SUBSPACE BASED DIRECT ADAPTIVE CONTROL FOR A CLASS OF NONLINEAR SYSTEMS

OSAMA HOSNI SALEH WHIDA

A project report submitted in partial fulfilment of the
requirements for the award of the degree of
Master of Engineering (Electrical- Mechatronics & Automatic Control)

Faculty of Electrical Engineering
Universiti Teknologi Malaysia

JANUARY 2012

Dedicated,

To my beloved inspirational father *‘Hosni’*

To my beloved heartwarming mother *‘Jamila’*

I love you and I hope that I can serve you better.

To the soul of my beloved grandmother *‘Muna’*, and uncle *‘Muftah’*

May Allah s.w.t accept you with his grace and pardon you; I love you and miss you!

ACKNOWLEDGEMENT

In The Name Of Allah, Most Gracious, Most Merciful

First and foremost, I must be thankful to Allah s.w.t for everything especially for his guidance in accomplishing this level of study. Indeed, I’m nothing without your care, Alhamdolellah.

I would like to express my sincere thanks and appreciation to my supervisor Dr Norhaliza Binti Abd Wahab, for her precious guidance, encouragement, constructive criticisms, advice, knowledge and motivation. Without her continued support and interest, this project report would not have been presented as the same as it is presented here.

Besides that, my sincerely appreciation is extended to my family and especially to my father ‘Hosni’ for his unlimited love, support, encouragement, understanding and care. I would say “Thank you very much and may Allah s.w.t rewards you well for everything (Jazak Allah A’nni khiran kathira)”.

My special thanks is going also to my dearest brothers: Muhee, Shihab and Zakariya, as well as my beloved sisters: Amel, Inasse, Tassnim, Suhaila, Reemo and Ismahan. To Fatima with a great appreciation. To aunt, cousins and their families.

Also I would like to express my thanks to all my fellow postgraduate master course maters, colleagues and friends in UTM for their support and assistance at various occasions. Their views and tips are useful indeed. Unfortunately, it is not possible to list all of them in this limited space, love and care.

ABSTRACT

This project exploits subspace prediction methods in order to apply a novel direct "predictive" control design technique, which can be viewed as an extension of model free subspace based linear quadratic Gaussian (LQG) control, and in the class of adaptive control. The main purpose of this study is to design a much simpler control approach for a wastewater treatment plant using a data driven direct adaptive predictive controller based on subspace identification of prediction matrices. The general direct control design problem requires the engineer to collect experimental data, and choose a performance objective. With these design choices, it is then possible to calculate a control law that optimizes expected future performance. Recently, there has been significant interest in developing a direct control design methodology producing a more reliable and automated control design technique. Effective control of wastewater treatment plants (WWTPs) has been receiving rising attention during the last decade due to increasing concern about environmental issues. In this sense, the importance of studies concentrating on control and simulation of WWTP is remaining intact. Activated sludge process is commonly used in biological wastewater treatment. The applied methodology of this project is supposed to regulate the substrate concentration and dissolved oxygen concentration to specified values, compensate the disturbances may occur in the load influents as well as to track any mechanistic or kinetic parameter variation immediately in the shortest possible time. Hence, direct adaptive predictive control (DAMPC) can provide simplicity, good performance and stability robustness of an activated sludge process.

