

LIMIT EQUILIBRIUM METHOD AND FINITE ELEMENT METHOD IN SLOPE
STABILITY ANALYSIS

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For you, Dad, Mum, and Brothers

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ABSTRACT

Limit equilibrium method (LEM) has been applied for decades in slope stability analysis due to its simplicity. Various methods of analysis are available as foundations for limit equilibrium approach such as Ordinary, Bishop, Janbu, Morgenstern-Price, Spencer and others. Between these methods, there are differences in terms of assumptions made regarding interslice forces. As the technology developed and the need for a more advance method of analysis grows, finite element method (FEM) was introduced. This method is able to run rigorous analysis on complex problems. With the differences both methods possess, sensitivity analyses and comparisons of result were done using Slope/w and Plaxis software for LEM and FEM respectively. Analyses were done based on a case study of a slope located in Universiti Teknologi Malaysia (UTM) Varying parameters used in this study are cohesion, friction angle and depth of groundwater table. It is found that both programs are sensitive towards changes in friction angle where Slope/w and Plaxis indicated an increment of 32.38% and 28.20% increment respectively in terms of factor of safety. Results also showed that factors of safety obtained from LEM are higher than FEM. This is generally because finite element method can effectively calculate stresses at the crest and toe of the slope. Generated mesh using finite element method also results in a more accurate calculation as each deformation that occurs is analysed by the mesh.

ABSTRAK

Kaedah '*limit equilibrium method (LEM)*' sudah digunakan sejak berdekad lamanya dalam menganalisa kestabilan cerun kerana kaedahnya yang mudah. Beberapa kaedah dalam penggunaan LEM sudah tersedia seperti Ordinary, Bishop, Janbu, Morgenstern-Price, Spencer dan banyak lagi. Terdapat beberapa perbezaan di antara kaedah-kaedah ini seperti pengambilkiraan '*interslic force*'. Pembangunan teknologi telah melahirkan kaedah yang lebih maju dalam penganalisaan cerun iaitu '*finite element method (FEM)*'. Kaedah ini mampu untuk menganalisa masalah yang rumit dengan lebih jitu. Perbezaan di antara dua kaedah ini menyebabkan perlunya analisis sensitiviti dilakukan dengan menggunakan program Slope/w (LEM) dan Plaxis (FEM). Analisis dilakukan berdasarkan kes yang terdapat di cerun yang terletak di Universiti teknologi Malaysia (UTM). Pembolehubah-pembolehubah yang digunakan di dalam kajian ini adalah nilai kelekitan, sudut geseran dan kedalaman aras air. Didapati bahawa kedua-dua program tersebut sensitif terhadap perubahan nilai sudut geseran di mana Slope/w menunjukkan peningkatan terhadap factor keselamatan sebanyak 32.38% manakala Plaxis menunjukkan peningkatan sebanyak 28.20%. Keputusan juga menunjukkan nilai factor keselamatan bagi LEM adalah lebih tinggi berbanding dengan FEM. Ini mungkin kerana kaedah FEM lebih efektif dalam mengambil kira pengiraan tekanan pada bahagian bawah dan atas cerun. '*Mesh*' yang terbentuk melalui program Plaxis juga membantu dalam memberikan keputusan yang lebih tepat kerana setiap perubahan yang berlaku pada cerun tersebut dikambil kira oleh '*mesh*' yang terbentuk.

TABLE OF CONTENTS

CHAPTER	TITLE	PAGE
	TITLE	i
	DECLARATION	ii
	DEDICATION	iii
	ACKNOWLEDGEMENT	iv
	ABSTRACT	v
	ABSTRAK	vi
	TABLES OF CONTENTS	vii
	LIST OF TABLES	x
	LIST OF FIGURES	xi
	LIST OF SYMBOLS	xiv
1	INTRODUCTION	1
	1.1 Introduction	1
	1.2 Background of Study	2
	1.3 Objectives	3
	1.4 Scope of Study	3
	1.5 Significance of Study	3
2	LITERATURE REVIEW	5
	2.1 Causes of Slope Failure	5
	2.1.1 Decrease in Shear Strength	6
	2.1.2 Increase in Shear Stress	7

