ITERATIVE PROCESS TO IMPROVE SIMPLE ADAPTIVE SUBDIVISION SURFACES METHOD FOR TRIANGULAR MESHES

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To my beloved parent

Husain Bin Sarkan and RamlahBinti Ahmad

who taught me never give up

Thank you

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بِسْمِ ٱللَّهِ ٱلرَّحْمَٰنِ ٱلرَّحِيمِ

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ABSTRACT

Subdivision surface is a refinement method applied to the entire polygon mesh in order to produce a smooth surface in any 3D object. This method has issues in terms of time and memory consumption due to the fact that it computes and renders all of the vertices of the mesh during the subdivision process. To overcome this issue, adaptive subdivision surface method is used because it would subdivide only at the required vertices of selected areas and decrease the number of polygons on the mesh. However, a related issue in the use of this method has risen,which is the determination of a suitable threshold value to be used for selecting the subdivision area. Besides that, the use of a higher level of subdivision will lead to an increase in the number of polygons and this would lead to heavy computational load and raise high undulation on the curve surface. To address these issues, Iterative Adaptive Subdivision Surface (IteAS) method is proposed. In this method, the area to be subdivided will be identified by using the threshold value. To get the optimal threshold value, a new formula based on statistical evaluation was embedded in the proposed method. Here, the threshold value is defined as the average value of a normal vector between the rates of 0° to 180° in a 3D object. The value will be compared with the angle between normal vectors, if the threshold value is greater than the angle, the surface will be subdivided by using Butterfly subdivision scheme. The results from this process will determine the number and levels of iteratives in the subdivision surface. The number of iteratives relieson the surface shape of the 3D object which is either a curve or flat surface. The number of iteratives will be higher for a flat surface as compared to a curve surface.In this research, IteAS can reduce 18% to 25% number of polygons as well as 1% to 3% use of computational memory whilst retaining the smoothness of the surface. This IteAS method has been proven to improve the present enhancement process.

ABSTRAK

Subbahagian permukaan merupakan kaedah pembaikan digunakan pada seluruh jaringan poligon objek tiga dimensi (3D) untuk menghasilkan permukaan yang halus. Kaedah ini mempunyai isu berkaitan penggunaan memori dan masa disebabkan ia menghitung dan menjana semua titik pada jaringan ketika proses subbahagian.Bagi menyelesaikan isu ini, kaedah adaptif subbahagian permukaan digunakan kerana ia hanya melakukan subbahagian pada jaringan poligon yang diperlukan sahajaiaitu pada kawasan terpilih dan mengurangkan bilangan poligon pada jaringan. Namun, isu lain yangtimbul ialahmenentukan nilai ambang yang sesuai untuk digunakan ketika memilih kawasan subbahagian. Disamping itu, penggunaan proses subbahagian ke peringkat tertinggi akan membawa kepada peningkatan bilangan poligon dan bebanan pengiraan serta menimbulkan gelembung yang ketara pada lengkungan permukaan. Bagi mengatasinya,kaedah Ulangan Adaptif Subbahagian Permukaan (IteAS) dihasilkan. Dalam kaedah ini, kawasan yang dipilih untuk proses subbahagian akan dikenalpasti dengan menggunakan nilai ambang. Untuk mendapatkan nilai ambang yang optimum, formula baru berdasarkan penilaian statistikal telah digabungkan dalam kaedah cadangan ini. Di sini, nilai ambang diperolehi daripada nilai purata vektor normal di antara kadar 0° hingga 180° dalam objek 3D. Nilai ambangakan dibandingkan dengan sudut di antara vektor normal, jika nilai ambang lebih besar daripada sudut, maka permukaan akan melakukan proses subbahagian menggunakan skima subbahagian *Butterfly*. Hasil dari proses akan menentukan bilangan dan peringkat pengulangan subbahagian. Bilangan pengulangan bergantung pada bentuk permukaan objek 3D samada ia berlengkung atau rata. Bilangan pengulangan lebih tinggi pada objek berpermukaan rata berbanding yang berlengkung. Dalam kajian ini, IteAS boleh mengurangkan 18% hingga 25% bilangan poligon dan 1% hingga 3% penggunaan memori,pada masa yang sama mampu mengekalkan kehalusan permukaan. Kaedah IteAS ini telah terbukti dapat memperbaiki proses peningkatan terkini