ABSTRAK

Tujuan penyelidikan ini dijalankan adalah untuk menghasilkan teknik kawalan yang lebih mudah bagi loji rawatan kumbahan air. Laporan ini mencadangkan kawalan dengan ramalan adaptasi terus atau juga dikenali sebagai kawalan dengan ramalan subspace sebagai alternatif kepada kaedah yang sedia ada. Struktur kawalan adaptasi adalah berdasarkan kepada model berkadar terus sistem tersebut dan digabungkan bersama algoritma angka untuk sistem subspace state space yang berperanan sebagai pengesan untuk anggaran langsung matriks ramalan dan matriks kawalan dalam proses-bio, serta model kawalan ramalan bagi mendapatkan jujukan kawalan yang optimum. Prestasi kedua-dua algoritma anggaran dan kawalan digambarkan menerusi keputusan simulasi. Analisis kestabilan dijalankan bagi menyiasat tindak balas sistem yang dicadangkan apabila wujudnya perubahan parameter. Projek ini membuktikan bahawa kaedah adaptasi subspace memiliki banyak kelebihan yang penting dan berguna, terutama aplikasi keupayaannya dikendalikan bersama sistem pelbagai masukan dan pelbagai keluaran serta keperluan yang rendah mengenai maklumat sistem. Berdasarkan kelebihan tersebut, bidang aplikasi yang sesuai bagi algoritma yang dicadangkan adalah proses dengan pembulehubah berbagai, di mana hanya sedikit info diketahui seperti loji rawatan kumbahan air ini. Dengan itu, pendekatan baru teknik kawalan dengan ramalan adaptasi terus mampu menyediakan kaedah yang mudah, prestasi yang baik serta kestabilan yang teguh dalam mengawal proses larutan aktif.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

LIST OF ABBREVIATIONS

ASP	-	Activated Sludge Process
DAMPC	-	Direct Adaptive Model Predictive Controller
IAMPC	-	Indirect Adaptive Model Predictive Controller
IVM	-	Instrumental variable method
LQR	-	Linear-Quadratic Regulator
LTI	-	Linear Time Invariant
M-PRBS	-	Multi level PRBS
MIMO	-	Multiple Input Multiple Output
MOESP	-	Multivariable Output-Error State-space model identification
MPC	-	Model Predictive Control\Controller
MRSE	-	Mean Relative Squared Error
MVAF	-	Mean Variance-Accounted-For
N4SID	-	Numerical algorithm for Subspace State Space System Identification
PEM	-	Prediction Error Method
PI	-	Proportional Integral
PID	-	Proportional Integral derivative

PRBS	-	Pseudo Random Binary Signal
SVD	-	Singular Value Decomposition
WWTP	-	Wastewater Treatment Plant

LIST OF SYMBOLS

U_p	-	Block Hankel past input matrix
U_f	-	Block Hankel future input matrix
Y_P	-	Block Hankel past output matrix
Y_f	-	Block Hankel future output matrix
W_p	-	Block Hankel matrices consisting of past inputs and past outputs
W_f	-	Block Hankel matrices consisting of future inputs and future outputs
Γ_i	-	Observability matrix
Δ_i	-	Controllability matrix
H_i	-	Block triangular Toeplitz matrix
X_i	-	State sequence
O_i	-	Oblique projection
$\Gamma^\dagger$	-	Moore-Penrose pseudo-inverse of the Observability matrix

LIST OF APPENDICES

CHAPTER 1

INTRODUCTION

This project presents a method of control approach which combines the functions of traditional system identification with that of control design, enabling synthesis of Model Predictive Controllers (MPC) in a single direct process. The method utilizes principles from the recently developed field of subspace identification in order to reduce huge amounts of experimental data to a much smaller "subspace predictor", which is then applied to apply a control law. The approach is referred to as "model free" because at no time in the process is an explicit model of the plant formulated. In addition, an efficient method of recursively updating the subspace predictor is developed, thereby allowing online adaptation of the controller as new experimental data are collected.

In the first section, a general overview of the project is given; showing the main characteristics of the applied techniques and when it is necessary to use such advanced methods. The problem statement is illustrated in the second section of this chapter. The objectives and scope of this research study are presented in the third and fourth sections, respectively. The methodology adopted is outlined in points and as a project flow chart in the sections 1.5 and 1.6.

1.1 Background of the Project:

For a class of nonlinear systems whose current states can be reconstructed with N past measurements, a new subspace-based predictive controller is designed based directly on input-output data. This technique combines the characteristics of subspace identification method with predictive control, such as the minimum request of prior knowledge, applicability for multi-input multi-output (MIMO) process, and the minimization of multi-step prediction errors, to result in the model-free subspace predictive controller. To add an adaptive mechanism to predictive control, this project uses receding window mechanism to capture nonlinear dynamic characteristics, updates subspace predictor at every time step, and implements the predictive control. Activated Sludge process of Wastewater Treatment Plants is used to prove the efficiency of the proposed algorithm.

