TWO DIMENSIONAL ACTIVE CONTOUR MODEL ON MULTIGRIDS FOR EDGE DETECTION OF IMAGES

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To my family, thanks for the never ending love.

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ABSTRACT

Low-level tasks have been widely regarded as autonomous bottom-up processes in computational vision research. Examples of low-level tasks are edge detection, stereo matching, and motion tracking. In medical imaging, active contours have also been widely applied for various applications. In fact, active contours is one of the most popular PDE-based tools and powerful tool in performing object tracking. Active contour model, also called classical explicit snake was first introduced by Kass, Witkin and Terzopoulos. The main weaknesses of this method relate to not only the intrinsic characteristics of the contour, but also the parameterization, in which it is unable to handle topological changes. To solve these problems, a different model for active contours based on geometric partial different equation is proposed which is independent of parameterization, intrinsic and very stable. The important development has been the introduction of geodesic active contours. Level-set method was introduced for the moving fronts capture, where the active contour method is given implicitly as the zero level-set of a scalar function defined on implementing the entire image domain. This allows for a much more natural changes in the topology of the curve than parametric snakes. However, the main weakness of level set methods is that the complexity of the computational cost is high. A fast algorithm using semi-implicit addictive operator splitting (AOS) technique is used to restrict the computational cost. Edge detection based on semi-implicit is implemented for the edge detection on medical images such as medical resonance image (MRI). Multigrid is a numerical method that has a good accuracy and stability even with big time step. Exploiting these properties, multigrid was adopted for implementation of the geodesic active contour model. MATLAB has been chosen as the development platform for the implementations and the experiments since it is well suited for the kind of computations that are required. Besides it is widely used by the image processing community. Experimental results demonstrate the multigrid is the most appropriate method that can applied with AOS implementation for medical imaging to detect the location of the tumor which can decrease number of iterations.

ABSTRAK

Tugasan aras rendah digunakan secara meluas sebagai proses berautonomi bawah ke-atas dalam kajian penglihatan perkomputeran. Beberapa contoh tugasan aras rendah ialah seperti pengesanan pinggir tepi, pemadanan stereo, dan penjejakan gerakan. Kontur aktif juga telah banyak digunakan dalam pelbagai aplikasi seperti imej perubatan. Salah satu alat berasaskan PDE adalah yang paling popular dalam perlaksanaan penjejakan objek. Kontur aktif model juga dinamakan kaedah klasik tidak tersirat yang pertama kali diperkenalkan oleh Kass, Witkin dan Terzopoulos. Kelemahan kaedah ini tidak hanya bergantung pada sifat hakiki kontur tetapi juga pada parameter. Ia tidak boleh dikendalikan dalam perubahan topologi. Dalam mengatasi masalah ini, model yang berbeza telah diperkenalkan untuk kaedah kontur aktif berdasarkan persamaan geometri berbeza separa. Kaedah ini mempunyai parameter bebas, intrinsik dan stabil. Kontur aktif geodesik telah menjadi suatu pembangunan yang penting. Kaedah set paras merupakan kaedah tangkapan gerakan terkehadapan, di mana kontur aktif secara tersirat diperkenalkan sebagai set paras sifar fungsi skalar tersembunyi dan dianggap sebagai domain keseluruhan imej. Perubahan dalam topologi yang lebih melengkung secara semulajadi dibenarkan berbanding kaedah kontur aktif tradisional. Namun, kelemahan utama kaedah set paras ialah kos pengiraan sudah cukup tinggi. Algoritma pantas ialah satu teknik pemisahan agihan separa-tersirat (AOS) digunakan untuk mengurangkan kos pengiraan. Pengesanan sempadan tepi berdasarkan separa tersirat mampu mengesan sempadan tepi pada imej perubatan seperti pengimejan resonans magnetik (MRI). Multigrid adalah suatu algoritma dalam kaedah berangka yang mempunyai ketepatan dan kestabilan yang lebih baik walapun dalam langkah masa yang besar. Berdasarkan sifat ini, kaedah multigrid dilaksanakan dalam melaksanakan kontur aktif geodesik. Perisian MATLAB dipilih sebagai platform pembangunan kerana ia sesuai untuk semua pengiraan yang diperlukan. Malah ia digunakan secara meluas dalam komuniti imej pemprosesan. Daripada hasil kajian menunjukkan kaedah berangka paling baik untuk dilaksanakan bersama AOS untuk mengesan lokasi tumor dalam bidang imej perubatan dimana ia mengurangkan bilangan lelaran.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF APPENDICES

