APPLICATION OF SMOOTHED PARTICLE HYDRODYNAMICS METHOD IN SOLVING TWO DIMENSIONAL SHEAR DRIVEN CAVITY PROBLEMS

NADIRAH BINTI ZANAL ABIDIN

UNIVERSITI TEKNOLOGI MALAYSIA

APPLICATION OF SMOOTHED PARTICLE HYDRODYNAMICS METHOD IN SOLVING TWO DIMENSIONAL SHEAR DRIVEN CAVITY PROBLEMS

NADIRAH BINTI ZANAL ABIDIN

A dissertation submitted in partial fulfillment of the
requirements for the award of the degree of
Master of Science (Mathematics)

Faculty of Science
Universiti Teknologi Malaysia

JANUARY 2013

DEDICATION

To my beloved father and mother

Zanal Abidin Nordin & Nur Unaiza Abd Razak

To my supervisor

Dr. Yeak Su Hoe

To my dearest sisters and brothers

Nadia Zanal Abidin, Nurul Aina Zanal Abidin

Muhammad Firdaus Zanal Abidin &

Muhamad Fahmi Zanal Abidin

To my friends

my infinite element

in gratitude

for encouraging me to

persist in the completion of this dissertation

and for loving companionship and support

through my whole life

ACKNOWLEDGEMENT

IN THE NAME OF ALLAH, THE MOST GRACIOUS, THE MOST MERCIFUL.

I would like to express my deepest gratitude to Almighty God because of His Blessing, I am able to complete this dissertation successfully within the stipulated time. I would like to thank my supervisor Dr. Yeak Su Hoe for his support and belief in me and also very thankful for his invaluable guidance and advice throughout completing my dissertation. His encouragement, thoughtfulness, and willingness to help are deeply appreciated. It has been my plesure to have them as my lecturer as well as their brilliance, resourcefulness, and patience are greatly admired.

I would also like to thank to all my coursemates and friends for their friendship and inspiring ideas who had given me a lot of moral and mental support. A special thank dedicated to the librarians at Perpustakaan Sultanah Zanariah for their assistance in guiding and supplying me with reading sources and relevant literature studies.

Last but not least, I would like to thank my family without whose support, I would never reached this point; my father, Zanal Abidin bin Nordin, whose accomplishments, boundless energy, and words of wisdom are great sources of inspiration to me; my mother, Nur Unaiza binti Abd Razak, to whom I am forever grateful for love, kindness, sensibility, and devotion she showed from day one; and my siblings who throughout the years tolerated their sister, filled her life with fun and happiness, encouraged her and stood by her through thick and thin.

ABSTRACT

Solution to the two-dimensional (2D) shear driven cavity problem has been done by many researchers earlier. Numerical methods are always being used in solving 2D shear driven cavity problem. The usual numerical method being chosen is the grid-based method such as finite difference method (FDM), finite element method and alternating direction implicit method. However, in this research, the smoothed particle hydrodynamics (SPH) method is chosen and being studied to be applied in solving the 2D shear driven cavity problem. The algorithm for SPH method is also being developed. As for making the comparisons to study on the accuracy of SPH method, 2D shear driven cavity also being solved using FDM. MATLAB and FORTRAN programming are used as a calculation medium for both the FDM and SPH method respectively.

ABSTRAK

Penyelesaian terhadap kaviti gerakan kekacip dua-dimensi telah banyak dijalankan dalam kajian-kajian oleh para penyelidik terdahulu. Kaedah berangka seringkali digunakan bagi menyelesaikan kaviti gerakan kekacip dua-dimensi. Antaranya adalah dengan menggunakan kaedah berdasarkan grid terhingga seperti kaedah beza terhingga (FDM), kaedah unsure terhingga dan kaedah arah ulang-alik implisit. Namun begitu, dalam kajian ini kaedah zarah hidrodinamik lancar (SPH) dipilih dan dikaji untuk diaplikasikan dalam menyelesaikan masalah persamaan haba dua-dimensi tersebut. Algoritma bagi kaedah SPH turut dirumuskan. Sebagai perbandingan bagi mengkaji tentang ketepatan kaedah SPH, kaviti gerakan kekacip dua-dimensi dalam kajian ini akan turut diselesaikan dengan menggunakan kaedah FDM dan pengaturcaraan Matlab dan Fortran digunakan sebagai medium pengiraan bagi kedua-dua kaedah ADI dan SPH tersebut.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF SYMBOLS

f	-	Function
$\lim_{h \to 0} W(x - x', h)$	-	Definition of continuous for function W at point "x'"
κ	-	Constant
N	-	Number of particles
x	-	Vector
u	-	Velocity field
v	-	Velocity field
$\mathbf{G}^{n+1}$	-	Gradient of pressure
W_{ij}	-	Smoothing kernel
α_d	-	Dimensional space