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LIST OF ABBREVIATIONS

CAD	-	Computer-Aided Design
2D	-	Two-Dimensional
3D	-	Three-Dimensional
NURBS	-	Non-Uniform Rational Basis Splines
FPS	-	Frame per-second
DOI	-	Degree of Interest (DOI
CA	-	Conical Angle
RGB	-	Red Green Blue
LOD	-	Level of Detail
LMR	-	Local Mesh Realignment
GPU	-	Graphic Processing Unit
PC	-	Personal Computer
VF	-	Vertex Flatness
RAM	-	Random Access Memory
ABF	-	Angle Between Faces
MB	-	Megabyte (1,000,000 Bytes)
TXT	-	Text File

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CHAPTER 1

INTRODUCTION

1.1 Overview

Computer graphics areas had widely used because it made computers easier to interact with, better for understanding and interpreting such as graphics presentation, computer-aided design (CAD), image processing, simulation & virtual reality, and entertainment. The most attractive and widely used application in this field is computer games development and 3D animation industry. In fact, the needs of computer graphicsdevelopment such as modelling and rendering phase had revolutionized became more sophisticated. Fortunately, development of the latest technology has given more benefit to this field to deliver visual appearances and produce real-time environment with 3D contents (Edward, 2008).

Nowadays, subdivision surface is a great surface modelling technique for creating 3D contents. Previously, nonuniform rational basis splines (NURBS) technique was widely used for generating and representing curves and surfaces. Nevertheless, NURBS have some disadvantages that have already been identified. First, NURBS can be ensured that modification on NURBS surface was quite expensive in term of difficulty to handle arbitrary topology so well. Second, it was difficult to keep smoothness of the NURBS surface in seams. Third, the animation, which uses NURBS technique could cause deep hole patchwork (Peter and Zorin,1998). Subdivison surface method has become increasingly popular because it can overcome weaknesses in

NURBS especially in handling arbitrary topology. Smoothness of subdivision surface can be easily maintained because it dependent to subdivision algorithm.

A basic idea of subdivision method had been proposed by G de Rham which uses corner cutting to produce a smooth curve. G de Rahm described a 2D subdivision scheme in 1947 and was rediscovered by Chaikin (1974). The concept is extended to subdivision schemes by two separate groups in 1978 which were Catmull Clark and Doo Sabin. They implied the beginning of subdivision for modelling surface. Subsequent works were with Loop (1987) and Dyn (1990).Recently, subdivision surface has found the way to expand its wing in computer graphic applications and computer aided geometric design (CAGD)(Zorin,2000).

Subdivision surfaces had already been used in many areas especially including the animation movies development. NURBS was applied in the first Toy Story movie in 1995. Then, subdivision surfaces were contributed in a Bug's Life movie in 1998. Pixar started demonstrating subdivision surfaces in 1997 in a short film called Geri's Game (DeRose, 1998). From 1999 onwards, everything they worked on was with subdivision surfaces, which are Toy Story 2, Monsters Inc and Finding Nemo.Maya, Rhino, 3D Max and Light Wave modelling and animation software also used subdivision as a great tool with the same purpose(Peter and Zorin, 1998; Peter, 2009).

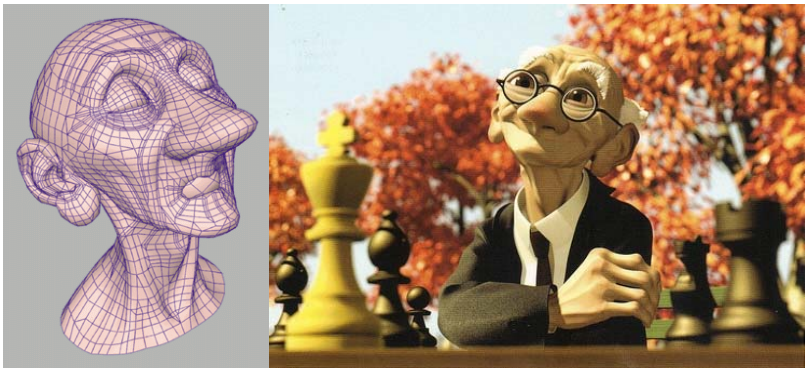

Figure 1.1 A Geri's Game (DeRose, 1998)