LIST OF SYMBOLS

f	-	Reduced stream function
f_0	-	Reduced stream function for $n = \frac{1}{2}$
x	-	Coordinate along the plate
y	-	Coordinate normal to the plate
e^0	-	Initial error vector
G	-	Gaussian
ω	-	Frequency domain
D^2	-	Second directional derivative operator
s	-	Normalized index of the control points.
E^*_{snake}	-	Energy functional of snake
E_{int}	-	Internal spline energy
E_{image}	-	Image forces
E_{con}	-	External constraint forces
u_0	-	Initial of level set
u	-	Level set
R_r	-	Relative residual convergence criterion
R_i	-	Relative error-norm criterion
ε	-	Convergence criterion
$\bigtriangledown^+$	-	Forward approximations of the spatial derivatives
$\bigtriangledown^-$	-	Backward approximations of the spatial derivatives
$\Sigma\Delta$	-	Average of the accuracy
k	-	Constant forces
τ	-	Speed of convergence
$\Sigma\Delta$	-	Average of accuracy

LIST OF SYMBOLS

CHAPTER 1

INTRODUCTION

1.1 Introduction

Over the last few decades, it has been predicted that the research of computer vision is not likely to be harvested practically in the foreseeable future. Preliminary works have been completed as a part of the Artificial Intelligence that has been released in the 1970s. Researchers sought to use computer to reproduce human ability to see objects, recognize them and make sense of their movements.

It is proven to be very much difficult than anyone had anticipated. New technology in Computer Vision applications are evolving into many practical applications, especially in the field of robotics, medical imaging and video technology. This approach on active contour is a prime candidate for practical exploitation (Blake & Isard, 1997).

In computational vision research, the process of autonomous bottom-up has been widely used for low-level tasks such as line or edge detection, motion tracking and stereo matching. Image processing has one problem where the edge detection has difficulty in finding lines separating homogeneous regions.

Active contour model(ACM) is one of the most popular PDE(Partial Differential Equation) based tools in computer vision and powerful in object tracking.

1.2 Theory of Edge Detection

The edge detection theory has two parts (Marr & Hildreth, 1980). Firstly, intensity changes were detected separately at different scales which occurred over a wide range of scales in natural images. Secondly, intensity changes which are spatially localized in images arise from surface discontinuities or from reflectance or illumination of boundaries.

Changes that occur over a wide range of scales is a major difficulty with natural images. A method of dealing separately with the changes occurring at different scales is greatly needed. A basic idea is to capture local images at various resolutions and then detects the changes in intensity that occur in each one (Marr & Hildreth, 1980). Therefore two issues have to be determined, (a) the nature of the optimal smoothing filter, and (b) how to detect intensity changes at a given scale.

There are two combined physical considerations in order to determine the suitable smoothing filter. The first is to filter an image, which is to reduce the range of scales where intensity changes occur. Therefore, the spectrum of filter should be smooth and roughly band- limited in the frequency domain. The second consideration is the constraint of spatial localization or constraint in the spatial domain.

There are three properties that could give rise to intensity changes in an image. The first property is illumination changes such as shadows, visible light sources and illumination gradients. The second is changes in the orientation or distance from the visible surfaces and the third is changes in surface reflectance.

Only one distribution could optimize the localization requirements (Leipnik, 1960) that is a Gaussian. A Gaussian provides an optimal trade-off between the conflicting requirements, which is the spatial and the frequency domain.

$$G(x) = [\frac{1}{\sigma(2\pi)^{\frac{1}{2}}}]exp(\frac{-x^2}{2\sigma^2}), \tag{1.1}$$

with Fourier Transform where σ is the standard deviation derivation q the distribution,

$$G(\omega) = exp(-\frac{1}{2}\sigma^2\omega^2). \tag{1.2}$$

. In two dimensions,

$$G(r) = (\frac{1}{2}\pi\sigma^2)exp(\frac{-r^2}{2\sigma^2}). \tag{1.3}$$

.

When intensity occurs, intensity change needs to be defined to reduce the task of detecting these changes to that of finding the zero-crossings of the second derivative D^2. The representation at this point consists of zero-crossing segments and their slopes. The intensity variation near and parallel to the line of zero-crossings should locally be linear so zero-crossing has a maximum slope. This condition will be approximately true in smoothed images (Marr & Hildreth, 1980).

There are three main steps in the detection of zero-crossings. They are: (1) a convolution with D^2G, where D^2 stands for a second directional derivative operator; (2) the localization of zero-crossings; and (3) checking of the alignment and orientation of a local segment of zero-crossings.

1.3 Snake Active Contour Models(ACM)

There are various tasks in image processing such as image segmentation, object tracking or object edge detection using snakes active contour model. An active contour was proposed which represents a new approach to visual analysis of shape (Kass et al., 1988).

