### ON- AND OFF-LINE IDENTIFICATION OF LINEAR STATE SPACE MODELS

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To my beloved mother and father

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#### ABSTRACT

A geometrically inspired matrix algorithm is derived for the identification of state space models for multivariable linear time-invariant systems and using possibly noisy input- output measurements data only. In this project, only a limited number of input and output data are required for the determination of the system matrices. The algorithm can be best described and also understood in the matrix formalism and consists in the following two steps. First step, a state vector sequence is realized as the intersection of the row spaces of two block Hankel matrices which is constructed by apply input - output data. Then, the system matrices are obtained at once from the least squares solution of a set of linear equations. When dealing with noisy data, this algorithm draws its excellent performance from repeated use of the numerically stable and accurate singular value decomposition. The algorithm is easily applied to slowly time-varying systems using windowing or exponential weighting.

### ABSTRAK

Geometri yang di ilhamkan daripada algoritma matrik di terbitkan untuk pengenalpastian model state space untuk sistem pelbagai parameter linear masainvariant dengan hanya menggunakan input-output kebisingan. Dalam projek ini, ia hanya memerlukan number input-output yang di hadkan untuk mendapatkan sistem matrik. Algoritma ini boleh dijelaskan dan di fahami dalam bentuk matrik dan mempunyai dua langkah. Langkah pertama, susunan state vector di ketahui sebagai pertemuan ruang baris oleh dua blok Hankel matrik di mana ia dibina dengan menggunakan input-output data sahaja. Kemudian, sistem matrik ini sekali gus di perolehi daripada penyelesaian persamaan linear least square. Apabila berhadapan dengan data kebisingan, algoritma ini menghasilkan prestasi yang sangat baik daripada pengulangan penggunaan angka stabil dan kejituaan Singular Value Decomposition. Lebih-lebih lagi algoritma ini mudah diaplikasikan pada sistem masa perlahan dan menggunakan teknik tetingkap atau keberatan exponential.

.

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# LIST OF ABBREVIATIONS

| MOESP | - | Multivariable Output-Error State-Space model identification |
|-------|---|---|
| SVD   | - | Singular Value Decomposition                                |
| I/O   | - | Input-output  |
| Gbest | - | Global best   |
| MIMO  | - | Multiple Input Multiple Output                              |
| PEM   | - | Prediction Error Method                                     |
| IVM   | - | Instrumental Variable Method                                |
| LTI   | - | Linear Time Invariant                                       |
| PI    | - | Proportional Integral                                       |
| PID   | - | Proportional Integral Derivative                            |
| LQR   | - | Linear-Quadratic Regulator                                  |
| PSO   | - | Particle Swarm Optimization                                 |
|       |   |   |

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### **CHAPTER 1**

#### **INTRODUCTION**

#### 1.1 An overview

Identification aims at finding a mathematical model from the measurement record of inputs and outputs of a system. A state space model is a most obvious choice for a mathematical representation because of its widespread use in system theory and control. Still, reliable general purpose state space identification schemes have not become standard tools so far, mostly due to the computational complexity involved (Ho and Kalman 1965, Kung 1978, Zeiger and Mc Ewen 1974). The theory of canonical correlation analysis, independently developed in the midthirties by Hotelling (Hotelling 1936) and Obukhov, the idea of using SVD to compute the principal angles and vectors being due to Bjorck and Golub (Golub and Van Loan 1983), has been intensively applied to the stochastic identification problem, where as a major departure canonical variate analysis is used to choose linear combinations of the past of the random process to optimally predict the future of the process. The analysis of a system in terms of past and future naturally leads to a state space description (Akaike 1974, Akaike 1975, Baram 1981, Ramos and Verriest 1984, Larimore 1984). Nevertheless, the intensive use of covariance information is a major

drawback when it comes to practice, since finite data records reveal only poor approximations for covariance matrices.

#### **1.2 Objective of Project**

The objective of this project comprises of the following:

- i. To analyze the differences between On- and Off-line identification of a linear state space models by apply subspace algorithm.
- ii. Implement the algorithm to the suitable system.

#### **1.3** Scope of Study

Scopes of the project are listed:

- i. To study On-line and Off-line identification algorithm.
- Two methods of the subspace algorithm are used which is N4SID (numerical algorithm for Subspace State Space System Identification) and MOESP (Multivariable Output-Error State-Space model identification)

#### **1.4** Thesis outline

Chapter 1 discusses on the objective and scope of this project. It also covers an overview of this project.

Chapter 2 introduces a several literature reviews that have been done for this project.

Chapter 3 includes the methodology of this project with basic knowledge of subspace based algorithm, Off- and On-line algorithm and global best of PSO.

Chapter 4 presents the results and analysis for this project. It covers on Off- and Online identification results.

Chapter 5 discusses the conclusion that can be made from the results and recommendations for future work.

#### 1.5 Summary

In this chapter is, well planning is very important to make this project success. Every planning that is planned should be follow to make the project finished on the dateline or earlier before the dateline. The objective of the project is also important to make the project successfully and as aim of the project. In addition, the scope of the project needs to recognize before start the project.

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