objectives between what the lecturer wanted or believed in and what they wanted or believed in. For instance, we believed that understanding mathematical processes and developing thinking skills were important. Students believed that 'drill and practise' was enough for them to get through the course. All they needed was to do a lot of problems so that they could answer the tests and examinations papers. We were thinking of the skills that they might need as engineers, professional traits such as critical thinking, problem solving skills and the ability to work in a team. They only wanted to get through the course, and if they had to work in groups then they would work in groups. Some wanted 'formulas' for solving problems and we quote Midah's request, '*give a question, show the method, the formula. This question, this is the technique. This will help me remember the technique*'. Others had personal problems that hindered their progress in class and needed some personal support to help them face these issues. Some students were quite good but believed that they were not good enough. They set their own standards and against that, they labelled themselves, '*average*' or '*slightly above average*'. Through conversations, they were afraid to say 'good' or 'excellent' just in case they could not sustain their performance in the future.

Throughout our interactions with the different kinds of students, we had used their trust and their feelings of rapport to negotiate changes in their manner of study or the way they work on the mathematics. We will always bring back their

attention to the mathematics being taught. We needed to get them through the subject. Good students who were already coping very well and did not want to participate in exploring their thinking powers, were coaxed to try out some of the tasks or perhaps try to verbalise their understanding so that they could increase their own awareness of their own skills. Students who always needed answers at the back of the book were persuaded that they could look at the working and discuss with their peers to try to make judgements whether they thought their answers were reasonably right. In the case of Lily, a student who had a bevy of personal problems as well as problems with the mathematics, we persuaded her to spend more time revising and reviewing the mathematics with me so that she could pass her course. We also had made some arrangements for her to get professional counselling. Thus, we used the students' own concerns and their motivation to 'negotiate and mediate' for more mathematics learning in the environment that was initiated, sustained and established.

6. Intensification of the Input Model

Data from the research has added more information about the details in each of the categories of the Input Model. The following will present some the suggestions to strengthen the model.

6.1. Entry Mastery

At tertiary level, prior knowledge was an important base but it is not necessarily a fixed commodity. Students are able and can increase their mastery through self learning or during the learning sessions at university. As each new topic has to depend on earlier knowledge, some revision will be required and sometimes carried out in class. However, if there exist a significant misconception then some counteractive means must be put in place to help students overcome this particular difficulty.

6.2. Student Motivation

Students' motivation is within a private domain and shaped by various factors. A lecturer, can only react to demonstrated behaviour or information shared by students. However, students can reshape and modify their views, values and expectations. Generally, students who decide to come to university are highly motivated but might have some difficulties with certain personalities amongst lecturers, non-academic staff or colleagues. They might have some difficulties with some subjects or a transient difficulty in coming to terms with a new environment and knowledge. Success in the mathematics class in the sense of knowing what they were doing was sometimes enough to rejuvenate their interest and increase their participation. Concern for their academic and personal wellbeing was also important in getting students' trust. Thus, support and facilitating learning were important aspects to persuade students to participate in the lessons.

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who always needed answers at the back of the book were persuaded that they could look at the working and discuss with their peers to try to make judgements whether they thought their answers were reasonably right. In the case of Lily, a student who had a bevy of personal problems as well as problems with the mathematics, we persuaded her to spend more time revising and reviewing the mathematics with me so that she could pass her course. We also had made some arrangements for her to get professional counselling. Thus, we used the students' own concerns and their motivation to 'negotiate and mediate' for more mathematics learning in the environment that was initiated, sustained and established.

6.4. Educational opportunities

We had endeavoured to create learning sessions where students could engage with the mathematics, whilst developing an awareness of their own sense-making of the concepts and techniques. The teaching also included activities that required students to relate and work with their peers and communicate their ideas especially in verbal form. We found that you could design any task but it can only contribute to meaningful learning if the students participate. Students' participation is within their domain of influence, based on their attitudes and values. Thus an environment of learning where there is mutual respect and some form of relationship became an important impetus for better students participation and willingness to share their views about their learning and other personal information that could be important for the lecturers to know. Thus, in creating educational opportunities to learn, the lecturers plays various roles; teacher, facilitator, mediator as well as lay counsellor. For the students, rapport and trust were important to encourage participation and communication.