1.2 Problem Background

Generally, subdivision surface is a refinement operation that is uniformly applied to a polygon mesh to produce smooth surfaces. There are few steps needed to be made to subdivide and refine surfaces become smooth. Basically, a midpoint between two points from coarse mesh needs to be calculated. New vertices were inserted at the midpoint, and new edges were created to form a new mesh. Then, the mesh needs to be refined by applying a set of subdivision rules (Warren, 1995).

Subdivision surfaces consist of several schemes that are used to produce smooth surfaces which are Doo Sabin, Catmull Clark, Loop, Butterfly, Kobbelt subdivision scheme. Each scheme has its own rules and properties to produce smoothness such as mesh type, continuity, approximation or interpolation, and evaluation mask (Doo and Sabin, 1978; Catmull and Clark, 1978; Loop, 1987; Dyn et al, 1990; Kobbelt, 2000; Zorin, 2000).

However, there are two main issues that arose in subdivision surfaces while accomplishing the objective of subdivision surfaces. The issues are geometry modeling and rendering process.

In geometric modeling, two sub issues that arose were addressed to implementation of subdivision algorithms (Dyn and Levin, 2002). First, to compute the limit values and limit derivatives at dyadic points through the subdivision process for any refinement level.Second, to get the best possible approximation of desired surface from choosing initial control points from a given scheme with the highest possible approximation power.

Rendering issues in subdivision surfaces is the problem that happens when the process of subdivision surface is slow, and rendering process takes a lot of memory and time. The complexity of triangular surfaces can produce a lot of computation. At a higher level of subdivision, the number of polygons increases and will lead a heavy computational load and also rapidly exceeds the memory limitations (Pulli and Segal, 1996; Lomont, 2007).

The focus of this research is to address issues in the rendering process which is the process of subdivision surface is slow, and rendering process takes a lot of memory and time. One approach to solve these issues has been studied to find out if only necessary areas can be subdivided and can avoid unnecessary refinement on other areas of the mesh while approximated to the limit surface. In certain cases, subdivisions of the entire mesh are not necessary or required. Adaptive subdivision is an approach that provides a rule to find out whether a given polygon meshes required for being subdivided to further subdivision at the next step of subdivision (Amresh et.al, 2001).

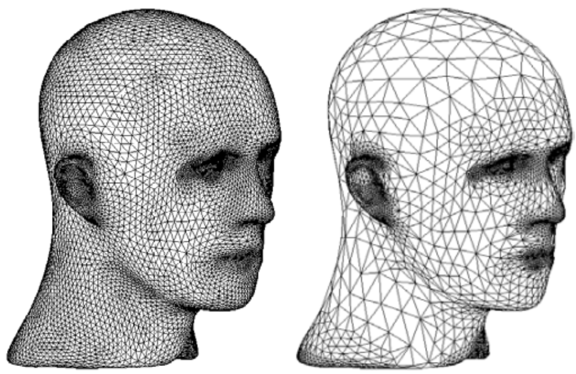

Figure 1.2 The comparison between regular subdivision and adaptive subdivision (Pakdel and Samavati, 2004)

The concept of adaptive subdivision surface is a refinement to the certain areas of the control mesh. Adaptive subdivision produces smooth surfaces precisely same as well as the regular subdivision to the entire mesh although only subdivides at the selected area. Therefore, all vertices must have the similar connectivity within the selected area as a regular subdivision.

Although adaptive subdivision was some of simplification and interest feature for subdivision surfaces, it has two drawbacks which is to compute and define a selection area to be subdivided while prevent unnecessary locations and to avoid cracks that are created from the differences subdivision depth of adjacent faces generations meet at an edge. Cracks must be removed because it will appear some artifact at the mesh (Pakdel and Samavati, 2004).