Active contours implement image processing selectively to regions of the image, instead of processing the entire image. The basic snake model is controlled continuity spline. It is operated under external constraint forces and influence of image forces.

An initial contour in ACM solves the image plane towards the problem of object boundaries. Suitable energy terms are added so the contour corresponds to a local minimum of the functional coincides.

The image forces push the snake towards the image features such as lines, edges, and subjective contours. The external constraint forces place the snake near

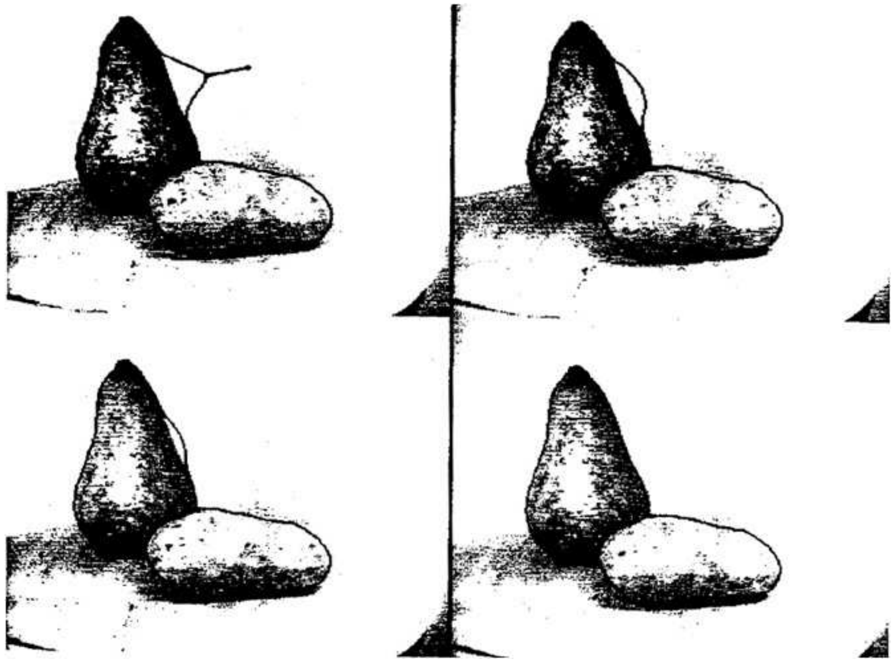

Figure 1.1: Edge detection using snakes active contour model (Kass et al., 1988)

the desired local minimum of the functional which coincides at the edge of the object as desired. These forces come from automatic attentional mechanisms such as a user interface or interpretation of high-level.

The contour is defined in the (x, y) plane parametrically as $v(s) = (x(s), y(s))$, where $x(s)$ and $y(s)$ are x, y coordinates of the contour and s is the normalized index of the control points. The function of energy consists of two components, internal energy and external energy. The energy functional is:

$$\begin{aligned} E^*_{snake} &= \int_0^1 E_{snake}(v(s))\, ds \\ &= \int_0^1 E_{int}(v(s)) + E_{ext}(v(s)) \\ &= \int_0^1 E_{int}(v(s)) + E_{image}(v(s)) + E_{con}(v(s)). \end{aligned} \tag{1.4}$$

.

The snake energy consists of three terms. The first term, E_{int} represents the internal spline energy due to bending. The second term, E_{image} gives rise to the image forces, and the last term, E_{con} gives rise to the external constraint forces.

The internal energy is defined as:

$$E_{int} = (\alpha(s)|v_s(s)|^2 + \beta(s)|v_{ss}(s)|^2)/2 \tag{1.5}$$

The spline energy is composed in first-order and second-order term where the first-order term is controlled by the coefficients $\alpha(s)$ and the second-order term is controlled by $\beta(s)$. Both terms are combined to make the snake act as a membrane and a thin plate.

The weight $\alpha(s)$and $\beta(s)$ should be adjusted to control the relative importance in terms of membrane and thin-plate. $\alpha(s)$ should be set to zero at a point to allow a snake to be a second-order discontinuous and develop a corner.

Each iteration takes implicit Euler steps to calculate the internal energy and explicit Euler steps to determine the image and external constraint energy. Three different energy functionals which attract the snake to the desired features in the image are present. The total image energy is as follows:

$$E_{image} = w_{line}E_{line} + w_{edge}E_{edge} + w_{term}E_{term}. \tag{1.6}$$

If set

$$E_{line} = I(x,y) \tag{1.7}$$

Then, the snake will be attracted if there is either a light or dark lines depending on the sign of w_{line}. Based on a very simple energy functional, it can search the edges in an image. If $E_{edge} = -|\bigtriangledown I(x,y)|^2$ is set then the snake is attracted to contours with large image gradients.