There are many types of systems where experimental data are particularly valuable in obtaining knowledge of process behavior. Examples include cases where the process is difficult or expensive to model, where the process is time-varying, or where the plant is well modeled but certain parameters must be determined experimentally. Examples of difficult or expensive plants to model include solid oxide fuel cells (X. Wang et al., 2007) and blast furnace ironmaking process (Zeng et al., 2010). Examples of time-varying processes include evaporator process (Yang, Li, 2005), and the process studied in this project in details the Activated Sludge Process (Nejjari et al., 1999) (Koumboulis et al., 2008). Which is an example of a plant that has features that can be well modeled from first principles, yet requires experimental data in order to obtain appropriate model parameters.

Methods of using experimental data can roughly be divided into four categories, as shown in Table 1.1. The techniques are distinguished by whether they operate "on-line" or "off-line", and whether a plant model is explicitly (Indirect) or implicitly (Direct) used to perform the control design. Plant model identification is perhaps the most popular method of using experimental data in the control design process. The engineer usually performs a number of experiments, and then uses the experimental data in conjunction with various optimization techniques to form a model of the plant. The plant model is then used with one of the well-known model based control design techniques to formulate a control law.

Typical identification techniques include the classical prediction error (PE), auto regressive with exogenous input (ARX), auto regressive moving average with exogenous input (ARMAX), output error (OE), and Box Jenkins (Ljung., 1987) techniques. More recently, subspace techniques such as eigensystem realization analysis (ERA) (Juang., 1994) and numerical algorithms for subspace state space system identification (N4SID) (VODM., 1996) have gained popularity.

The control design literature is vast, and includes simple proportional integral derivative (PID) as well as more advanced modern and postmodern techniques e.g. linear quadratic Gaussian (LQG), and μ-synthesis. When model based design is used "on-line", it is usually referred to as indirect adaptive control. The process typically begins by assuming a nominal plant model. As new experimental data are collected, the outputs are compared to the outputs predicted by the nominal plant model, producing a nominal error. The gradient of the error with respect to the plant parameters is used to modify the plant parameters to improve the plant model. Periodically, the control law is updated using the most recently developed plant model as the basis for control synthesis. An example of this approach is model reference adaptive control (MRAC) (Astrom & Wittenmark., 1995).

Table 1 Control System Design Techniques

	Plant Model	No Plant Model
Off-line	Model Based Design	Direct Control Design
On-line	Indirect Adaptive	Direct Adaptive

The "no plant model" column in Table 1.1 is somewhat of a misnomer: in some sense, a data set can be considered an empirical plant model, thus any simplified representation of the data set is also a plant model. The defining property of model free techniques is that a single integrated procedure derives the control law directly from experimental data and a performance specification. If the technique has a sufficiently low computational burden, it is generally straightforward to implement the "no plant model" design technique on-line. This produces a "direct adaptive" control technique where the controller attempts to improves its performance in response to newly available experimental data. Examples of direct control techniques include model free subspace based LQG control (Favoreel., 1998) (Favoreel., 2000), adaptive inverse control, LMS, and, FxLMS and its alternatives. The next two subsections describe the properties and main features of ‘model free’ direct control design and outline conditions under which it might be advantageous to apply these techniques.

1.1.1 **Properties of Direct Techniques**

The most important property of direct control design techniques is the close coupling of the "plant identification" and the "control design" steps.

In traditional model based control design, the development of a plant model requires great simplifications of the experimental data set in order to obtain a plant model.

In the direct technique, much more of the experimental information is retained throughout the control design process. If the controller is then simplified, the simplifications that are made are with respect to the controller's input-output relationship, rather than with respect to the plant input-output relationship. The simplifications of the controller are made with respect to what is important to the control law, rather than what is important to the plant model. Closer coupling of the identification and control design process should lead to increased automation of the control design process, however, this conjecture can only be confirmed by the experiences of control engineers who are able to try both model based and model free techniques in the field. The increased automation is expected to result from the removal of the intermediate design steps, thereby requiring the engineer to make fewer arbitrary choices of parameters during the design process. An additional advantage is realized in the iterative process between designing "identification" experiments and performing closed loop tests: the direct control design process naturally provides the engineer with an immediate estimate of closed loop performance.