Previously, adaptive subdivision methods have been observed which can be categorized by two ways which is identifying the surfaces by vertex split or face split (Amresh*et.al*, 2001). A crack was not produced when refinement process based on vertex split. However, adaptive subdivision method based on face split will produce the cracks. Then, related works about adaptive subdivision based on selection area and handling cracks also have been reviewed.

Selection area must be determined on which areas need to be subdivided to avoid unnecessary areas. Based on the idea, Amresh*et.al* (2001) used the angle between normal vectors of the faces and its adjacent to decide whether this faces required for being subdivided to further subdivision process. Meyer *et.al* (2003) have introduced Gaussian curvature analysis to define high curvature area. Higher curvature area needs more refinement because it contains more details than flat areas. The Degree of Interest (DoI) function is to decide refinements of the mesh that are required or not by comparing to a certain threshold value (Isenberg *et.al*, 2003). Liu and Kondo proposed a new rule called as conical angle (CA), which used error estimation of the largest angle between the normal vectors of adjacent faces of a vertex(Liu and Kondo, 2004). Wu *et.al* (2005) proposed an approach in adaptive subdivision method which utilizes local refinement for vertices or faces by local flatness which set a reasonable tolerance limits. The RGB (Red-Green-Blue) subdivision proposed by Panozzo and Puppo (2009). This method extends red-green triangulation with the Modified Butterfly subdivision schemes to a fully dynamic adaptive scheme supporting both local refinement and coarsening. They further proposed an adaptive interpolation scheme that can be used effectively and efficiently in selective editing of meshes and amenable with the same subdivision scheme. They also present a method to support both adaptive refinement and coarsening (Panozzo and Puppo, 2010).

After the selection area work efficiently, another problem that will occur needs to be highlighted. While a subdivision process working based on face split, cracks are created between subdivided and non-subdivided areas. Based on ideas for handling cracks, a new method had been proposed that called as red-green triangulation which removing cracks by inserting new edge into the triangular mesh was developed. This method consists of two conditions which are green triangulation and red triangulation. Green triangulation is bisecting for faces with one crack, while red triangulation is quadrisect faces with more than one cracks (Bank *et.al*, 1983). According to Amresh*et.al* (2001), a simple triangulation method was introduced to remove cracks by bisecting an adjacent face that has not been subdivided. Seeger *et.al*, (2001) have discussed how to apply simplest mesh modification with triangle mesh subdivision by vertex splitting. The dyadic refinement is decomposed into atomic local operations based on the popular vertex split operation which called as quarks. Liu and Kondo (2004) proposed method that was identified which vertex in a face is labeled as '*dead*'. They also proposed appropriate mesh refinements based on the three vertices properties. They address these cracks problem and provide a solution which is called local mesh realignment (LMR).

Pakdel and Samavati (2004) proposed a new adaptive subdivision algorithm that subdivides the adjacent faces around the selected area. This method used Loop subdivision scheme to handle cracks and more efficient rather than two previous methods which are red-green triangulation and restricted mesh in effect creating a surface that gradually increases in subdivision depth. In 2007, they introduced a new incremental adaptive subdivision method for triangular mesh. It produces surfaces from coarse mesh to fine model which have consistent connectivity and geometry with steadily increases in subdivision depth. The combination methods of restricting mesh and limiting the difference depth of adjacent faces are potential to obtain better behaved adaptive subdivision especially for handling cracks.

There are existing methods that deal with issues of the selection area and handling cracks. However, they are not seriously looking towards on addressing memory consumption issue in rendering. The related works regarding to the memory consumption issue in adaptive subdivision were observed.

Amor *et.al* (2000) presented architecture to implement the adaptive subdivision of triangular meshes according to the surface complexity which the coarse triangle meshes is tessellated described by the displacement map. This was a standard architecture and characterized by data management efficiency that minimizes the data storage. It also avoids remaining cycles that would be linked with the multiple data accesses needed for each subdivision step. In 2001, this adaptive displacement mapping algorithms in hardware was improved. They presented a meshing scheme and new architecture that practicable in hardware that brings for speedy access using a small memory.