(Kass et al., 1988) experimented with a slightly different edge functional to demonstrate the continuity of the relationship of scale-space to the edge detection theory. The edge energy functional is:

$$E_{edge} = -(G_\sigma * \bigtriangledown^2 I)^2, \tag{1.8}$$

where G_σ is a standard deviation σ of Gaussian. If the term of energy is added to a snake, it means that the snake is attracted to zero-crossings. However it is still constrained by the smoothness of its own (Kass et al., 1987). The curvature of the level lines is used to seek line segments and corners termination and can be defined by:

$$E_{term} = \frac{\partial \theta}{\partial \mathbf{n}_\perp} \tag{1.9}$$

where $\mathbf{n} = (cos\ \theta, sin\ \theta)$, $\mathbf{n}_\perp = (-sin\ \theta, cos\ \theta)$ and $\theta = tan^{-1}(\frac{C_y}{C_x})$ be the gradient angle. Let $C(x, y = G_\sigma(x, y) * I(x, y)$ be a slightly smoothed version of the image.

Snake is attracted to edges or terminations created by combining E_{edge} and E_{term}.

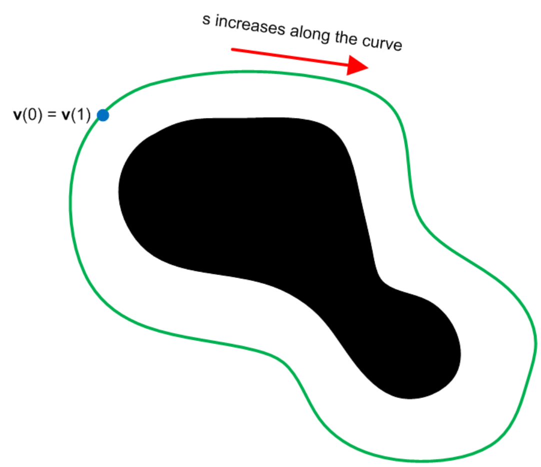

Figure 1.2: Parametric snake curve v(s)

A formulations of the image energy has also been proposed to improve the original model, including the Balloon force field, together with the potential forces comprising the external forces. The external forces on the original model are modified to give more stable results to push the the curve to the edges (Cohen,1991).

1.4 Fast ACM Algorithm

The traditional snake ACM has two significant weaknesses (Caselles & Coll,1996). Firstly, it depends on the intrinsic characteristics of the contour and parameterization because the model is non-geometrical. Secondly, it cannot naturally handle topology of the evolving contour changes because of situations where no prior knowledge of the number of objects to be detected is available.

Geometric ACM has addressed some weaknesses of the standard active contours. Geometric ACM was introduced based on the mean curvature motion equation (Caselles et al.,1993). The implicit geometric ACM, that can be associated with the explicit snake model is represented by function, is embedded into an energy functional.

(Weickert & Kuhne,2002) considered, which combined geometric and geodesic model and introduced two additional functions, a and b:

$$\frac{\partial u}{\partial t} = a(x)|\bigtriangledown u|div(\frac{b(x)\bigtriangledown u}{|\bigtriangledown u|} + |\bigtriangledown u|kg(x), \tag{1.10}$$

- $|\bigtriangledown u|div(\frac{\bigtriangledown u}{|\bigtriangledown u|})$, is the mean curvature term of ACM which smoothes level sets,
- $k|\bigtriangledown u|$ describes motion in normal direction, i.e. dilation or erosion depending on the sign of k. Also called as balloon force for pushing a level set into concave regions or to create convex regions.
- g is a stopping function such as the PeronaMalik diffusivity.

$a := g$, $b := 1$ are set for the results in the geometric model, or $a := 1$, $b := g$ for the results in the geodesic model. Discretizations of space and time have to be considered to provide a numerical algorithm.

1.4.1 Geometric ACM

Based on the curve evolution theory and geometric flows, the level sets are used in geometric ACM implementation to allow automatic changes in the topology(Caselles et al.,1993).

The geometric ACM equation is expressed by (Weickert & Kuhne,2002):

$$\frac{\partial u}{\partial t} = g(x)|\bigtriangledown u|div(\frac{\bigtriangledown u}{|\bigtriangledown u|} + k),\ \Omega \times (0, \infty), \tag{1.11}$$

$$u(x,0) = u_0(x) \ on \ \Omega, \tag{1.12}$$

where

$$g(x) = \frac{1}{1+(\bigtriangledown G_\sigma * g_o)^2} \tag{1.13}$$

$G_\sigma * g_o$ is the convolution of the image where the contour of an object O is searched with the Gaussian $G_\sigma(x)$ and u_0 is the initial data.

