THE HOMOLOGICAL FUNCTORS OF SOME FINITE TWO-GENERATOR TWO-GROUPS OF NILPOTENCY CLASS TWO

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To my beloved mother, father and family

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ABSTRACT

The homological functors are originated in homotopy theory. One of the homological functors, namely the nonabelian tensor square, was introduced by Brown and Loday in 1987. Meanwhile, the rest of the homological functors were introduced by Bacon and Kappe in 2003. Some of the homological functors are $J(G)$, $\nabla(G)$, exterior square, the Schur multiplier, $\Delta(G)$, the symmetric square and $\tilde{J}(G)$. The nonabelian tensor square and some homological functors of some finite two-generator two-groups of nilpotency class two are determined in this research. A software named Groups, Algorithms and Programming (GAP) is used to compute and verify the nonabelian tensor square and some homological functors of some finite two-generator two-groups of nilpotency class two.

ABSTRAK

Fungtor homologi adalah berasal daripada teori homotopi. Salah satu fungtor homologi adalah kuasa dua tensor tak abelan telah diperkenalkan oleh Brown dan Loday pada tahun 1987. Sementara itu, fungtor homologi yang lainnya telah diperkenalkan oleh Bacon dan Kappe pada tahun 2003. Di antara fungtor homologi adalah $J(G)$, $\nabla(G)$, kuasa dua peluaran, pendarab Schur, $\Delta(G)$, kuasa dua simetrik dan $\tilde{J}(G)$. Kuasa dua tensor tak abelan dan beberapa fungtor homologi untuk sebahagian kumpulan-dua dengan penjana-dua yang mempunyai nilpoten kelas dua terhingga telah ditentukan dalam penyelidikan ini. Perisian "*Groups, Algorithms and Programming"* (GAP) telah digunakan untuk mengira dan menentusahkan kuasa dua tensor tak abelan dan beberapa fungtor homologi untuk sebahagian kumpulan-dua dengan penjana-dua yang mempunyai nilpoten kelas dua terhingga.

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LIST OF SYMBOLS

1	Identity element
$1_{\otimes}$	Identity of tensor square
GAP	Groups, Algorithms and Programming
G	A group G
$\lvert G\rvert, \lvert x\rvert$	Order of the group G, the order of the element x
$\mathbb{Z}$	Set of integers, the finite cyclic group
$\mathbb{Z}_n$	Cyclic group of order n
$\langle x \rangle$	Group generated by x
$[g,h]$	The commutator of g and h
$H \leq G$	H is a subgroup of G
$G \cong H$	G is isomorphic to H
$G \times H$	Direct product of G and H
$G \otimes H$	Tensor product of G and H
$G \otimes G$	Tensor square of G
$G \tilde{\otimes} G$	Symmetric square of G
$G \wedge G$	Exterior square of G
$M(G)$	Schur multiplier of G
$Z(G)$	Center of the group G
$\ker(\kappa)$	Kernel of the homomorphism κ
${}^{g}h$	The conjugate of h by g
G'	The commutator subgroup of G
$\subseteq$	Subset
$\in$	Element of
$\wedge$	Wedge product
$\times$	Direct product

$>$	Greater than
$<$	Less than
$\geq$	Greater than or equal
$\leq$	Less than or equal
$\rtimes$	Semidirect product

CHAPTER 1

INTRODUCTION

1.1 Introduction

The homological functors of a group including $J(G)$, $\nabla(G)$, the exterior square, the Schur multiplier, $\Delta(G)$, the symmetric square and $\tilde{J}(G)$ are closely related to the nonabelian tensor square of the group. The nonabelian tensor square of a group G, denoted as $G \otimes G$, is generated by $g \otimes h$ for all $g, h \in G$ subject to the relations $gg' \otimes h = \left({}^{g}g' \otimes {}^{g}h\right)(g \otimes h)$ and $g \otimes hh' = (g \otimes h)\left({}^{h}g \otimes {}^{h}h'\right)$. Throughout the years, many researchers have been conducted to investigate the nonabelian tensor square and homological functors of various groups.

This research begins on viewing the history of the nonabelian tensor square and the homological functors for some groups that have been done by previous researchers. In this research, we will focus on some finite two-generator two-groups of nilpotency class two. The nonabelian tensor square and some homological functors of some finite two-generator two-groups of nilpotency class two have been determined in this research. Groups, Algorithms, and Programming (GAP) software, is used to compute and verify the results.