Due to the increased automation, direct techniques are easily implemented as part of an adaptive framework. It is believed that direct techniques will be of great utility when solving adaptive control problems.

1.1.2 **When One Might Use Direct Techniques**

Control issues with certain attributes are likely to receive the most benefit from direct method. These attributes include:

- Experimental data are plentiful, are representative of the important system dynamics, and are inexpensive to obtain.

- Iteration between design and closed loop experiment is possible.

- Some aspects of the system are difficult to quantify by analytic modeling, e.g. time- varying nonlinearities.

- The plant has many inputs and many outputs, such that modeling each input-output relationship might be prohibitively tedious.

The adaptive methodologies are of course applicable to cases where the plant is time-varying. In many time-varying problems, the control law adapts to compensate for changing system parameters, such as the kinetics parameters of a process. However, if the plant structure is changing, such as addition of new modes, a model free technique may be better suited than a model based adaptive controller in which the model structure is determined a priori.

1.2 Problem Statement:

Generally, nonlinear system's control design has many problem statements. Moreover, for this study the main difficulties are:

1. The dynamic model obtained for real plants is most often highly complex and high order non-linear system which makes the prediction of process behavior into the future is quite difficult.
2. Selecting a method of extrapolation of plant behavior into the future is not an easy task due to the lack of cheap and reliable sensors for on-line measurement of the key state variables.
3. Laboratory analysis with delays of several days cannot be used for on-line monitoring which require the proper development of a disturbance uncertainty model.
4. Selection process of the appropriate performance objective function should be applied in order to meet the low effluent quality and high energy consumption of wastewater treatment plants.
5. The model-free design approach presented in the literature so far does not include all the important predictive control features such as inclusion of an integrator for offset-free control, constraint handling, feedforward option and

a means of tuning the controllers through the disturbance model; these features are important for practical applications.

1.3 Objectives of the project:

The main objectives of this project are:

1. To study the operation of Activated Sludge Process (ASP) in wastewater treatment plants (WWTP).
2. To estimate Prediction Matrices and Control Matrices.
3. To design Subspace Predictive Controller (SPC).
4. To analyse the stability issues occur on the system proposed.

1.4 Scope of the project:

1. To control ASP, in order to provide good performance and stability robustness in controlling: biomass X (t), substrate S (t), dissolve oxygen C (t) and recycled biomass X_r (t) concentrations in the activated sludge process.
2. To study on-line identification using N4SID. In this case, the prediction matrices and controller matrices can be retrieved on-line and used for MPC controller.

3. To design adaptive model predictive controller, this would have the ability to adjust its control order signal for any parameter variations occurring in a plant model.

4. To use the direct method only. This project only focuses on the direct method which gives better performance with respect to input disturbance and also gives better tracking properties as well as disturbance rejection, with less design and computational efforts compared to the indirect method.

5. Stability analysis to investigate the effects of the user defined parameters such as Hankel matrices block sizes on identification results.

1.5 Methodology

With those issues discussed in problem statement section (1.2) in mind, the following design choices were made for this project.

- ❑ The predictive control applied is completely data based, since it only requires a set of input–output open-loop data.
- ❑ Through online updating of predictor using receding window, a nonlinear process is approached in the surrounding of working points and adaptive control is implemented.
- ❑ In addition, unlike other data-driven predictive control designs, the proposed approach can deal with systems without complete on-line measurement of all output variables.

These design choices result in what has been termed direct subspace based adaptive predictive control.

As a member of a class of controllers known as predictive control (Qin & Badgwell., 2003), (Clarke., 1996) Predictive control refers to any technique that employs the following steps:

1. A predictor is used to determine the plant input that will optimize a specified cost function over a future time horizon.

2. The first time step of the control is implemented. The plant output at this time step is recorded.

3. The new input-output data are added to the predictor, and steps 1-3 are repeated.

These steps are collectively known as a "receding horizon" implementation.