New adaptive rendering method for general Catmull-Clark subdivision surfaces is based on direct evaluation of the limit surface to produce an inscribed polyhedron. Inscribed approximation typically provides quicker convergent rate that produces less polygons at the last rendering process rather than circumscribed approximation. This method implements evaluation of the limit surface at the points that are required at the final rendering process. Therefore, this method is efficient in memory and speed (Lai *et.al*, 2005).

1.3 Problem Statement

Generally, adaptive subdivision is a method that subdivides only at certain areas of the mesh. While, the rest were retaining original polygons. Although adaptive subdivision occurs at the selected areas, the quality of produced surfaces which is their smoothness can be preserved similar as well as regular subdivision. The main advantage of adaptive subdivision is that it can reduce the number of polygons compared to regular subdivision. Nevertheless, adaptive subdivision process burdened from two issues; calculations need to be done to define areas that are required to be subdivided while preventing unnecessary locations and cracks occurring from the difference of subdivision depth between the subdivide and nonsubdivide areas should be avoided.Appropriate mesh refinements based on its neighboring faces properties should be proposed. Several methods have been proposed to overcome these issues. The methods proposed to determine face requirement for subdivision and can be efficiently computed for selection area such as Dihedral Angle and Gaussian curvature. Several methods also have been proposed to handle crack in adaptive subdivision surfaces with steadily change of resolution all over the surface with consistent connectivity and geometry such as Simple Triangulation, Red Green Triangulation and Incremental Adaptive. Unfortunately, the result of adaptive subdivision when it reaches the higher level of subdivision still brings the problem with memory consumption (Pulli and Segal, 1996; Wu *et.al*, 2005; Lomont, 2007; Patney and Owens, 2008; Zhao, F. and X. Ai. 2010). There are needs of enhancement process which can reduce cost of memory consumption.

Thus, the issue on how the memory could be optimised to reduce the rendering time even adaptive subdivision process reaches the higher level was discussed. This research is based on Wu *et.al*, 2005 and Zhao, F. and X. Ai, (2010) methods. Their method proposed a process of adaptive subdivision surfaces method but not focused on rendering process. This research is aimed to enhance their methods on reduce memory consumption and to improve the process of adaptive subdivision surfaces method.

1.4 Research Goal

The aim of this research is to enhance current process of adaptive subdivision surfaces method that can reduce the number of polygons and the cost of memory usage in optimal results.

1.5 Research Objective

To achieve the goal of this research, the following objectives are stated:

Objective:

1) To enhance the adaptive subdivision method by the iterative process for improving speeds and maintains smooth graphic appearances
2) To propose the new methods of adaptive subdivision method by determining threshold value for selection area.
3) To produce an enhanced process of adaptive subdivision with prototype development.

1.6 Research Scope

The scopes for this research are:

1. The proposed method focus on the memory consumption for subdivision surfaces rendering process.
2. This research used a triangular meshes data (data triangulation file format).
3. To produce a smooth surfaces from coarse mesh using Butterfly subdivision scheme.
4. Other features such as texturing, lighting and shading were not discussed.
5. This research is not focused on subdivision sharp feature.
6. This research is not focused for handling cracks.
7. This research only focused on basic adaptive subdivision method and does not covers advanced method such as incremental adaptive and RGB triangulation.

1.7 Research Justification

This research is particularly important in the computer graphics field. Some of the famous applications such as animation, film production and games development are taken into account the modeling process that related with the subdivision process. For example is a short film story, Geri's Game (Zorin, 2000). Recent interest in adaptive subdivision surfaces has inspired many groups to conduct research in this area. With the adaptive subdivision surfaces, the developer can represent details of polygonal mesh in a way to looks smooth and realistic such a real world while a number of the polygon can be reduced that it may be easy for rendering problems. This research was discussed several approaches that focused on selection criteria and handling cracks that appear while an adaptive approach adapted to the subdivision process (Bank *et.al*, 1983; Amresh*et.al*, 2001; Pakdel and Samavati, 2004; and Puppo and Panozzo, 2009). An enhancement process in adaptive subdivision method has been proposed and lead to the efficiency for better result than the previous method.