2.2	Limit Equilibrium Principles	8
2.3	Limit Equilibrium Methods	11
2.3.1	Ordinary or Fellenius Method	13
2.3.2	Bishop's Simplified Method	14
2.3.3	Janbu's Simplified Method	15
2.3.4	Spencer's Method	15
2.3.5	Morgenstern-Price Method	16
2.3.6	Corps of Engineers Method	17
2.4	Finite Element Analysis	17
2.4.1	Constitutive Models	18
2.4.1.1	Linear Elastic Model	18
2.4.1.2	Mohr-Coloumb Model	19
2.4.1.3	Hardening Soil Model	19
2.4.1.4	Modified Cam Clay Model	19
2.4.1.5	Soft Soil Creep Model	20
2.4.2	Summary of Finite Element Method	20
2.5	Limit Equilibrium Method (LEM) Versus Finite Element Method (FEM)	21
2.6	Case Study: Slope Failure in Skudai, Malaysia	21
3	METHODOLOGY	25
3.1	Introduction	25
3.2	Soil Model and Parameters Used	26
3.3	Software Used in Slope Stability Analyses	27
3.3.1	Slope/w	27
3.3.2	Plaxis	28
3.4	Computation of Factor of Safety using Plaxis	29
3.5	Sequence of Study	30

4	RESULTS AND DISCUSSIONS	31
4.1	Sensitivity Analysis	31
4.1.1	Sensitivity Analysis Using Slope/w	31
4.1.1.1	Effect of Cohesion	31
4.1.1.2	Effect of Angle of Friction	36
4.1.1.3	Effect of Groundwater Table	39
4.1.2	Sensitivity Analysis Using Plaxis	43
4.1.2.1	Effect of Cohesion	43
4.1.2.2	Effect of Friction Angle	49
4.1.2.3	Effect of Groundwater Table	56
4.2	Comparison of Results from LEM and FEM	59
4.2.1	Analysis of Varying Cohesion Values	60
4.2.2	Analysis of Varying Friction Angle	61
4.2.3	Analysis of Varying Depth of Groundwater Table	62
5	CONCLUSIONS	63
5.1	Conclusion	63
5.2	Recommendation	64
	REFERENCES	65

LIST OF TABLES

TABLE NO.	TITLE	PAGE
2.1	Interslice force assumptions used in force equilibrium procedures (Duncan and Wright, 2005)	10
2.2	Equations of static satisfied (Krahn, 2004)	12
2.3	Interslice force characteristics and relationships (Krahn, 2004)	13
2.4	Summary of differences between LEM and FEM	21
2.5	Groundwater table and depth of blockage	24
2.6	Results of peak shear strength values	24
3.1	Values of input parameters used in the analysis	26
3.2	Other Parameters Used in Plaxis Analyses	29
4.1	Factor of safety of various methods of analysis by Slope/w	32
4.2	Values of FOS with respect to the changes of cohesion by Slope/w	35
4.3	Values of FOS with respect to changes of friction angle friction by Slope/w	38
4.4	Values of FOS with respect to changes of groundwater table depth by Slope/w	42
4.5	Values of FOS with respect to changes in cohesion by Plaxis	48
4.6	Values of FOS with respect to changes of angle of friction by Plaxis	55
4.7	Values of FOS with respect to changes of groundwater table depth by Plaxis	58