1.2 Research Background

The classification of finite two-generator two-groups of nilpotency class two and their nonabelian tensor square have been determined by Kappe et al. [1] in 1999. In [2], by adding a new parameter, the classification for Type 2.5.3 has been corrected. Therefore, the nonabelian tensor square for Type 2.5.3 given in [1] is not valid. The new nonabelian tensor square together with some homological functors of some finite two-generator two-groups of nilpotency class two of those types (later will be called Type 3) are computed in this research. In [2], some homological functors of finite two-generator two-groups of nilpotency class two of Type 2.4 (later will be called Type 1) are determined. Extending the research done in [1], some homological functors of finite two-generator two-groups of nilpotency class two of Type 1 for parameter $\alpha = \beta = \gamma$ have been determined in this research by following the definition of homological functors given in [3]. The results are then verified using Groups, Algorithms, and Programming (GAP) software. The GAP software is also used for the computation of some homological functors of some finite two-generator two-groups of nilpotency class two.

1.3 Problem Statement

What are the nonabelian tensor square for finite two-generator two-groups of nilpotency class two of Type 3 and what are the homological functors of some homological functors for finite two-generator two-groups of nilpotency class two of Type 1 and Type 3?

1.4 Research Objectives

The main objectives of this research are:

1. to study the classification of finite two-generator two-groups of nilpotency class two,
2. to determine the homological functors of some finite two-generator two-groups of nilpotency class two for Type 1,
3. to compute the nonabelian tensor square and some homological functors of some finite two-generator two-groups of nilpotency class two for Type 3,
4. to verify all the results by Groups, Algorithms and Programming (GAP) software.

1.5 Scope of Research

This research focuses only on Type 1 and Type 3 in the classification of finite two-generator two-groups of nilpotency class two. The homological functors computed include only the nonabelian tensor square, $J(G)$, exterior square, the Schur multiplier, and $\tilde{J}(G)$.

1.6 Significance of Research

This research will lead to the development of new theorems with proofs. The results of this research can be presented in a conference and can be sent for publication in a journal. Furthermore, the results of this research can be used for further research that related to two-generator two-groups of nilpotency class two and related research in homological functors. This research also enhances contribution from mathematicians in Malaysia especially in Pure Mathematics fields.

1.7 Research Methodology

This research has been carried out according to the following steps:

1. Study the classification of finite two-generator two-groups of nilpotency class two.
2. Study on nonabelian tensor square of finite two-generator two-groups of nilpotency class two.
3. Study on the homological functors of finite two-generator two-groups of nilpotency class two of Type 1, Type 2, and Type 4.
4. Compute some homological functors of some finite two-generator two-groups of nilpotency class two of Type 1.
5. Compute the nonabelian tensor square and some homological functors of finite two-generator two-groups of nilpotency class two of Type 3.
6. Write up of dissertation.
7. Presentation of dissertation.

1.8 Dissertation Report Organization

This dissertation is organized into five chapters. Chapter 1 is the research framework. This chapter includes the research background, problem statement, research objectives, scope of research, significance of study, and the research methodology.

In Chapter 2, some definitions and basic concepts that will be used throughout the dissertation are included. Among these are some basic concepts in group theory and homological functors and also the classification of two-generator two-groups of nilpotency class two. Some introduction of Groups, Algorithms and Programming (GAP) software is also included in this chapter.

Chapter 3 reviews on some results obtained by previous researchers. By using the complete classification of finite two-generator two-groups of nilpotency class two, the homological functors of some finite two-generator two-groups of nilpotency class two of Type 1 are presented. These include the proof for each homological functors calculated. The results are then verified by using Groups, Algorithms and Programming (GAP) software.

Chapter 4 discusses the use of Groups, Algorithms and Programming (GAP) software in the computation of the results. Some GAP programmes that generate the general code of the nonabelian tensor square and the homological functors of some finite two-generator two-groups of nilpotency class two of Type 3 are provided in this chapter. The general formula of the nonabelian tensor square and some homological functors for some cases are also included.

Lastly, in Chapter 5, the obtaining results are summarized. Suggestions for further research are also included.

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