The summary of our findings is visualised in a modified version of the Input Model as shown in Figure 2, which shows influences of lecturers' actions to the students' motivation.

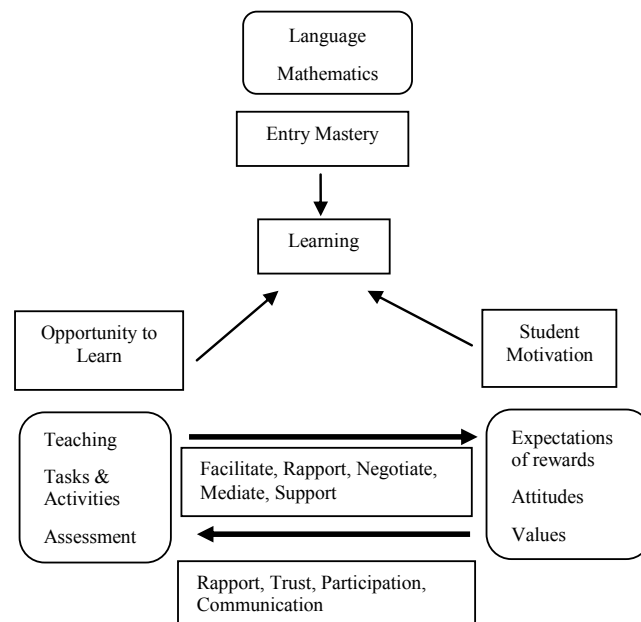


Fig. 2. Input Model of the teaching and learning of Engineering Mathematics.

7. Conclusion

Changes in the teaching practice was made to the presentation of mathematical content, class tasks and activities, as well as the teaching and learning environment by encouraging active participation of the students in inquiry, thinking, problem solving and communicating the mathematics learnt. Fortunately, current trends towards outcome-based education, Problem-Based Learning and giving students some exposure to real problems are providing some of the means to achieve these. However, it was not easy to convince students to participate and explore new ways of learning more suitable for advanced education and professional development. We have described some of the factors that contributed towards their participation

and motivation. As they studied Engineering Mathematics early in their course, we felt that we were providing some support in helping students to overcome the transition from pre-university learning towards tertiary learning.