LIST OF FIGURES

FIGURE NO.	TITLE	PAGE
2.1	Various definitions of the factor of safety (FOS) (Abramson et al. 2002)	9
2.2	Cut slope failure in Skudai (Liew, 2004)	22
2.3	Slope profile before and after failure (Liew, 2004)	22
2.4	Layout of boreholes and instrumentations plan (Liew, 2004)	23
3.1	Slope profile used in the analysis	26
3.2	Node elements and Gauss points	28
3.3	Methodology in slope stability analysis	30
4.1	Critical failure surface by Slope/w for $c'=3.5\text{kPa}$	32
4.2	Critical failure surface by Slope/w for $c'=2.5\text{kPa}$	33
4.3	Critical failure surface by Slope/w for $c'=4.5\text{kPa}$	33
4.4	Critical failure surface by Slope/w for $c'=5.5\text{kPa}$	34
4.5	Critical failure surface by Slope/w for $c'=6.5\text{kPa}$	34
4.6	Factor of safety against the range of cohesion by Slope/w	35
4.7	The critical failure surface by Slope/w for $\phi'=34^\circ$	36
4.8	The critical failure surface by Slope/w for $\phi'=36^\circ$	37
4.9	The critical failure surface by Slope/w for $\phi'=38^\circ$	37
4.10	The critical failure surface by Slope/w for $\phi'=40^\circ$	38
4.11	Factor of safety against the range of angle of friction using Slope/w	39
4.12	Critical failure surface for groundwater table using Slope/w with average depth of 5.8m	40
4.13	Critical failure surface for groundwater table using Slope/w with average depth of 4.8m	40

4.14	Critical failure surface for groundwater table using Slope/w with average depth of 3.8m	41
4.15	Critical failure surface for groundwater table using Slope/w with average depth of 2.8m	41
4.16	Factor of safety against the range of groundwater table depth using Slope/w	42
4.17	Deformation of slope by Plaxis for $c'=2.5\text{kPa}$	43
4.18	Factor of safety by Plaxis when $c'=2.5\text{kPa}$	44
4.19	Deformation of slope by Plaxis for $c'=3.5\text{kPa}$	44
4.20	Factor of safety by Plaxis when $c'=3.5\text{kPa}$	45
4.21	Deformation of slope by Plaxis for $c'=4.5\text{kPa}$	45
4.22	Factor of safety by Plaxis when $c'=4.5\text{kPa}$	46
4.23	Deformation of slope by Plaxis for $c'=5.5\text{kPa}$	46
4.24	Factor of safety by Plaxis when $c'=5.5\text{kPa}$	47
4.25	Deformation of slope by Plaxis for $c'=6.5\text{kPa}$	47
4.26	Factor of safety by Plaxis when $c'=6.5\text{kPa}$	48
4.27	Factor of safety against the range of cohesion by Plaxis	49
4.28	Deformation of slope by Plaxis for $\phi'=32^\circ$	50
4.29	Factor of safety when $\phi'=32^\circ$	50
4.30	Deformation of slope by Plaxis for $\phi'=34^\circ$	51
4.31	Factor of safety when $\phi'=34^\circ$	51
4.32	Deformation of slope by Plaxis for $\phi'=36^\circ$	52
4.33	Factor of safety when $\phi'=36^\circ$	52
4.34	Dformation of slope by Plaxis for $\phi'=38^\circ$	53
4.35.	Factor of safety when $\phi'=38^\circ$	53
4.36	Deformation of slope by Plaxis for $\phi'=40^\circ$	54
4.37	Factor of safety when $\phi'=40^\circ$	54
4.38	Factor of safety against the range of angle of friction using Plaxis	55
4.39	The critical failure surface by Plaxis for groundwater table with average depth of 2.8m	56
4.40	The critical failure surface by Plaxis for groundwater table with average depth of 3.8m	57

4.41	The critical failure surface by Plaxis for groundwater table with average depth of 4.8m	57
4.42	The critical failure surface by Plaxis for groundwater table with average depth of 5.8m	58
4.43	Factor of safety against the range of depth of groundwater table using Plaxis	59
4.44	Comparison of results obtained from Slope/w and Plaxis in term of varying cohesion values.	60
4.45	Comparison of results obtained from Slope/w and Plaxis in term of varying friction angle values.	61
4.46	Comparison of results obtained from Slope/w and Plaxis in term of varying depth of groundwater table	62

LIST OF SYMBOLS

SYMBOL	DESCRIPTION
LEM	Limit equilibrium method
FEM	Finite element method
FOS	Factor of safety
M_{sf}	Incremental multiplier used in Plaxis
CIU	Isotropically Consolidated Undrained
c'	Effective cohesion
ϕ'	Effective friction angle
γ_{dry}	Unit weight above groundwater table
γ_{wet}	Unit weight below groundwater table
k_x	Hydraulic conductivity in x-direction
k_y	Hydraulic conductivity in x-direction
E	Young's modulus
ν	Poisson's ration