1.6 Project Flow chart

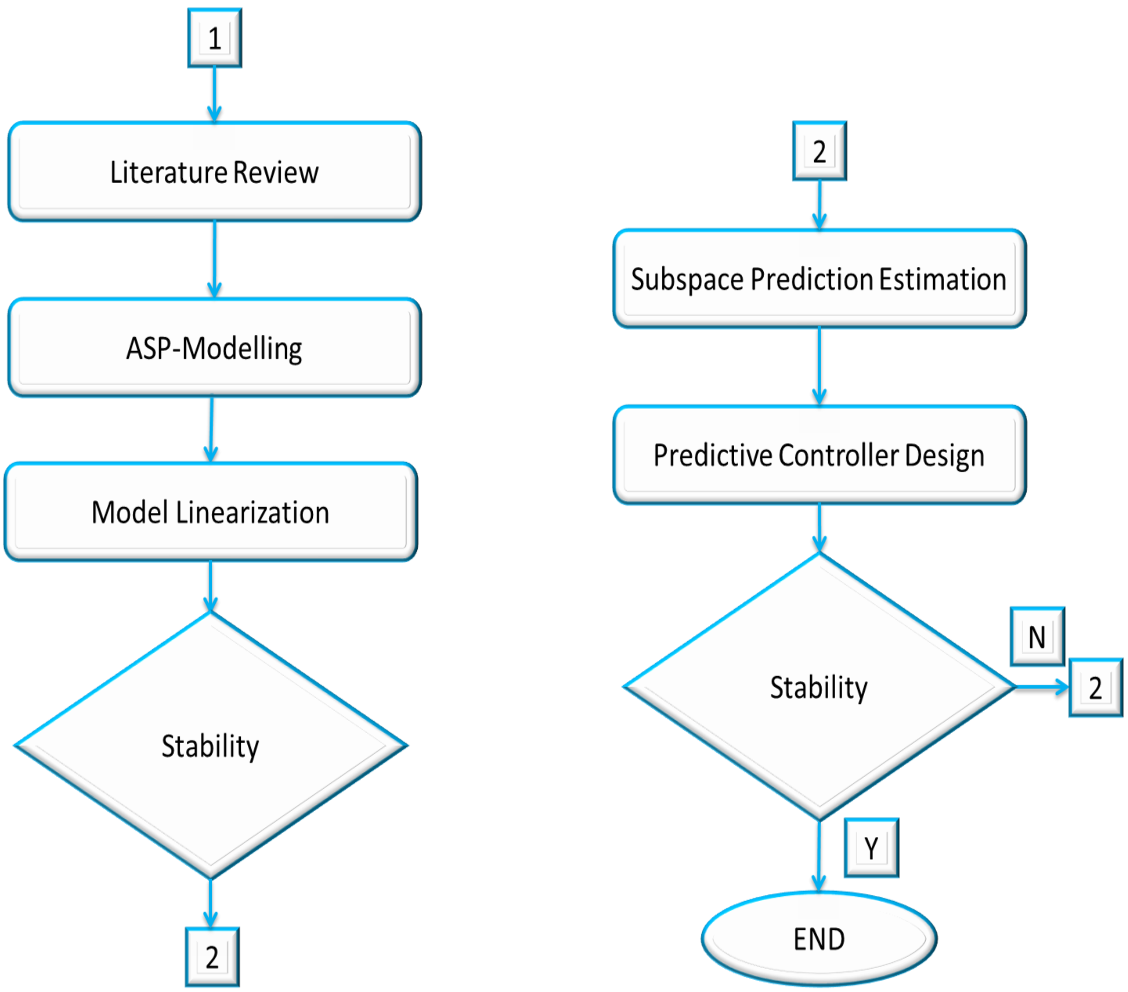

Figure 1.1 The Project Flow Chart

REFERENCES

Aguero, J. C. and G. C. Goodwin (2004). A recursive algorithm for MIMO stochastic model estimation. Control Conference, 2004. 5th Asian.

