FREE VIBRATION OF SYMMETRIC ANGLE-PLY LAMINATED TRUNCATED CONICAL SHELLS USING SPLINE FUNCTION

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To my beloved husband and family members who gave me the loves, encouragement and endless support

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ABSTRACT

Free vibration of symmetric angle-ply laminated truncated conical shells using spline function technique is studied. The equations of motion for conical shells can be obtained by deriving and substituting the proper Lamé parameters of conical shells in the general shell equations. The displacements are assumed in the separable form. Since no closed form solutions are generally possible, a numerical solution procedure is adopted in which the displacement functions are approximated by cubic and quintic splines. A generalized eigenvalue problem is obtained which is solved numerically for an eigenfrequency parameter and an associated eigenvector of spline coefficients. Parametric studies are made to investigate the effect of circumferential mode number, length ratio and cone angle using different number of layers with different ply orientations on the frequency parameter under two different boundary conditions.

ABSTRAK

Getaran bebas bagi simetri laminar kon yang dipotong dengan menggunakan teknik splin dikaji. Persamaan gerakan untuk rangka kon boleh diperolehi dengan mendapatkan dan menggantikan parameter Lamé bagi rangka kon dalam persamaan rangka umum. Anjakan kon ini diandaikan dalam bentuk pengasingan. Oleh kerana ianya tiada penyelesaian dalam bentuk tertutup, prosedur penyelesaian berangka digunakan di mana fungsi anjakan dianggarkan oleh splin padu dan kuintik. Satu masalah nilai eigen teritlak diperolehi di mana ianya dapat diselesaikan secara berangka bagi parameter frekuensi eigen dan vektor eigen yang dikaitkan dengan bilangan pokok pekali splin. Kajian parametrik dibuat untuk mengkaji kesan bilangan lilitan mod, nisbah panjang dan sudut kon dengan menggunakan bilangan dan orientasi lapisan yang berbeza ke atas parameter frekuensi di bawah dua keadaan yang berbeza.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF SYMBOLS

x, y, z	Longitudinal, rotational and transverse displacement
u, v, w	Longitudinal, rotational and transverse displacements
$\dot{\varepsilon}_x, \dot{\varepsilon}_\theta, \dot{\varepsilon}_{x\theta}$	Midplane strains
$\kappa_x, \kappa_\theta, \kappa_{x\theta}$	Midplane curvature
β_x, β_θ	Angular displacements of normal to middle surface
$N_x, N_\theta, N_{x\theta}$	Stress resultants
$M_x, M_\theta, M_{x\theta}$	Moment resultants
Q_x, Q_θ	Transverse shear force
q_x, q_θ	Surface load
R_1, R_2	Principal Radii
A, B	Lamé parameters
A_{ij}	Extensional stiffness matrices
B_{ij}	Coupling stiffness matrices
D_{ij}	Bending stiffness matrices
ρ	Mass density
h	thickness
λ	Frequency parameter
ω	Angular frequency

ℓ	length of the cone
γ	ratio of thickness to radius
γ'	ratio of thickness to a length
n	circumferential mode number
α	cone angle
t	time

LIST OF APPENDICES

CHAPTER 1

INTRODUCTION

1.1 Project background

Over the past few decades, thin shells as a structural element has been widely used in many engineering fields particularly in civil, architectural, aeronautical and marine engineering (see Figure 1.1). The extensive use of shell structures in engineering is because of their advance properties of efficiency of load-carrying behavior, high degree of reserved strength and structural integrity, high strength-to-weight ratio, very high stiffness-to-weight ratio and containment of space. Examples of shell structures in civil and architectural engineering are large-span roofs, elevated water tanks and arch domes. In aeronautical, the use of shells structures can be seen in aircrafts, missiles and rocket. Other than that, ships and submarines are the examples of the use of shells in marine engineering.