Unlike the method of snakes that depends on many adjustment parameters, geometric ACM is stable and allows a rigorous mathematical analysis. The model allows simultaneous extraction of smooth shapes and detects several contours. As a result of the stability, it can be engineered as a method of zero parameter in the applications.

A novel algorithm using multigrid for the rapid evolution of geometric ACM implementation was proposed in order to retain accuracy and demonstrate excellent stability and rotational invariance properties even with big time steps. Internal forces are treated with implicit schemes while external forces with explicit schemes to keep the curve smooth (Papandreou & Maragos(2004).

1.4.2 Geodesic ACM

Based on energy minimization, geodesic ACM permits connections of snakes for object segmentation. The energy of the snakes model is equivalent to finding geodesic curves in a Riemannian space, which is defined by the content of image (Caselles et al.,1997).

In the implicit geodesic ACM (Weickert & Kuhne,2002)

$$\frac{\partial u}{\partial t} = |\bigtriangledown u|(div(g(x)\frac{\bigtriangledown u}{|\bigtriangledown u|}) + kg(x), \ \Omega \times (0,\infty), \tag{1.14}$$

$$u(x,0) = u_0(x) \ on \ \Omega, \tag{1.15}$$

where k is a positive real constant.

This scheme is derived from the classical energy-based active contours and geometry curve evolution. Formulation of geodesic is to detect the edge to find a minimum weighted length of curves, improving the edges detection with large differences in their gradient. Then, this geodesic ACM is assumed to represent the zero level- set of a 3D function, and the computation of geodesic curve is reduced to a geometric flow.

To improve those models, this geodesic flow includes a new velocity curves component. The new velocity component allow accurate tracking of boundaries of the high variation in their gradient, including small gaps, a task that is difficult to achieve with the previous curve evolution models. In (Caselles et al.,1997), the level sets approach is used in order to find the geodesic curve (Osher & Sethian,1988).

The purpose of $g(x)$ is to stop the evolving curve when it arrives at the object boundaries. (Caselles et al.,1993) & (Malladi,R., Sethian & Vemuri,1995) chose

$$g(x) = \frac{1}{1 + (\bigtriangledown G_\sigma * g_o)^2}, \tag{1.16}$$

where x is a smoothed version of x and x was computed using Gaussian filtering. $G_\sigma * g_o$ is the convolution of the image g_o where we are looking for the contour of an object O with the Gaussian $G_\sigma(x) = C\sigma^{-1/2}exp(-|x|^2/4\sigma)$.

Geodesic ACM are widely used in many applications, including object segmentation and tracking in movie sequence (Goldenberg et al.,2001). The AOS scheme has been adapted to the geodesic ACM, motivated by level set approach and fast marching for re-initialization.

1.5 ACM for MRI Application

ACM has also been widely used to detect edge and segmentation of medical imagery of magnetic resonance imaging (MRI), computed tomography (CT), and ultrasound medical imagery application.

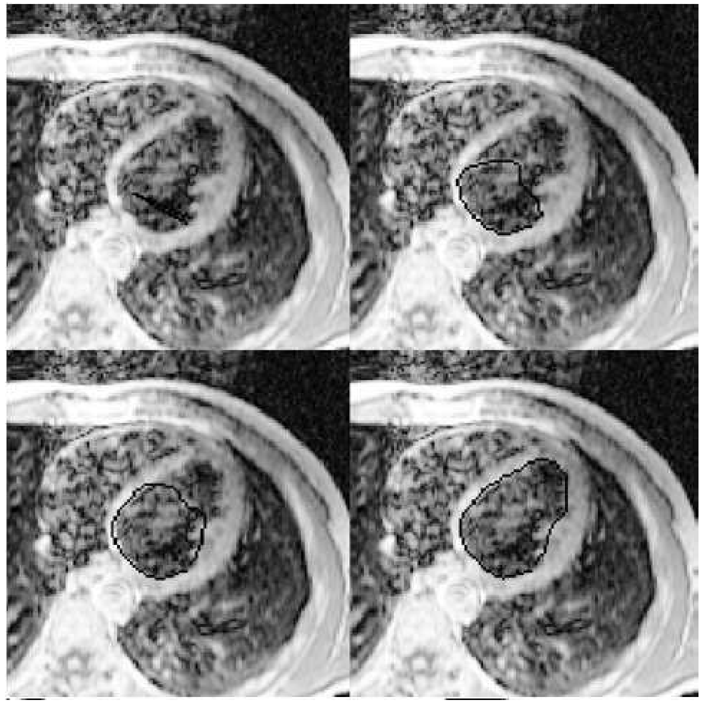

Figure 1.3: MRI application using ACM

A novel technique that combines MRI of hyper-polarized helium gas and conventional MRI facilitates high resolution imaging of lung function (Ray & Acton (2002). This application which computes the total volume of the lung cavity and the cavity volume from the proton imagery were used to calculate ventilation percentage.