Akyurek, E., M. Yuceer, et al., (2009).“Comparison of Control Strategies for Dissolved Oxygen Control in Activated Sludge Wastewater Treatment Process”. Computer Aided Chemical Engineering. J. Jacek and T. Jan, Elsevier. Volume 26: 1197-1201.

Astrom, K.J, Å. (1983). "Theory and applications of adaptive control—A survey." Automatica 19(5): 471-486.

Astrom & Wittenmark., (1995) Astrom, K.J. and B. Wittenmark, (1995) “Adaptive Control”, Addison Wesley, Reading, MA.

Bo Wahlberg, (2007): MIT Optimal Control Course in Principles of optimal control

Camacho, E. F., & Bordons, C. (1999). Model predictive control. London: Springer-Verlag.

Carl-Fredrik Lindberg (1997). “Control And Estimation Strategies Applied To The Activated Sludge Process”. Phd. Thesis, Uppsala University.

D.W. Clarke (1996) "Adaptive predictive control." Annual Reviews in Control 20(0): 83-94.

Favoreel,W., De Moor, B., & Gevers, M. (1999). SPC: Subspace predictive control. In Proc. 14th World Cong. IFAC , pp. 235.

Favoreel, D. M. B. a. W., (1998). "SPC: Subspace predictive control." Technical report, Katholieke Universiteit, Leuven.

Favoreel, W, B. De Moor, M. Gevers, and P. Van Overschee (1999). Closed-loop model-free subpace-based lQG-design. In Proceedings of the 7th Mediterranean Conference on Control and Automation (MED99), Haifa, Israel, 1999.

Favoreel W., B. D. Moor, P. V. Overschee, and M. Gevers (1999). “Model-free subspace-based LQG-design”. In Proc. of the American Control Conference, pages 3372-3376, San Diago,California, 1999.

Favoreel, W., B. De Moor and P. Van Overschee (2000). “Subspace state space system identification for industrial processes, Journal of Process Control, vol.10, no.2-3, pp.149-155, 2000.

Ferkl, L.; Široký, J.; Prívara, S., (2010) "Model predictive control of buildings: The efficient way of heating,"Control Applications (CCA), 2010 IEEE International Conference on, vol., no., pp.1922-1926, 8-10 Sept. 2010 doi: 10.1109/CCA.2010.5611114

Gambler, A., (2004) "Multivariable adaptive state-space control: A survey," Control Conference, 2004. 5th Asian , vol.1, no., pp.185-191 Vol.1, 20-23 July 2004

Garcia C.E., D.M. Prett and M. Morari, (1989) “Model predictive control: theory and practice-a survey”, Automatica, **25** (3), pp. 335–348

Gernaey, K. V., M. C. M. van Loosdrecht, et al. (2004). "Activated sludge wastewater treatment plant modelling and simulation: state of the art." Environmental Modelling & Software 19(9): 763-783.

Golub, G. H. and C. Loan, (1996) Matrix Computations, The Johns Hopkins University Press, Baltimore, MD, 1996.

Hale, E. T. and S. J. Qin, (2002). “Subspace model predictive control and a case study”. American Control Conference, 2002. Proceedings of the 2002.

Holenda B., E. Domokos, A. Redey and J. Fazakas, (2008) “Dissolved Oxygen Control of the Activated Sludge Wastewater Treatment Process Using Model Predictive Control,” Computers and Chemical Engineering, Vol. 32, 2008, pp. 1270-1278.

Huang and Kadali, (2008) B. Huang and R. Kadali, Dynamic Modeling, Predictive Control and Performance Monitoring: A Data-Driven Subspace Approach.London: Springer, 2008.

Isaacs S. H. and M. Henz (1995). Controlled carbon source addition to an alternating nitrification-denitrification wastewater treatment process including biological P removal. Wat. Res. 29(1) 77-89

Juang., (1994) J.N. Juang. Applied System Identification. PTR Prentice-Hall, 1994.

Kadali, R., Huang, B., & Rossiter, A. (2003). "A data driven subspace approach to predictive controller design." Control Engineering Practice 11(3): 261-278.