CHAPTER 1

INTRODUCTION

1.1 Introduction

Slope instability has been a major concern among Geotechnical engineers. Analysing such problems can sometimes be time consuming and challenging to engineers due to its complexity. For decades, researches have done numerous works on understanding the complex behaviour of slopes and the pattern of failure that comes with each of them. Since then, soil analysis have propelled to the next level where computers are being used as a tool to investigate the behaviour of a soil mass which is subjected to stresses. From the data obtained, one can predict how the slope fails and the location of the slip failure.

There are several factors that contribute to the instability of slope. In construction areas, instability may result due to rainfall, increase in groundwater table and change in stress conditions. Similarly, natural slopes that have been stable for many years may suddenly fail due to changes in geometry, external forces and loss of shear strength (Abramson *et al.* 2002).

The unpredictable behaviour of the soil leads to the development of tools or software to predict the response of slopes under the influence of stresses and surrounding conditions. The conventional limit equilibrium method (LEM) has been known to be the pioneer of analysing problems on slopes. It is a method based on the assumptions about the sliding surface. This method remains popular because of its simplicity and the reduced number of parameters they require, which are slope

geometry, topography, geology, static and dynamic loads, geotechnical parameters and hydrogeologic conditions. However it does not take into account the ground behavior and the safety factors are supposed to be constant along the failure surface (Khadijah Baba *et al.* 2012).

As the technology advances and the needs to study more on the complexity of slope behaviour are highly required, another method of analysis has been developed which is the finite element method (FEM). It is a form of numerical technique for finding approximate solutions of partial differential equations (PDE) and integral equations. While this method is of course useful in dealing with complex geometry, they are also designed to analysis problems which cover sequences of loading, presence of material for reinforcement, action of water and laws for complexes soil behaviour.

According to Abramson *et al.* (2002), the primary purpose of slope stability analysis is to contribute to the safe and economic design of excavations, embankments, earth dams, landfills, and spoil heaps. Slope stability evaluations are concerned with identifying critical geological, material, environmental, and economic parameters that will affect the project, as well as understanding the nature, magnitude, and frequency of potential slope problems. When dealing with slopes in general and slope stability analysis in particular, previous geological and geotechnical experience in an area is valuable.

1.2 Background of Study

In slope stability analysis, two methods have been developed which are the conventional limit equilibrium method and finite element method. These two concepts have helped Geotechnical Engineers to solve instability of slope problems for many years. Due to their differences in terms of the analysis itself and the parameters used, the result obtained would definitely be different. Therefore, a performance study should be made to compare these two methods by observing the factor of safety results that they produce.

1.3 Objectives

The aim of this study is to find the differences in results when analysing a particular slope stability problem with both limit equilibrium method and finite element method. The objectives of this matter are:

- 1) Sensitivity analysis for shear strength parameters and groundwater table for Slope/w and Plaxis software.
- 2) Analysis of slope stability methods for both limit equilibrium and finite element method and comparisons of results.

1.4 Scope of Study

This research is focused on applying and comparing the limit equilibrium and finite element method in analysing slope stability. The method of carrying out this study is by using LEM and FEM softwares which are Slope/W and Plaxis respectively. The soil model is to be done in 2-dimensional. Evaluations and investigations of shear strength parameters in slope stability analysis are to be made. The condition of the soil is taken to be fully saturated.

Comparisons of results obtained are to be done with respect to the results of analysis done from a true case study. The selected case study is a slope profile located at Universiti Teknologi Malaysia in Skudai, Malaysia.

1.5 Significance of Study

Limit equilibrium approach has been widely used for most slope stability analyses alongside with the emergence of finite element method. This is due to its simplicity and that has made it to be reliable in most analyses. However, the application of finite element method is still new and that the knowledge towards its

reliability is still very limited. This study provides the information needed in understanding both techniques in a clearer view. Information on advantages and disadvantages of both methods are also included in this thesis. Understanding the limitations of each method is crucial in order to get the most reliable results.

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