LIST OF APPENDICES

CHAPTER 1

RESEARCH FRAMEWORK

1.1 Introduction and Background of the Problem

The numerical solution of partial differential equations is dominated by finite difference method (FDM), finite element method (FEM), boundary element, spectral methods and others [1].

The finite difference method is the simplest method for solving boundary value problems and it is a universally applicable numerical method for the solution of differential equations. However, FDM can be difficult to analyze and suffers the problem of low accuracy solution, in part because its applicability is quite general [2]. The underlying idea of the FDM is to approximate differential equations by appropriate difference quotients, hence reducing a differential equation to an algebraic system. There are a variety of ways to do the approximation such as a forward difference, a backward difference and a centered difference. Much of the convergence and existing stability analysis is limited to special cases, especially to linear differential equations with constant coefficients. Then, these results are used to predict the behaviour of difference methods for more complicated equations [3].

The finite element method is introduced as a variationally based technique of solving differential equations and also provides a systematic technique which can represent a geometrically complex region by deriving the approximation functions (piecewise polynomials) for simple subregions [4]. The basic idea in FEM is simple which is starting by the region of physical interest or subdivision of the structure into smaller pieces. Hence, these pieces must be easy for the computer to record and identify; they maybe rectangles or triangles. Then the extremely simple form of trial function are given, normally they are polynomials, of at most the third of fifth degree and the boundary conditions are infinitely easier to impose locally along the edge of a triangle or rectangle [5].

The accuracy of FEM can be easily increased by increasing the degree of shape function. However, FEM will suffers the problem of ill condition element shape function and fail to generate reliable solutions for the boundary value problems in two dimension and three dimension when the geometry of the problems are complex with large deformation of structures. Thus, it requires remeshing process [2].

Meshless methods (MMs) are the next generation of computational methods development which are expected to be premier to the conventional grid-based FDM and FEM in many applications. The objectives of the MMs is to eliminate part of the difficulties associated with the accuracy and stable numerical solutions for partial differential equations or integral equations with all kinds of boundary conditions without using any mesh to solve that problems. One of the main idea in these meshless methods is to modify the internal structure of mesh-based FEM and FDM to become more versatile and adaptive [6].

One of the first meshless methods is the smoothed particle hydrodynamics (SPH) method by Lucy, and Gingold and Monaghan [7; 8]. It was born to solve problems in modelling astrophysical phenomena and later on, its applications were extended to problems of fluid dynamics and continuum solid. A few years later, Liszka and Orkisz [9] proposed a generalized finite difference method which can deal with arbitrary irregular grid. In 1992, diffuse element method (DEM) was born which formulate by applying moving least square approximations in Galerkin method [10]. Based on DEM, the element free Galerkin (EFG) method was advanced remarkably and it is one of the most famous meshless methods. EFG was applied to many solid mechanics problems with the help of a background mesh for integration [11].

Meshless Local Petrov-Galerkin (MLPG) was introduced by Atluri and Zhu [12] which requires local background cells for the integration only. It has been used to the analysis of beam and plate structures, fluid flow and other mechanics problems. A few years before, Liu and Chen [13] proposed a reproducing kernel particle method (RKPM) which improves the accuracy of the SPH approximation especially around the boundary through reproducing conditions and revisiting the consistency in SPH. Another meshless methods are the point interpolation method and meshfree weak-strong form [14].

The effort of MMs is focused on solving the problem which the conventional FDM and FEM are difficult to apply such as problems with deformable boundary, free surface (for FDM), large deformation (for FEM), mesh adaptivity and complex mesh generation (for both FEM and FDM) [6]. There are some major advantages in using MMs which are problems with moving discontinuities such as shear band and crack propagation can be treated with ease, higher-order continuous shape functions, large deformation can be handle more robustly, no mesh alignment sensitivity and non-local interpolation character [15].

1.2 Statement of the Problem

The conventional mesh or grid based numerical methods such as finite element method (FEM) and finite difference methods (FDM) have been widely used to various areas of computational solid mechanics and fluid dynamics. These methods are currently the main contributor in solving problem of science and engineering.