1.8 Thesis Organization

The state of the art in the areas of computer graphics especially in subdivision surfaces method was described in Chapter 1. The problem on this field was stated; alongside with the objectives, goal, scope and justification of this research.

Chapter 2 reviews the literature review of previous related to this research study. This chapter consists of (1) Subdivision Surfaces (2) Adaptive Subdivision: and (3) Comparative Study.

Research methodology of the adaptive in subdivision surfaces technique for the smoothness of the 3D object was discussed in Chapter 3. This chapter also describes the research methodology and proposed design method. Both hardware and software specification requirements are discussed here.

Chapter 4 is a detailed description on the proposed method which is the iterative process on basic adaptive subdivision method based on framework given in Chapter 3. The underlying ideas are explained in detail and mathematical formulations are derived. All necessary mathematical formulas are given and explained.

In chapter 5, an iterative adaptive subdivision method is implemented. One prototype has been developed to implement and test the objects. Testing and evaluation for several objects was done to justify the efficiency of the new method based on comparison to the previous one.

Finally, the conclusion all the chapters and the contribution, findings and future works were discussed in Chapter 6.

REFERENCES

Amor, M.,B´oo, M., Doggett,M.,Hirche, J., and Strasser, W. (2000).*A Meshing Scheme for Memory Efficient Adaptive Rendering of Subdivision Surfaces*.*WSI/GRIS*.Universit¨atT¨ubingen.

Amresh, A., Farin, G. andRazdan, A. (2003).AdaptiveSubdivision Schemes for Triangular Meshes.*Hierarchical and Geometric Methods in Scientific Visualization*.319–327.

Bank,E.R., Sherman,A. H., and Weiser,A. (1983).Some Refinement Algorithms and Data Structures for Regular Local Mesh Refinement.*Scientific Computing, IMACS/North-Holland*.3-17.

Bates, D.(2009).*Adaptive General-Degree Subdivision for 3D Surfaces*. Robinson College. 42.

Bernardini, F. and Rushmeier, H. (2002).The 3D Model Acquisition Pipeline.*Computer Graphics Forum :TheEurographics Association and Blackwell Publishers Ltd.* 21(2), 149–172.

Bolz, J. and Schr¨oder, P.(2002).Rapid Evaluation of Catmull-Clark Subdivision Surfaces.*Proceedings of the Seventh International Conference on3D Web Technology 2002*. Tempe, Arizona, USA.

Booy,M.,Amoryy,M.,Doggett, M.,Hirche, J.,andStrasser,W. (2001).Hardware Support for Adaptive Subdivision Surface Rendering. *Proceedings of the ACM Siggraph/Eurographics2001, New York USA*.

Catmull, E. and Clark, J. (1978).Recursively Generated B-Spline Surfaces on Arbitrary Topological Meshes.*Computer Aided Design*.10:350-355.

Chen, Z.,Luo, X., and Ling, R. (2007).An Adaptive Subdivision Method Based On Limit Surface Normal.*Computer Aided Design and Computer Graphic 10th IEEE International Conference.*

Cope, K. (2002).Female head (ear). A *study in subdivision surfaces in Mirai*.http://www.ozcot.com/mod_ears.htm.

DeRose, T., Kass, M., and Truong, T. (1998). Subdivision Surfaces in Character Animation.*Proceedings of the 25th Annual Conference Computer Graphics and Interactive Techniques*,*New York, NY: ACM Press*. 32:85–94.

Doo, D., and Sabin, M. (1978).Behaviour of Recursive Division Surfaces near Extraordinary Points.*Computer Aided Design*. 10:356-360.

Dyn, N., Gregory, J., and Levin, D. (1990).AButtery Subdivision Scheme for Surface Interpolation with Tension Control.*ACM Trans. Graph*.9:160-169.

Edward A. (2008). *Interactive Computer Graphic; ATop-Down Approach Using OpenGL*. Pearson International Edition, Fifth Edition.

Funkhouser, T. 2002.*Overview of 3D Object Representations*.Princeton University.