Parametric snake is used in the segmentation method to measure the amount of lung air space and a classification approach to calculate the functional air space. It has a lower association with computational complexity compared to the geometric counterpart. The main weakness associated with parametric snakes is it is difficult to merge or split.

ACM is developed to find and map the outer cortex of the brain images (Davatzikos & Prince, 1995) and then to determine the spine of such ribbons. ACM has an external force derived from an integration of the data and internal elasticity forces.

(Yezzi et al., A.,1997) used a new geometric ACM, based on Riemannian metrics depending on the image and related to gradient flows for ACM. This is done by combining curve evolutionary approach to active contours and classical snake methods.

Their idea is based on Euclidean curve shortening evolution which determines the gradient direction where the Euclidean perimeter shrinks as fast as possible. The numerical methods from evolutionary level set techniques were developed by (Osher Sethian, 1982, 1984,1986, 1989), and (Malladi et al.,1995).

In (Derraz et al.,2007), coupling of geometrical ACM for image segmentation using homogenization of edge stopping map function based on the anisotropic diffusion PDE was carried out. This homogenization provides a regular velocity propagation, unique viscous solution and unique segmentation for low contrasted images. However, geometrical ACM is based on gradient information.

1.6 Direct Method and Iterative Method

Much research has focused on the development algorithms, both direct and iterative. Direct and iterative methods was used to solve a linear system of equations, in the form of $Au = f$ with A an n by n nonsingular matrix, that form a subspace. The convergence of Krylov subspace methods depends strongly on the eigenvalue distribution of A, and on the angles between eigenvectors of A.

Classical direct search methods was developed during the period 1960-1971. The direct method used here is a Gaussian elimination and Thomas method applied to sparse symmetric matrices. Gaussian elimination is an algorithm for solving systems of linear equations. It can also be used to find the rank of a matrix, to calculate the determinant of a matrix, and to calculate the inverse of an invertible square matrix.

Comparison between direct and iterative methods show that the computation rate and the number of iterations are both important factors influencing the CPU time required for each solver. The computation rate was influenced by the algorithm and the data structure used for each method. While the number of iterations was influenced by the structure of the matrices and convergence rate.

The iterative methods require much less memory than direct methods and show the most promising for very large finite element models, especially if the element aspect ratios are near unity. Rapid convergence of the iterative methods makes them faster than the direct solvers (Poole,1991).

Many researchers have considered to apply left pre-conditioning methods applied to linear system of equations in order to ensure that the associated Jacobi and

Gauss-Seidel methods converge faster than the original ones. Such modifications or improvements are based on pre-chosen pre-conditioning method which eliminates the elements of the first column of A below the diagonal (Milaszewicz,1987).

In 1975, a 3-block SOR method was proposed for solving least-squares problems based on a partitioning scheme for the observation matrix A. Preliminary works have been completed for correcting and extending the SOR convergence interval. Problem of the 3-block formulation, leading other alternative formulation to a 2-block SOR method and it is shown that the method always converges for sufficiently small SOR parameter ω (Markham et al.,1985).

The successive over-relaxation (SOR) method is a variant of the Gauss-Seidel method to solve a linear system of equations and have more faster convergence than the Gauss-Seidel method. The idea is to choose a value for ω that will accelerate the rate of convergence of the iterations to the solution(David & Frankel, 1950).

1.7 Multigrid Method (MG)

The boundary value problems in numerical solution are absolutely necessary in almost all development fields of physics and engineering sciences. Numerical solution is crucial to solve dimensional huge system where the system has become larger and possesses larger number of equations although the computers have become faster and vector computers are available.

Brandt, McCormick and Ruge introduced Multigrid in 1982. In 1983, it was further explored by (Stuben,1983), and then popularized by Ruge and Stuben in 1987. The basic idea of multi-level adaptive technique (MLAT) is not to work with a single grid, but with order grids of rising fineness. Brandt has shown the actual efficiency of the multigrid methods (Brandt,1997).