FDM is one of grid-based method, has never been an easy task in constructing a regular grid for complex or irregular geometry. It is usually requires additional complex mathematical transformation that can be even more expensive than solving the problem itself. Several of the problems in applying FDM are determining the precise locations of the in homogeneities, free surfaces and deformable boundaries. Besides, FDM is also not well suited to problem that need monitoring the material properties in fixed volumes such as, particulate flows.

However, mesh-based methods such as FEM suffer from some difficulties which limit their applications to many problems. One of the problem arise is a costly process of generating and regenerating the mesh due to large deformation of structures. In efforts to overcome this time consuming problem, meshless method is developed which successfully avoid the process that consume a lot of time. One of the meshless methods is smoothed particle hydrodynamics (SPH) method which can solve problem of large deformation without remeshing process.

The aim of this research is to solve two dimensional shear driven cavity flow problem using SPH method. In order to study on the accuracy of the SPH method, the finite difference method has first being applied to solve the two dimensional shear driven cavity flow problem. From the solution obtained by finite difference method, a comparison has been made with the solution of SPH method in order to study the accuracy of the SPH method.

1.3 Objectives of the Problem

The main objectives of this research are:

a. to study the basics concepts of SPH methods.
b. to solve a two dimensional shear driven cavity problem by using SPH method.
c. to solve problem of two dimensional shear driven cavity numerically by using finite difference method.
d. to compare the accuracy of solution between SPH method with the results of finite difference method.

1.4 Scope of the Study

The main focuses in this research is on the concept of SPH method. Numerical algorithm of the SPH method will be constructed in order to make use of the method later. After that, the SPH method is then be applied to solve a two dimensional shear driven cavity problem. Validation of SPH method will also be done by doing the comparison with the results of finite difference method. The solution of the problem will be focused on the accuracy of the different material properties of the shear driven cavity.

1.5 Outline of the Research

This research has been divided into five chapters. **Chapter 1** discusses about the background of the study, problem statement, objectives, scope, outline of the research and research methodology.

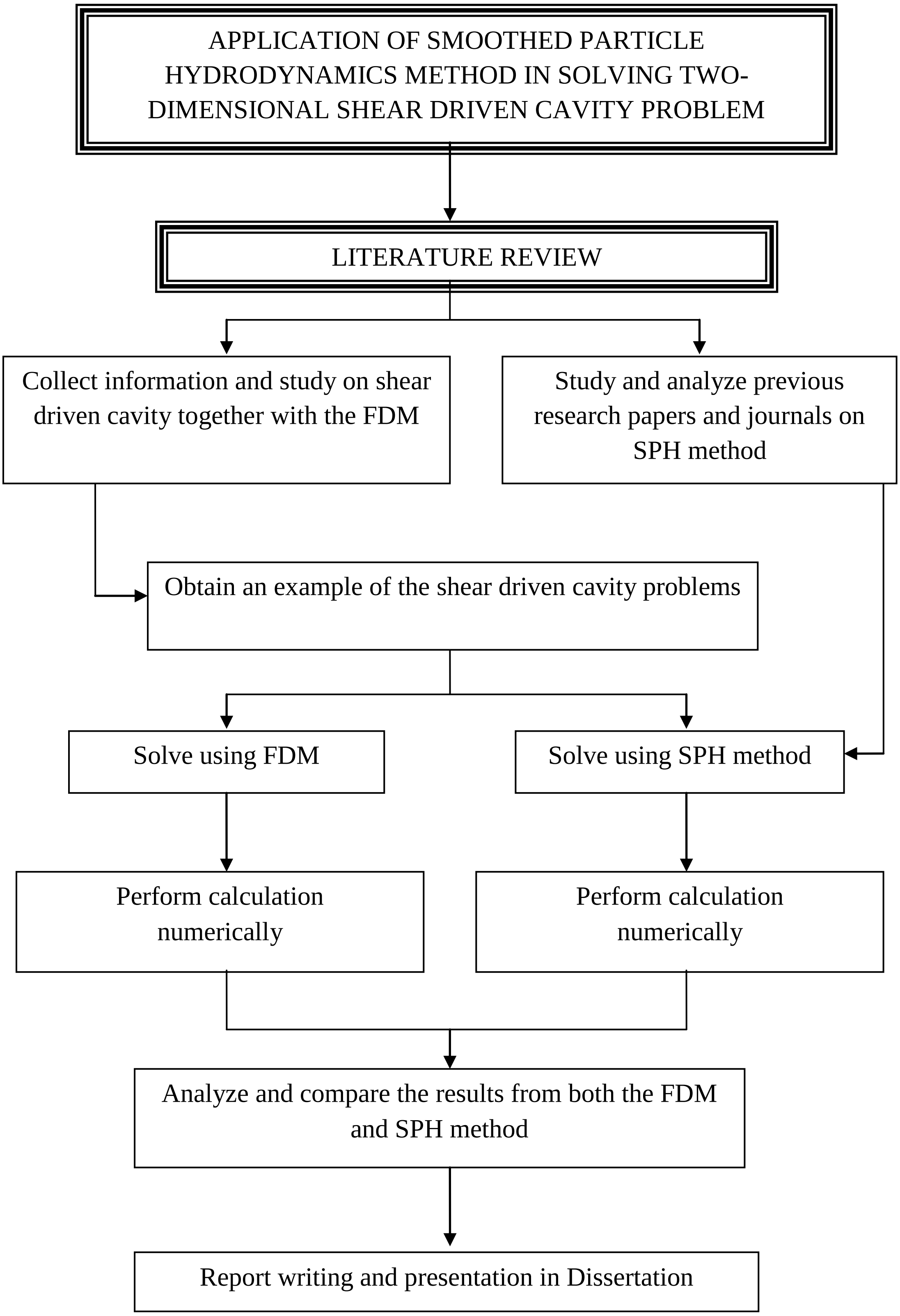
Chapter 2 provides some information on the literature review that is related to this study. This chapter starts with the historical development in solving the shear driven cavity problem followed by some introduction on SPH method and finite difference method.