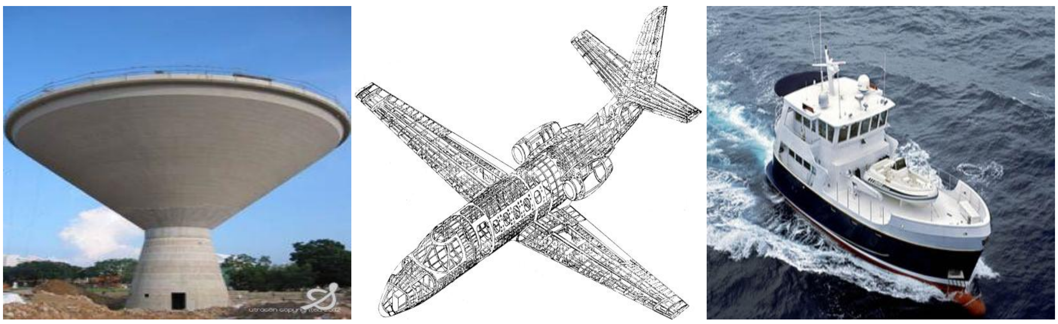

Figure 1.1: Examples of shells structure in engineering

Shell structures made up of composite materials always meet the demands of today's advanced technologies. Composite materials are becoming an essential part of latest material because of their advantages over non composites materials. They are used in everyday applications such as squash racket, fishing poles, helicopter blades, propeller, reservoirs and many more. The characteristics of composite material can be enhanced if the material is stacked by a proper orientation. If the material consists of several unidirectional layers, this may be optimum for unidirectional loads, but very poor to withstand complex loading and stiffness. This problem can be overcome by layering them at different angles for a given loading and stiffness requirements. This advantage is usually not found in other types of structure. In addition, laminated structures also offer the advantage of combining the appropriate constituent layers to achieve better properties for the whole structures.

The most common shell theories are those based on linear elasticity concepts. In three dimensional (3D) theory of elasticity, the fundamental equations occurs in three broad categories; equation of motion, strain-displacement relations and Hooke's Law. However, the solution of problem in three dimensional theory of elasticity is very difficult and complicated. Thus, an alternative simplified method is implemented. According to this method, the 3D problem of shell equilibrium and straining may be reduced to the analysis of its middle surface only where the given shell as a thin plate and may be regarded as 2D body. The problem now can be reduced to 2D problem. In the development of thin shell theories, simplification is accomplished by reducing the shell problems to the study of deformations of the middle surface. The group of simplifying assumptions that provide a reasonable description of the behavior of thin elastic shells known as Love's first approximation theory and the three categories equations mentioned earlier has a reduced counterpart in the theory of thin elastic shells. According to the theory:

1. The thickness of the shell is small compared with other dimensions.
2. The transverse normal stress is small compared with other normal stress components and may be neglected.
3. Normals to the undeformed middle surface remain straight and normal to the deformed middle surface and undergo no extension. This assumption implies

that all the strain components (normal and shear) in the direction of the normal to the middle surface vanish.

To analyze the free vibration problem of truncated conical shells, there are many computational methods available. These methods can be classified as analytical methods and numerical methods. The examples of analytical methods are Rayleigh-Ritz method and power series expansion method. Numerical method is referring to finite element method. Comparing these two method, both have advantages and drawbacks. Analytical method require less computational effort yet they are not easy to implement. Finite element method is easy to implement but requires a lot of virtual storage and compuatational effort. By using numerical methods, the partial differential equations are reduced to a set of algebraic equations. In this project, the problem is solved numerically by using Bickley spline function.

1.2 Problem statement

In this thesis, we investigate the free vibration of symmetric angle-ply laminated truncated conical shells using spline function method. The governing equations of motion for laminated conical shells can be obtained using Love's first approximation theory. In this problem, we consider three and five layers of conical shells to analyze the vibration behavior of the shell using two types of materials under two types of boundary conditions. The governing equations of motion consist of the displacement functions u (longitudinal displacement), v (circumferential displacement) and w (transverse displacement). The displacements are assumed in the separable form and substituted into the governing equations. The governing differential equations of motion are obtained in terms of the reference surface displacement components which are coupled in the longitudinal, circumferential, and transverse directions. By approximating the displacements using cubic and quintic spline and applying collocation technique, one can get a system of algebraic equations in terms of spline coefficients. The system of equations is a generalized eigenvalue problem with spline coefficients as eigenvectors. This eigenvalue problem can be solved using numerical technique to get many numbers of required eigenvalues and corresponding eigenvectors. These eigenvalues are the frequency parameter for the conical shell.

1.3 Objectives

1. To model the free vibration behaviour of symmetric angle-ply laminated truncated conical shells.
2. To formulate the governing equations of motion for conical shells from the general shell equations.
3. To approximate the displacement using spline method.
4. To analyze the effects of circumferential mode number, length ratio and cone angle using different number of layers with different ply orientation on the frequency parameter under two different boundary conditions.

1.4 Scope of study

In view of the large number of parameter combinations that could be studied, it is decided to limit the scope of the current work to an investigation of the effect on frequency by varying the circumferential mode number (n), length ratio (β) and cone angle (α) of the truncated conical shells with two different number of layers with different ply angle. Two types of boundary conditions are imposed and the effect of these boundary conditions is also studied. Numerical technique, spline method is used to approximate the displacement functions.

1.5 Significance of study

The study of free vibration has been of considerable interest over the past few decades. It has been pursued with great intensity in many engineering fields. This problem helps us to analyze the effect of circumferential mode number (n), length ratio (β) and cone angle (α) with different number of layers and ply orientation on the frequency parameter which in return will help engineers to construct good structures and the most important is the safety of operation.

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