Katayama, T. (1996) Special issue on subspace methods for system identification. Signal Processing 52

Koumboulis et al., (2008) F.N. Koumboulis, N.D. Kouvakas, R.E. King, and A. Stathaki. Two-stage robust control of substrate concentration for an activated sludge process. ISA Transactions 47 (2008) 267–278.

Lalo Magni and Riccardo Scattolini, (2009) An Overview of Nonlinear Model Predictive Control

Larimore, W.E. (1996). Statistical optimality and canonical variate analysis system identification. Signal Proc., 52, 131–144.

L. Ljung. System Identification: (1987) Theory for the User. Prentice-Hall, 1987

Ljung, L., & McKelvey, T. (1996). Subspace identification. Signal Processing, 52, 209–215.

Lennart, L. (2010). "Perspectives on system identification." Annual Reviews in Control 34(1): 1-12.

Maciejowski et. al., (2007) J.M. Maciejowski, P.J. Goulart and E.C. Kerrigan Constrained Control Using Model Predictive Control. Lecture Notes in Control and Information Sciences, 2007, Volume 346/2007, 273-291

Mardi & Wang, (2008) N. A. Mardi and L. Wang, “Subspace-based model predictive control with data prefiltering,” in Proc. of the UKACC International Conference on Control 2008, (University of Manchester, UK), 2008.

Mardi and Wang, (2009) Mardi, N. A. and W. Liuping (2009). “Subspace-based Model Predictive Control of time-varying systems”. Proceedings of the IEEE Conference on Decision and Control, art. no. 5400285, pp. 4005-4010.

Mardi, (2010) Noor Azizi Mardi (2010) “Data-driven Subspace-based Model Predictive Control”, Phd-Thesis, RMIT University.

Mastronardi, N., D. Kressner, et al., (2001) "A fast algorithm for subspace state-space system identification via exploitation of the displacement structure." Journal of Computational and Applied Mathematics 132(1): 71-81.

Nejjari et al., (1999) F. Nejjari, A. Benhammou, B. Dahhou and G. Roux, Non-linear multivariable adaptive control of an activated sludge wastewater treatment process, Int. J. Adapt. Control Signal Process. Vol. 13, pp. 347-365, 1999.

Olsson, G.,(1976) State of the art in sewage treatment plant control', A.I.Ch.E. Symp. Ser. 72,52}76 (1976)

Olsson G, Andrews JF (1987) The dissolved oxygen profile—a valuable tool for control of the activated sludge process. Water Res 12:986–1004

Olsson G. and Newell R.B. (1999) Wastewater Treatment Systems - Modelling, Diagnosis and Control. IWA Publishing, London, United Kingdom. pp. 742.

Oscar A.Z. Sotomayor, Song Won Parka, Claudio Garcia, (2003) “Multivariable identification of an activated sludge process with subspace-based algorithms”, Control Engineering Practice 11. pp: 961–969

Overschee, Van. P., De Moor, B. (1994) “N4SID: Subspace algorithms for the identification of combined deterministic-stochastic systems” Automatica, 30 (1), pp. 75-93. Cited 497 times.

Overschee, Van. P., B. De Moor, (1995) “Choice of state space basis in combined deterministic-stochastic subspace identification”, Automatica 31 (2) (1995) 1877-1883.

Overschee, Van. P., B. De Moor, (1996) “Subspace Identification for Linear Systems: Theory-Implementation-Applications”. Kluwer Academic Publishers.

Pannocchia, G. and M. Calosi (2010). "A predictor form PARSIMonious algorithm for closed-loop subspace identification." Journal of Process Control 20(4): 517-524.

Qin, S. J., Thomas A. Badgwell. (2003) “A survey of industrial model predictive control technology”. Control Engineering Practice 11 (2003) 733–764.

Qin, S. J., W. Lin, et al. (2005). "A novel subspace identification approach with enforced causal models." Automatica 41(12): 2043-2053.

Qin, S. J., (2006). "An overview of subspace identification." Computers & Chemical Engineering 30(10–12): 1502-1513.