Gardere, L. and Landry,B. (2002). Project 2: Subdivision Surfaces. http://www.cc.gatech.edu/~lgardere/graphics/SubdivisionSurfaces.html.

Isenberg, T., Hartmann,K., and K¨onig, H. (2003).Interest Value Driven Adaptive Subdivision.*Simulation und Visualisierung, SCS European Publishing House*.139-149.

Jean E., and Schweitzer.(1996). *Analysis and Application of Subdivision Surfaces*. Department of Computer Science and Engineering, University of Washington, August 1996.

Kobbelt, L.P., Bischoff,S., and Botsch, M.,(2000).Geometric Modeling Based on Polygonal Meshes.*TheEurographics Association 2000*.

Kobbelt, L. (2000). √3 Subdivision.*Computer Graphics Proceedings,SIGGRAPH 2000*.103-112.

Kovacs,D.,Mtchell, J., Drone, S., and Zorin,D. (2009). Real Time Creased Approximate Subdivision Surfaces. *Proceedings of the 2009 Symposium on Interactive 3D Graphics and Games.*155-160.

Lai, S. and Cheng, F.F. (2005).Adaptive Rendering of Catmull-Clark Subdivision Surfaces.*Ninth International Conference on Computer Aided Design and Computer Graphics (CAD/CG 2005)*.

Lavou´e, G.(2008). *Basic Background in 3D Object Processing*.John Wiley & Sons, Ltd.

Li, G., and Ma, W. (2009).Adaptive Refinement for Unified Subdivisions with Sharp Features.*Computer-Aided Design and Applications*.

Liu, W., and Kondo,K. (2004). An Adaptive Scheme for Subdivision Surfaces based on Triangular Meshes. *Journal for Geometry and Graphics*.8(1),69-80.

Loop, C. (1987).*SmoothSubdivision Surfaces Based on Triangles*.Master'sThesis.University of Utah, Department of Mathematics.

Lomont, C.(2007).*High Performance Subdivision Surfaces*.www.lomont.org.

Meyer, M., Desbrun, M.,Schr¨oder, P., and Barr,A.H. (2003).Discrete Differential-Geometry Operators for Triangulated 2-Manifolds.*Visualization and Mathematics III: Heidelberg: Springer-Verlag.*

Muller, H. and Jaeschke,R. (1998).Adaptive Subdivision Curves and Surfaces.*Proceeding Computer Graphics International*.48-58.

Pakdel, H.R., and Samavati, F. (2004).Incremental Adaptive Loop Subdivision.*International Conference on Computational Science and Its Applications 2004*.Springer-Verlag Berlin Heidelberg.

Pakdel, H.R., and Samavati, F. (2005).IncrementalCatmull-Clark Subdivision.*3-D Digital Imaging and Modeling.*

Pakdel, H.R., and Samavati, F. (2007).IncrementalSubdivision for Triangle Meshes.*International Journal Computational Science and Engineering*.3(1) 13.

Panozzo, D. and Puppo,E. (2010).Adaptive LOD Editing of Quad Meshes.*The 7th International Conference on Virtual Reality, Computer Graphics, Visualization and Interaction*.Franschhoek, South Africa: ACM Siggraph.

Patney, A. and Owens,J.D. (2008).Real-Time Reyes-Style Adaptive Surface Subdivision.*ACM Transactions on Graphics*.27(5).

Peters, J.O., and Ni,T. (2009).*Efficient Substitutes for Subdivision Surfaces*.University of Florida.

Piegl, L. (1991).On NURBS: A Survey.*ComputerGraphics and Applications, IEEE*.11(1).

Pulli, K., and Segal, M.(1996).*Fast Rendering of Subdivision Surfaces*.University of Washington, Seattle.

Puppo, E. and Panozzo, D. (2009).RGBSubdivision.*IEEE Transactions on Visualization and Computer Graphics (TVCG)*. 15(2) 295-310.

Razdan,A. and Bae, M. (2002).*A Hybrid Approach to Feature Segmentation of Triangle Meshes*.PRISMand Computer Science and Engineering, Arizona State University.Elsevier.