Chapter 3 focused on the computational methods. The methodology including the procedures is discussed in detail. There are some sub sections in this chapter that will describe the different case study. All solutions for numerical method (FDM and SPH method) were obtained by using MATLAB programming and FORTRAN programming respectively.

Chapter 4 is mainly about the result and discussion. The solution for each case study will be put under each section. There will be a discussion on the solution for each case study that been obtained from numerical method. Comparisons of the solution from both the FDM and SPH method will be compared in order to look at the accuracy.

Finally, the last chapter, **chapter 5** will be the conclusion that summarizes this study and also some suggestion that might be useful for further study.

1.6 Research Methodology



REFERENCES

1. Press, W.H., Teukolsky, S. A., Vettering, W. T. and Flannery, B. P. *Numerical Recipes in C*. 2nd Ed. Cambridge University Press. 1992.

2. Yeak, S. H. *Numerical Methods in Engineering Lecture Notes*. University Teknologi Malaysia. 2009.

3. Atkinson, K. And Han, W. Texts in Applied Mathematics. *Springer : Theoretical Numerical Analysis*. 2009. 39: 253-275.

4. Reddy, J. N. *An Introduction to the Finite Element Method*. 2nd Ed. United State of America: McGraw Hill, Inc. 1993.

5. Gilbert, S. and George, F. *An Analysis of the Finite Element Method*. 2nd Ed. United State of America: Wellesley-Cambridge Press. 2008.

6. Liu, G. R. And Liu, M. B. *Smoothed Particle Hydrodynamics: A Meshfree Particle Method*. Singapore: World Scientific Publishing Co. Pte. Ltd. 2003.

7. Lucy, L. B. A Numerical Approach to the Testing of the Fission Hypothesis. *Astron. J*. 1977. 82(12): 1013-1024.

8. Gingold, R. A. and Monaghan, J. J. Smoothed Particle Hydrodynamics: Theory and application to Non-Spherical Stars. *Mon. Not. R. Astron. Soc.* 1977. 181: 375-389.

9. Liszka, T. and Orkisz, J. The Finite Difference Method at arbitrary irregular grids and its applications in Applied Mechanics. *Computer and Structures*. 1980. 11: 83-95.

10. Nayroles, B., Touzout, G., and Villon, P. Generalizing the Finite Element Methods: Diffuse Approximation and Diffuse Elements. *Computational Mechanics*. 1992. 10: 307-318.

11. Belytschko, T., Lu, U. U., and Gu, L. Element-Free Galerkin methods. *International Journal for Numerical Methods in Engineering*. 1994. 37: 229-256.