Rawlings, J. B. (2000). "Tutorial overview of model predictive control." Control Systems, IEEE 20(3): 38-52.

Rossiter. J.A., “Model-based predictive control, a practical approach”. Prentice Hall Int., 2003

Ruscio, D. (1997) On subspace identification of the extended observability matrix. In: Proceedings of the 36th Conference on Decision and Control, pp. 1841–1847.

Ruscio, D. D. (1997). Model based predictive control: an extended state space approach. Decision and Control, Proceedings of the 36th IEEE Conference on.

Sergiu, C., S. Mihaela, et al., (2008). “Predictive Control of a Wastewater Treatment Process”. International Journal of Computers, Communications & Control Vol. II (2007), No. 2, pp. 132-142.

Song, In-Hyoup., Kee-Youn Yoo, and Hyun-Ku Rhee (2001) “Analysis and Design of a Linear Input/Output Data-Based Predictive Control” Ind. Eng. Chem. Res. 2001, 40, 4292-4301.

Song, I.-H., S.-B. Lee, et al. (2006). "Identification and predictive control of a simulated moving bed process: Purity control." Chemical Engineering Science 61(6): 1973-1986.

Stare, A., D. Vrecko and N. Hvala, (2007) “Comparison of Control Strategies for Nitrogen Removal in an Activated Sludge Process in Terms of Operating Costs: A Simulation Study,” Journal of Process Control, Vol. 41, 2007, pp. 2004-2014.

Tchobanoglous, G. and E. D. Schroeder (1985). Water Quality. Addison-Wesley Publishing Company.

Torben, K., (2001). "Consistency analysis of subspace identification methods based on a linear regression approach." Automatica 37(1): 81-89.

Verhaegen, M. and E. Deprettere (1991). “A fast, recursive MIMO state space model identification algorithm”. Decision and Control, 1991., Proceedings of the 30th IEEE Conference on.

Verhaegen, M., P. Dewilde, (1992) Subspace model identification. Part 1. The output-error model identification class of algorithms, Int. J. Contr. 56 1187–1210.

Verhaegen, (1994) Verhaegen, Michel, Identification of the deterministic part of MIMO state space models given in innovations form from input-output data, Automatica, Vol. 30. No. 1. pp. 61-74 (1994)

Vilanova, et. al., (2011) Vilanova R., Rojas J.D., Alfaro V.M., Digital Control of a Waste Water Treatment Plant, International Journal of Computers Communications & Control, ISSN 1841-9836, 6(2):367-374, 2011.

Vlad, G., R. Crisan, et al., (2010). Development and application of a predictive adaptive controller to a wastewater treatment process. Automation Quality and Testing Robotics (AQTR), 2010 IEEE International Conference on.

Wahab, N. A., R. Katebi, et al., (2011). "Data-driven adaptive model-based predictive control with application in wastewater systems." Control Theory & Applications, IET 5(6): 803-812.

Wang. Liuping, (2009) “Model Predictive Control System Design and Implementation Using MATLAB”. Springer-Verlag London Limited, 2009.

Wang et al., (2007) X. Wang, B. Huang, and T. Chen (2007). "Data-driven predictive control for solid oxide fuel cells." Journal of Process Control 17(2): 103-114.

Woodley, (2001) B. R. Woodley, J. P. How, and R. L. Kosut. (2001) Subspace based direct adaptive H∞ control. International Journal of Adaptive Control and Signal Processing, 15:535561, 2001.

Yang, H., & Li, S. Y. (2005). Subspace-based adaptive predictive control of nonlinear systems. In Int. J. Innovative Comp., Inf. Control, 4 , pp. 743.

Yang, H., & Li, S. Y. (2010) Hua, Y. and L. Shaoyuan (2010). A data-driven bilinear subspace predictive controller design. Control and Automation (ICCA), 2010 8th IEEE International Conference on Control and Automation (ICCA).

Zeng et al.,(2010) Jiu-sun Zeng, Chuan-hou Gao, Hong-ye Su. Data-driven predictive control for blast furnace ironmaking process. Computers and Chemical Engineering 34 (2010) 1854–1862