Rossignac, J., Safonova, A., and Szymczak, A. (2001). 3D Compression Made Simple: Edgebreaker on a Corner-Table.*Proceedings of Shape Modeling International Conference, Genoa, Italy.*

Samavati, F.(2009).*Modeling for Computer Graphics* CPSC 589/689 Course Notes. University of Calgary.

Seeger, S., Hormann, K., H¨ausler, G.,andGreiner,G. (2001).A Sub-Atomic Subdivision Approach.*Proceedings of the Vision Modeling and Visualization Conference 2001*. Stuttgart, Germany: Aka GmbH.

Severn, A. andSamavati, F. (2006).*Fast Intersections for Subdivision Surfaces*.Department of Computer Science, University of Calgary.*ICCSA*.Springer-Verlag Berlin Heidelberg,

Smith, C. (2006).*On Vertex-Vertex Systems and Their Use in Geometric and Biological Modelling*. Department of Computer Science, University of Calgary: Calgary, Alberta. 216.

Stam, J. (1998).ExactEvaluation of Catmull-Clark Subdivision Surfaces at Arbitrary Parameter Values.*Proceedings of SIGGRAPH 98*.395–404.

Suresh, S. and Agam, G. (2007).Incremental Adaptive Subdivision of Mesh Surfaces.*Proceedings of SPIE-IS&T Electronic Imaging.*

Theoharis, T., Papaioannou,G.,Platis,N.,andPatrikalakis,N.M. (2008).*Graphics & Visualization – Principles and Algortihms*, A K Peters Ltd. 191-299.

Tobler, R.F. and S. Maierhofer,. 2006.A Mesh Data Structure for Rendering and Subdivision.*Short Communications Proceedings WSCG'2006*. Plzen, Czech Republic.

Umlauf, G. (2003).A Technique for Verifying the Smoothness of Subdivision Schemes.*Geometric Modeling and Computing*.1-4.

Warren, J.(1995).*Subdivision Methods for Geometric Design*.Department of Computer Science, Rice University.

Warren,J.,andSchaefer,S. (2004). A Factored Approach to Subdivision Surfaces.*IEEE Computer Graphics and Applications.* 24(3)74-81. May/June 2004.

Wu, J., Liu, W.,andWang, T. (2005). Adaptive Refinement Scheme for Subdivision Surfaces based on Triangular Meshes. *Ninth International Conference on Computer Aided Design and Computer Graphics. 2005: IEEE*.

Wu, X., and Peters, J. (2005).An Accurate Error Measure for Adaptive Subdivision Surfaces.*Proceedings of the International Conference on Shape Modeling and Applications.*Cambridge, Massachusetts IEEE Computer Society.

Zhao, F. and X. Ai.(2010).An Adjustable Adaptive Subdivision Surface Modeling Based onTriangle Meshes. *Applied Mechanics and Materials*. 26(28) 702-705.

Zhen-Yu, Y., Shao-Wen, X., and Wei-Yong, W.(2010).An Adaptive Loop Subdivision Algorithm for Keeping Local Features With Mesh Segmentation. *IEEE*.

Zorin, D. (1996).*C^k Continuity of Subdivision Surfaces*.Department of Computer Science, California Institute of Technology.

Zorin, D., and Kristjansson, D. (2002).Evaluation of Piecewise Smooth Subdivision Surfaces.*The Visual Computer*. 18, 5–6, 299–315.

Zorin, D.(1998).*Stationary Subdivision and Multiresolution Surface Representations*. California Institute of Technology: Pasadena, California.

Zorin, D., Schr¨oder, and P., Sweldens, W.(1996). Interpolating Subdivision for Meshes with Arbitrary Topology.*Computer Graphic Proceeding (SIGGRAPH 1996).*189-192.

Zorin, D., Schr¨oder, P.,andSweldens, W., (1997).InteractiveMultiresolution Mesh Editing.*Proceedings of the 24th International Conference on Computer Graphics and Interactive Techniques*. ACM Press/Addison-Wesley Publishing Co. 259-268.

Zorin, D. and Schr¨oder,P. (2000*).Subdivision for Modeling and Animation.ACM SIGGRAPH*. Course Notes: New York University Caltech.