12. Atluri, S. N. and Zhu, T. A new meshless local Petrov- Galerkin (MLPG) approach in computational mechanics. *Computational Mechanics*. 1998. 22: 117-127.

13. Liu, W. K. and Chen, Y. Wavelet and Multi-Scale Reproducing Kernel methods. *International Journal for Numerical Methods in Fluids*. 1995. 21: 901-931.

14. Liu, G. R. and Gu, Y. T. *Advance in Mesfree and X-FEM Methods: A truly meshless method based on weak-strong*. 2002. 259-261.

15. Nguyen, V. P., Rabczuk, T., Bordas, S., and Duflot, M. Meshless methods: A review and computer implementation aspects. *Mathematics and Computers in Simulation.* 2008. 79: 763–813.

16. Fudhail, A. M., Sidik, N. A. C., Rody, M. Z. M., Zahir, H. M. and Musthafah, M. T. Numerical Simulations of Shear Driven Square and Triangular Cavity by using Lattice Boltzmann Scheme. *World Academy of Science, Engineering and Technology*. 2010. 69: 190-194.

17. Ghia, U., Ghia, K. N. and Shin, C. T. High Re solutions for Incompressible flow using the Navier-Stokes equations and a Multigrid method. *Journal of Computational Physics*. 1982. 48: 387-411.

18. Valdehi, A. SIMPLE Solver for Driven Cavity Flow Problem. Purdue University, Indiana. 2010.

19. Strang, G. *Computational Science and Engineering*. 1st Ed. Wellesly-Cambridge Press. 2007.

20. Girault, V. Theory of a Finite Difference Method on Irregular Networks. *SIAM: J. Numer. Anal*. 1974. 11: 260-282.

21. Douglas, J. Finite Difference Method for Two-Phase Incompressible Flow in Porous Media. *SIAM: J. Numer. Anal*. 1983. 20: 681-696.

22. Vesenjak, M. and Ren, Z. Application of Smoothed Particle Hydrodynamics Method in Engineering Problem. *Springer: Comp. Methods in Appl. Sc. – Progress on Meshless Method*. 2009. 2(16): 273-290.

23. Benz, W. Applications of Smoothed Particle Hydrodynamics (SPH) to Astrophysical Problems. *Comp. Physics Comm*. 1988. 48: 97-105.

24. Monaghan, J. J. Smoothed Particle Hydrodynamics. *Annu. Rev. Astron. Astrophys*. 1992. 30: 543-574.

25. Frederic, A. R. and James C. L. Smoothed particle hydrodynamics calculations of stellar interactions. *Journal of Computational and Applied Mathematics*. 1999. 109: 213-230.

26. Hoover, W. G. *Smooth Particle Applied Mechanics: The State of the Art*. World Scientific. 2006.

27. Kristof, P., Banes, B., Krivanek, J. and Stava, O. Hydraulic Erosion Using Smoothed Particle Hydrodynamics. *The Eurographics Association and Blackwell Publishing Ltd*. 2009. 28(2): 219-228.

28. Wolfe, C. T. and Semwal, S. K. Acoustic Modeling of Reverberation using Smoothed Particle Hydrodynamics. *16th International Conference in Central Europe on Computer Graphics Visualization and Computer Vision*. 2008. 191-198.

29. Harada, T., Koshizuka, S. and Kawaguchi, Y. *Smoothed Particle Hydrodynamics on GPUs*. The University of Tokyo. 2007.

30. Monaghan, J. J. Why Particle Methods Work (Hydrodynamics). *SIAM: J. Sci. Stat. Comput*. 1982. 3(4): 422-433.

31. Monaghan, J. J. and Kocharyan, A. SPH simulation of multi-phase flow. *Comp. Physics Comm*. 1995. 87: 225-235.

32. Chorin, A. J. Numerical solution of the Navier-Stokes equations. *Math. Comput*. 1968. 22: 745-762.

33. Morris, J.P., Fox, P. J. and Zhu, Y. Modelling low Reynolds number incompressible flows using SPH. *Journal of Computational Physics*. 1997. 136: 214-226.

34. Wan Lokman bin Wan Bedurdin. *Penyelesaian Masalah Matematik Menggunakan Pengaturcaraan Selari BerasaskanModel OpenMP*. Projek Sarjana Muda. Universiti Teknologi Malaysia. 2005

35. V. Horak and P. Gruber. Parallel Numerical Solution of 2-D Heat Equation. *Parallel Numerics'*. 2005. pp.47-56.