References

- Anthony, G. (2000). Factors influencing first year students' success in mathematics. *Int Jnl of Math Edn in Sc & Tech*, Vol. 31 (1), 3-14.
- Artigue, M., Analysis. In Tall (ed) (1991), *Advanced Mathematical Thinking*, Kluwer Academic Publishers, Dordrecht.
- Bennet, A. A., (1977). Concerning the Function Concept. In Grinstein, L. S. and Michaels, B. (eds), *Calculus. Readings from the Mathematics Teacher*, NCTM, Virginia.
- Eisenberg, T. (1991). Functions and Associated Learning Difficulties. In Tall (ed), *Advanced Mathematical Thinking*, Kluwer Academic Publishers, Dordrecht.
- Cocking, R. R. & Chipman, S. (1988). Conceptual Issues Related to Mathematics Achievement of Language Minority Children. In Cocking, R. R & Mestre, J. P. (eds.), *Linguistic and cultural influences on learning mathematics*, p 17-46, Hillsdale, NJ: Lawrence Erlbaum Associates.
- Gray, E. M. & Tall, D. (2001). Duality, Ambiguity and Flexibility in Successful Mathematical Thinking, *Proceedings of PME 15*, Assisi, 2, 72-79.
- Maselan Ali. (1996). *Symbolic Manipulation Related To Certain Aspects Such As Interpretation Of Graphs*. Unpublished PhD thesis. University of Warwick, UK.
- Mason, J. H. (1999) *Learning and Doing Mathematics*. QED, UK.
- Mason, J. H. (2002). *Mathematics Teaching Practice: Guide for university and college lecturers*. Horwood Publishing in association with The Open University, UK.
- Mason, J., Burton, L. & Stacey, K. (1982). *Thinking Mathematically*. Addison-Wesley Publishing Company, Inc, Wokingham, England.
- Mason, J. H & Johnston-Wilder, S. (2004). *Designing and Using Mathematical Tasks*. Open University, Milton Keynes, UK.
- McNiff, J., Lomax, P. & Whitehead, J. (1996). *You and Your Action Research Project*, Routledge & Hyde Publications, London & New York.
- Md. Nor Bakar, (1991). *What Do Students Learn About Functions? A Cross Cultural Study in England and Malaysia*. Unpublished PhD thesis. University of Warwick, UK.
- Meyers, C. & Jones, T.B. (1993). *Promoting Active Learning: Strategies or the College Classroom*. San Francisco: Jossey-Bass Publishers.
- Mohd. Yusof, Y. & Tall, D. (1999). *Changing Attitudes to University Mathematics Through Problem Solving*. Educational Studies in Mathematics, 37, 67-82.
- Mohd. Yusof, Y. (1995). *Thinking Mathematically: A Framework for Developing Positive Attitudes amongst Undergraduates*. Unpublished PhD Thesis, Univ. of Warwick, UK.
- Norman, F. A., & Prichard, M. K. (1994). Cognitive Obstacles to the Learning of Calculus: A Kruketskiian Perspective. In Kaput, J. J. & Dubinsky, E. (eds.) *Research Issues in Undergraduate Mathematics Learning. Preliminary Analyses and Results*, MAA Notes No 33, MAA.
- Roselainy A. Rahman, Yudariah Mohd. Yusof & Mason, J. H. (2002(a)). Invoking Students' Mathematical Thinking in the Classroom: The Teaching of Differentiation. *Prosiding Persidangan Kebangsaan Pendidikan Matematik 2002*, Kuala Lumpur, 287-291.
- Roselainy Abd. Rahman, Mohd Rashidi, Md Nor, Yudariah, Ali Hassan, Maslan, Ong Chee Tiong & John H. Mason. (2002(b)). Students' Appreciation of Generality: What is general about a general technique? *Proceedings Tenth National Symposium of Mathematical Sciences*, Johor Bahru, 23-24 December, 563 – 567.
- Roselainy Abdul Rahman. (2009). *Changing My Own and My Students Attitudes Towards Calculus Through Workin on Mathematical Thinking*, Unpublished PhD Thesis, Open University, UK
- Schoenfeld, A. H. (1985). *Mathematical Problem Solving*, Academic Press, Inc., Orlando.
- Schoenfeld, A. H. (1989). Explorations of Students' Mathematical Beliefs and Behavior, *Jnl. For Research in Mathematics Education*, Vol. 20, No. 4, 38-355.
- Selden, J., Mason, A., & Selden, A. (1994). Even Good Calculus Students Can't Solve Non-routine Problems. In Kaput, J. and Dubinsky, E. (eds), *Research Issues in Undergraduate Mathematics Learning*, MAA, 3, 19-26.
- Skemp, R. (1987). *The Psychology of Learning Mathematics*. Lawrence Erlbaum, Mahwah, NJ.
- Tall, D. (1991). The Psychology of Mathematical Thinking. In Tall, D. (ed.), *Advanced Mathematical Thinking*, Kluwer Academic Publishers, Dordrecht.
- Tall, D. (1995). Cognitive Growth in Elementary and Advanced Mathematical Thinking, Plenary Lecture, Conf. of International Group of PME, Recife, Brazil, Vol. 1, 161-175.
- Tall, D. (1996). Functions and Calculus. In Bishop, A.J, Clements, K, Keitel, C, Kilpatrick, J & Laborde, C. (eds), *International Handbook of Mathematics Education*, Kluwer Academic Publishers, Dordrecht, 289-325.
- Tall, D. O. & Vinner, S. (1981) Concept Image and Concept Definition in Mathematics with Particular Reference to Limits and Continuity, *Educational studies in Mathematics*, 12 (2), 151-169.
- Tall, D. & Razali, M. R. (1993). Diagnosing Students' difficulties in Learning Mathematics, *Int Jnl of Math Edn in Sc & Tech*, Vol. 24, No. 2, 209-222.
- Watson, A. & Mason, J. (1998). *Questions and Prompts for Mathematical Thinking*. ATM, Derby.