Multigrid has an extremely simple principle. Firstly, suitable relaxation methods are applied to obtain approximations with smooth errors. Secondly, because of the smoothness of the error, this corrections approximation can be computed on coarser grids.

The basic idea is to recursively take coarser and coarser grids. Finally, combination with nested idea of iteration for a suitable algorithmization in which the computational work required to achieve the accuracy of the discretisation is proportional to the number of discrete unknowns (Trottenberg et al., 1984).

The basic idea for convergence acceleration is to get error smoothing effect of relaxation methods. This idea can be found in the early literature by Southwell (1935,1946 & 1952). Scroder has then introduced the recursive application of coarser grids for an efficient solution of specific discrete elliptic boundary value problems. But, explicit error smoothing has not yet been performed. Finally, the self-suggesting idea of nested iterations has been known for a long time.

A theory of multigrid methods to find considerations of the problem in analysis of model type is presented. For smoothing, non-rectangular domains and non-linear problems treatment, red black and four colour relaxation methods are used (Hackbusch, 1985).

Since 1977, multigrid methods have grown. In recent years, the field of finite elements which has first been of a more theoretical interest to multigrid methods becomes attractive for practical investigations. Apart from linear and non-linear boundary value problems, eigenvalue problems and bifurcation problems, parabolic and other time-dependent and non-elliptic problems occur in numerical fluid dynamics.

Multigrid methods also can be solved by integral equations efficiently. Furthermore, multigrid methods are suitable to solve special systems of equations without continuous background. Extension of the field of applications of multigrid methods is the combination of the multigrid idea with other numerical and more general mathematical principles such as combination with extrapolation and defect method. Finally, multigrid methods are used on vector and parallel computers for optimal use as well as to approach within the computer architecture.

The main idea of multigrid is to use a global correction from time to time to accelerate the convergence of basic iterative methods, accomplished with solving a coarse problem. This principle is the same as the interpolation between coarser and finer grids. The typical application for multigrid is in the numerical solution of elliptic partial differential equations in two or more dimensions(Hackbusch, 1985).

In some cases iterative methods performed better than multigrid methods, based on the comparison with Borzi(1999) and Chan et al., (2009). Though, the number of iterations and root mean square error, $(rmse)$ of iterative methods is not as good as multigrid. The multigrid algorithm involves a new parameter (cycle index) which is the number of times the MG procedure is applied to the coarse level problem. According to Borzi(1999), the choice $N = 1$ in a multigrid cycle is suitable to solve the problem to second-order accuracy.

1.8 Performance Analysis

1.8.1 Convergence Criterion

Convergence criterion is usually sought in converged solution. The concept of the convergence rates has been developed for analyzing iterative methods for solving systems of simultaneous linear algebraic equations. Its rate of convergence sequence as well as the convergence, uniqueness of the solution and error in the approximate solution are obtained after a finite number of computations(Hamilton,1984).

Many convergence rates have been proposed and some are based on the residual-norm, while others are based on the error-norm. Consider a large sparse linear system of equations $Au = f$, where A is a $n \times n$ matrix.

(1)Relative residual convergence criterion. It is defined as

$$R_r = \frac{\| r_k \|_2}{\| r_0 \|_2} = \frac{\| b - Ax_k \|_2}{\| b - Ax_0 \|_2} \leq tol, \;\; k = 1, 2, ..., maxit \tag{1.17}$$

where *maxit* refers to maximum number of iteration.

Based on the relation $e_k = A^{-1} r_k$, it is know that the criterion has the following error bound,

$$\| e_k \|_2 \leq \| A^{-1} \|_2 . \| r_k \|_2 \leq tol. \| A^{-1} \|_2 . \| r_0 \|_2 \tag{1.18}$$

(2) Relative error-norm criterion. The relative error-norm criterion is defined based on the approximate solution as

$$R_i = \frac{\| x_k - x_{k-1} \|_\infty}{\| x_k \|_\infty} \leq, \ \ k = 1, 2, ..., maxit \tag{1.19}$$

in which $\| \ . \ \|_\infty$. The relative error-norm criterion is closely related to the relative residual convergence criterion, but this relation also depends on the properties of matrix A.

Iterative methods for solving this problem are

$$u_{(n+1)} = MU_{(0)} + Nf, \tag{1.20}$$

where M and N have to be constructed in such a way that given an arbitrary initial vector $u_{(0)}$, the sequence $u_{(v)}, v = 0, 1, ...$, converges to the solution $u = A_{-1}f$. M is called the iteration matrix. The following convergence criterion based on the spectral radius $\rho(M)$ of the matrix M. The following theorem is proved by Varga (1962).

