CONCEPTUAL MODEL OF NUMERICAL APPROXIMATIONS USING SENSORS FOR RADIATIVE HEAT TRANSFER IN ETHYLENE CRACKER FURNACE

Zuraida Abal Abas, Shaharuddin Salleh and Sakilah Sapar

Department of Mathematics, Faculty of Science
Universiti Teknologi Malaysia
81310 Johor Bahru, MALAYSIA.
zuraidaa@utem.edu.my, ss@utm.my, sakilah_sapar@yahoo.com.my

Abstract : A conceptual model of integrating a sensor network and the radiative heat transfer equation is developed and presented in this paper. The idea is to improve the numerical scheme of solving the radiative heat transfer equation by providing the exact values of temperature from the sensor network to obtain a set of intensity boundary values. We propose unisensor and multi-sensor models based on a triangular mesh model with points at its boundaries and coils as their pivot elements for the network. The paper concentrates on the two-dimensional cross-sectional model only with several U-bend reactor tubes in the ethylene furnace as an initial phase of constructing a complete simulation in the real furnace design.

Keywords: Finite element, triangular mesh, sensor, radiative heat and ethylene furnace.

1. Introduction

Ethylene, or olefin, is a chemical compound commonly produced in the petrochemical industry [1]. Ethylene is produced in mass production through a process known as the steam cracking process or pyrolysis in the ethylene cracker furnace. Feedstocks such as naphtha and ethane are required as input elements during the cracking process. The cracker furnace provides heat for the steam cracking process. In this process, heat is used for breaking or cracking down the saturated hydrocarbon or feedstocks into smaller hydrocarbons in the coils or reactor tube and eventually produce the desired products such as ethylene.

Initially, the process starts with combustion in the radiant box to produce heat for cracking the coil reactor. The cracking reaction occurs, and ethylene produced from this reaction is quenched in transfer line exchanger. The remaining heat from radiant box is utilized to preheat the feed, the boiler feed water and the steam in the convection section.

In recent years, many researchers are interested in studying the operating parameters including those related to heat transfer such as heat flux at the reactor tube, flue gas temperature and process gas temperature [2,3,4,5]. The existing commercial software such as FLUENT and SPYRO are used to conduct the simulation in order to obtain the distribution of the desired operating parameters. The problem involves some unknown boundary values of the intensity which are generated through approximation. Numerical solutions suggest the values are set to be zero everywhere in the first iteration, and an iterative approach is required since the initial intensity is unknown [3,6].

Since heat transfer plays a significant role in the daily operation of the ethylene furnace, it is of great importance to monitor the heat distribution right from the burner to the tubes in the radiant section. This can be achieved by obtaining the heat flux profile at the tubes along the firebox height as well as at the circumference of the tube at some selected height in the furnace. This paper concentrates on the radiation model only as the main objective is to produce a two-dimensional conceptual model for integrating the sensor network and radiative heat transfer equation. The conceptual model should improve the numerical solution by providing temperature values at some selected points of the furnace as their boundary values.

In this paper, we propose a conceptual model on the deployment of sensors as a means to monitor radiative heat transfer inside the furnace. Two models are proposed: one with unisensors and

another with multi-sensors. Both models are based on triangular mesh network as a platform for obtaining the numerical solution to the radiative heat transfer. The mesh network has its sensor nodes as its pivot elements along the inner boundaries of the furnace as well as the outer layer of the U-bend coils. The sensor nodes provide exact values of temperatures and flux from their periodical readings, and these values serve as inputs to the boundary value problem.

2. Radiative Heat Transfer in Ethylene Furnace

There are many models of reactor tubes used in the ethylene furnace around the world following the manufacturer design. Figure 1 shows a model proposed by Lan et al. [2] which has ten U-bend reactor tubes and four floor-fired burners. In the figure, radiative heat transfer occurs at the radiant section. The burners which are located at the floor will heat up the furnace as combustion in the burner results in flames with its hot flue gas released. During this time, the radiative heat from the furnace wall and hot flue gas is transferred to the reactor tubes and coils.
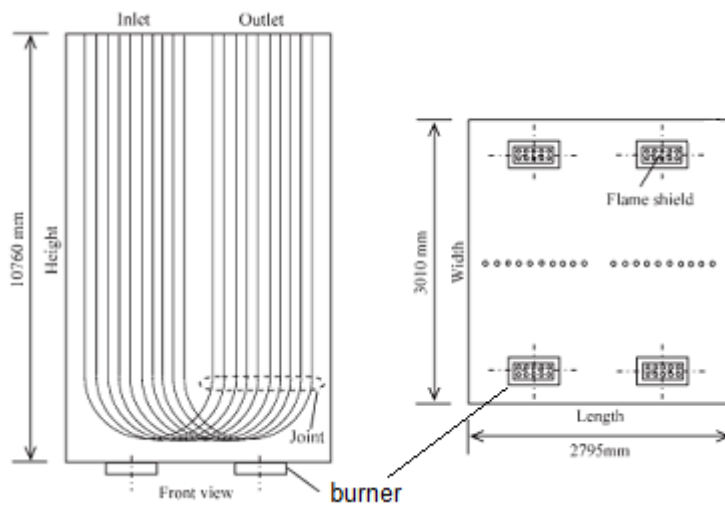

Figure 1. One fourth of the configuration in ethylene furnace in Lat et al. [2].

