EPILEPTIC SEIZURE AS A SYSTEM OF ORDINARY DIFFERENTIAL EQUATION

AMEEN OMAR ALI BARJA

UNIVERSITI TEKNOLOGI MALAYSIA

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Specially Dedicated

To my beloved family and friends

especially

my father, mother, brothers and sisters

To my beloved wife Mrwah and my beloved Asma and Amna

To my whole family

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In The Name Of ALLAH, The Most Beneficent, The Most Merciful

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ABSTRACT

One of the applications of differential equation is dynamic systems, where the description of a system in state space by first-order vector nonlinear. An epileptic seizure is a dynamic system since it's spends through time. Epilepsy is a collection of disturbances characterized by recurrent paroxysmal electrical discharges of the cerebral cortex that resulted in intermittent disorders of brain functions. Electroencephalography (EEG) is a test that measures and records the electrical activities of the brain from the scalp by using sensors. Our main interest in this dissertation is to model an epileptic seizure as a system of ordinary differential equation.

ABSTRAK

Salah satu aplikasi persamaan terbitan ialah di dalam sistem dinamik dimana penghuraian ruang keadaan sistem dengan vektor tak linear berperingkat pertama. Serangan sawan ialah satu sistem dinamik kerana ia berlaku merentangi masa. Serangan sawan merupakan himpunan gangguan nyahcas elektrik paroksisme di korteks serebral yang mengakibatkan ganggun pada fungsi otak pula. Elektroensifalorgafi (EEG) ialah satu ujian yang mengukur dan mencerap aktiviti elektrik di dalam otak melalui kulit kepala oleh alat pencerap. Di dalam disertasi ini, dipaparkan bagaimana suatu serangan sawan boleh dimodelkan sebagai satu sistem persamaan terbitan.

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GLOSSARY

ODEs	Ordinary Differential Equations
EEG	Electroencephalography

LIST OF SYMBOLS

y'	First derivative
$y^{(n)}$	nth derivative
t	Time
$\frac{dx}{dt}$	First derivative
f, g	Functions
%	Percentage
(R, d_R), (L, d_L)	Metric spaces
d	Metric
$\prec$	Precede
$(L, \prec)$	Linearly ordered
$(X, \prec)$	Partially ordered set
$\in$	Member of
$\geq$	Greater than or equal
$\leq$	Less than or equal
$=$	Equality
$\mathbb{N}$	Natural numbers
$\mathbb{Z}$	Integers
$\mathbb{Q}$	Rational numbers
$\mathbb{R}$	Real numbers
$\rightarrow$, $\Rightarrow$	Implies

$\{....\}$	Set consisting of
$f : X \rightarrow Y$	f is a mapping from X to Y
T^t	Transition mapping
$A(t)$	The amount of chemical at time t
$V(t)$	The volume of solution at time t
$x_{t+1} = f(x_t)$	Iteration of function
X	State space
$\dot{x} = G(x, t)$	First order differential equation
$\subset$	Proper set inclusion
$\subseteq$	Set inclusion
$[a, b)$	Interval of real numbers closed in a and open in b
$\forall$	For all
$\Leftrightarrow$	If and only if
X_t	An augmented dynamic trajectory of the seizure
A_t	Temporal set

CHAPTER 1

INTRODUCTION

1.1 Background and Motivation

Mathematical modeling is a key tool to analyze and explain a wide-range of real-world phenomena [1, 2, 3], and pose challenging mathematical problems. Among the different modeling approaches, ordinary differential equations (ODE) are particularly important and have led to significant advances [4, 5]. Ordinary differential equations model the temporal evolution of the relevant variables by describing their deterministic dynamics. The study of dynamical systems with ODEs is well developed in different fields and therefore, there is a rich literature devoted to their analysis [6, 7] and solution [8, 9, 10]. Such modeling of systems with ODEs bears a considerable degree of uncertainty and/or variability in both initial conditions and parameters [11, 12, 13]. The propagation of uncertainty and variability through the system dynamics can lead to considerable variations in the model outputs and neglecting these may lead to unreliable conclusions.

Differential equations have a great relationship to investigate a wide variety of problems in physics, medicine, engineering and other sciences. The field of differential equations is divided into ordinary and partial both of them divided into linear and nonlinear differential equations. [14].

Linear and nonlinear differential equations appear in many applications. For example, dynamic systems, where the description of a system in state space is by vector nonlinear differential equations of first order. We deal with physical systems whose state at a time t is completely specified by the values of n real variables $x_1, x_2, \ldots\ldots\ldots, x_n$ in dynamics. Accordingly the system is such that the rates of change of these variables, namely $\frac{dx_1}{dt}, \frac{dx_2}{dt}, \ldots\ldots\ldots, \frac{dx_n}{dt}$, merely depend upon the values of the variables themselves, therefore the laws of motion can be expressed by means of n differential equations of the first order $\frac{dx_i}{dt} = X_i(x_1, x_2, \ldots\ldots\ldots, x_n) \qquad (i = 1, 2\ldots\ldots, n)$ [15].

State space is the set of all possible states of a dynamical system. Each state of the system corresponds to a unique point in the state space. In general, any abstract set for some dynamical system could be a state space. A state space could be finite or finite-dimensional. If it is finite it just consists of a few points, but if it is finite-dimensional it consists of an infinite number of points forming a smooth manifold, such as in the case of ordinary differential equations and mappings. Furthermore, a state space could be infinite-dimensional, as in partial differential equations and delay differential equations. A state space is often called a phase space [16].

1.2 Problem Statement

How the epileptic seizure can be viewed as a system of ordinary differential equation (ODE).

1.3 Objectives of Research

The objectives of this research as follows:

i. To view an epileptic seizure as a process or system whereby the input and output objects cannot be distinguished.
ii. To describe an epileptic seizure as a system of ordinary differential equation (ODE).
iii. To prove the temporal set of the seizure is a chain of this seizure.
iv. To verify the temporal set of the seizure contain at least one maximal element.

1.4 Scope of Research

The scope of this research is describing an epileptic seizure as a system of ordinary differential equation (ODE).

1.5 Significance of This Research

The main purpose of this dissertation is to model an epileptic seizure as a system of ordinary differential equation (ODE). Hence, we can study and understand more of an epileptic seizure once the dynamic model is established.

1.6 Dissertation's Layout

This dissertation contains five chapters, which are divided as follows:

The first chapter is the introduction to the research. It discusses the background and motivation, problem statement, objectives, scope of the research and finally significance of the research. The second chapter is literature review of the research. It begins with mathematical model, a brief overview on differential equations, epileptic seizure, Electroencephalography, and some basic mathematical concepts. The third chapter presents mixing problems and operational framework. Chapter four present an epileptic seizure can be viewed as a system of ordinary differential equation (ODE). Finally, chapter five is the summary and conclusion.

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