Theorem 1.8.1. *If A is an* $(m \times m)$*, then A converges if and only if* $p(A) < 1$.

$$\|M_(k)\| < \varepsilon \tag{1.21}$$

Iteration process is determined by number of k iteration which satisfy the inequality below,

$$\frac{-log\varepsilon}{-\|A_(k)\|^{\frac{1}{k}}} \leq k \tag{1.22}$$

An optimum value of iteration, k_0 *at convergence criterion can be approximate by the following equation,*

$$k_0 \simeq \frac{-log\varepsilon}{\rho(A)} = \frac{log\varepsilon}{\tau(A)}, \tag{1.23}$$

where $\tau(A)$ *is an asymptotic mean of iteration rate which satisfies the theorem provided below,*

Theorem 1.8.2. *If $k \to \infty$, asymptotic mean of iteration rate is given by*

$$\tau(A) = \lim_{n\to\infty} \tau^k(A) = log\rho(A) \tag{1.24}$$

Practically, an iterative method will converge at k_0 iteration, if the convergence criterion $\varepsilon = 10^{-a}, \alpha \in Z^+$ is satisfied such that

$$e^{(k)} = max|u^{k+1} - u^k| < \varepsilon, \tag{1.25}$$

with $u^{(k)}$ and $u^{(k+1)}$ are previous iteration and latest iteration respectively. The best convergence rate for a comparison of iterative method is the one which gives the smallest maximum error $e^{(k)}$.

1.8.2 Root Mean Square Error (rmse)

The convergence rate for iterative methods is determined by computing root mean square error, $(RMSE)$ formula. The formula is given as the following,

$$RMSE = \sqrt{\sum_i^{\aleph}(u_j^{(k+1)} - u_j^{(k)})^2/\aleph}, \tag{1.26}$$

where u_i and u_i are approximation solution and exact solution respectively. For one dimensional problem $\aleph = m$, two dimensional $\aleph = m \times m$, and three dimensional $\aleph = m \times m \times m, i = x, i = (x,y)$, and $i = (x,y,z)$ respectively.

If the *rmse* is within the error, ε, then the procedure will stop. Otherwise, the procedure will continue until this condition is reached.

1.9 Background of the problem

Difficulty in image processing is the boundaries detection where the problem is to find lines separating the homogeneous regions. The original ACM has some significant weaknesses.

Firstly, it depends on its parameterization, in which the model is non-geometrically and intrinsic properties of the contour. Secondly, there is no prior knowledge of some of the detected object, so it cannot handle the topological changes in contour naturally.

In image analysis and computer vision, geometric ACM is a very popular PDE-based tool. Most of the geometric ACMs are built based on the level-set method. Computation cost can be high, which makes its utilization in time-critical applications problematic despite the advantages of the level-set method.

The Geodesic ACM's weakness is in its stability constraints on the time step size related to explicit numerical schemes.

1.10 Research Questions

The research will explore the following questions.

1. What is active contour and how does it work?
2. How to implement the active contour using direct, iterative and multigrid method?
3. Under which condition is each active contour method satisfactory?
4. How is the performance of the active contour methods for each numerical methods?
5. Which numerical methods is the best approaches to detect edge of object ?

1.11 Objectives of the study

The objectives of this research are:

1. To implement a Semi-implicit additional operator scheme (AOS) on geodesic ACM.
2. To apply a multigrid algorithm to the semi-implicit scheme for geodesic ACM.

3. To compare the performance of three algorithms (direct, iterative and multigrid method) on the geodesic ACM.
4. To analyze the numerical results from the sequential algorithms based on the computational complexity, accuracy and number of iterations.

1.12 Scope of the problem

The scope of this research revolves around detecting edges of the object using geodesic ACM based on semi-implicit additional operator scheme (AOS). This approach will be implemented with numerical methods such as direct, iterative and multigrid method to solve the tridiagonal linear system efficiently.

The methods under consideration are Gauss-elimination, Thomas, Jacobi, Gauss-Seidel and Successive-over-relaxation (SOR) and will be applied for medical image segmentation such as medical resonance image(MRI).

The experiment were run on Intel Core Duo Processor 2.0GHz and RAM 2GB. Matlab 7.6.0 (R2008a) were used as a tool to implement the algorithm. The analysis of the results are conducted in terms of numerical performance.

1.13 Originality of the research works

A number of original works have been carried out in this research. First, iterative methods were implemented for the edge detection on medical images unlike previous works where geodesic ACM based on AOS scheme use direct methods such as Thomas method and Gauss-elimination. Second, the multigrid implemented to enhance the iterative methods for improving the current results based on accuracy, number of iterations and root mean squared error.

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