Engineers are always interested in the heat flux profiles along the furnace height and the circumferential heat flux distribution at the reactor tubes and coils. Therefore, it is important to have accurate estimation of the heat flux for the correct design and operation in an industrial ethylene furnace. It must be noted that the product of combustion in ethylene furnace contains a large amount of carbon dioxide and water vapour. Therefore, the fluid is a participating medium and there is interaction between the participating medium and radiation. However, no scattering is involved in the case. The surface of the furnace wall and the skin of the reactor tubes/coils are assumed to be opaque and gray-diffuse [7]. Opaque wall or surface is a medium thick enough that no electromagnetic waves and radiation can penetrate through it [4].

3. The Computational Model

The discretization of the radiative heat transfer equation can be done by either finite volume or finite element approaches. The common practice of measuring the radiative heat transfer to the reactor tube along the furnace height as well as at the circumference reactor tube is by conducting simulation with existing commercial software such as FLUENT, SPYRO and GAMBIT. More recently, researchers are interested with the new version of finite element known as discontinuous finite element. This discretization scheme offers an advantage of no inter-element continuity and thus approximation functions are from a finite element broken space which means that the field variable may be considered discontinuous at the same geometric point [3]. However, it must be noted that the first iteration of calculating the intensity, the initial intensity value are unknown once the derived equation are needed to solve element by element. This is because the lack of information regarding

the temperature value or temperature distribution in the system. Therefore, the initial value of intensity is set to be zero in the calculation.

The conceptual model proposed in this paper fill the gap of getting the initial intensity value by deploying the sensor network in the ethylene furnace. The value that needs to be sensed by the sensor nodes are temperature and the heat flux. The main reason for selecting temperature and heat flux as the desired value from the sensor nodes is the dependencies from this mode of heat transfer are very high.

In [4], the intensity of the blackbody I_b can be calculated from the equation

$$I_b = \sigma\varphi^4 / \pi,$$

(1)

where $\sigma = 5.67 \times 10^{-8} (W/m^2.K^4)$ is the Stefan-Boltzmann's constant and φ is the absolute temperature. The intensity of the real surface I_s can be calculated from the equation of the product of its surface emissivity ε and black body intensity I_b, that is,

$$I_s = \varepsilon I_b = \varepsilon\sigma\varphi^4 / \pi.$$

(2)

Once the intensity distribution is generated at the reactor tube, the heat flux profile on the surface Ω with normal vector $\boldsymbol{n}$ can be calculated as follows:

$$q = \int \varepsilon I_b \ n \cdot s \ d\Omega.$$

(3)

It is obvious from Equation (3) that the deployment of sensors can assist in calculating the initial intensity at the boundary of the enclosure.

The general relationship that governs the changes in intensity at a point along the radiation ray due to emission, absorption and scattering in a fluid medium is [4]

$$\frac{dI(r,s)}{ds} = \kappa I_b(r) - \kappa I(r,s) - \sigma_s I(r,s) + \frac{\sigma_s}{4\pi} \int_{4\pi} I_-(s_i) \ \Phi(s_i,s) \ d\Omega_i.$$

(4)

Equation (4) represents the rate of change of the intensity I per unit length s along the radiation ray r. The first, second, third and fourth terms in the right-hand equation are the emitted intensity, absorbed intensity, out-scattered intensity and in-scattered intensity of the radiation, respectively.

Figure 2 shows the front view of one U-bend in a cracker furnace. In our model, sensors are placed along the wall of the furnace, shown as small circles in the figure. The sensor nodes should provide exact values of the temperatures at their positions along the furnace wall. These values will serve as inputs in the form of boundary values for approximating the temperature and flux values at other points in the furnace.
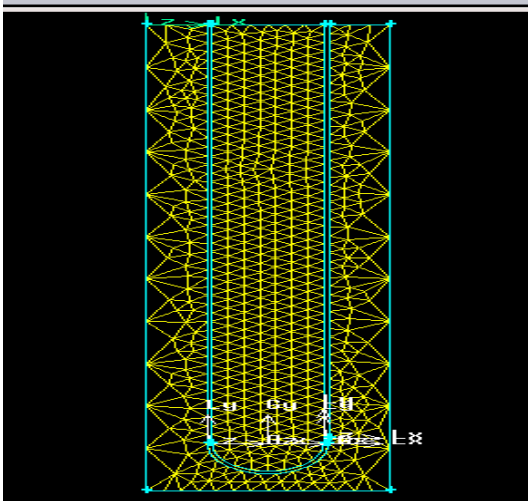
Figure 2. Vertical mesh formed from sensors placed along the inner wall of the furnace.
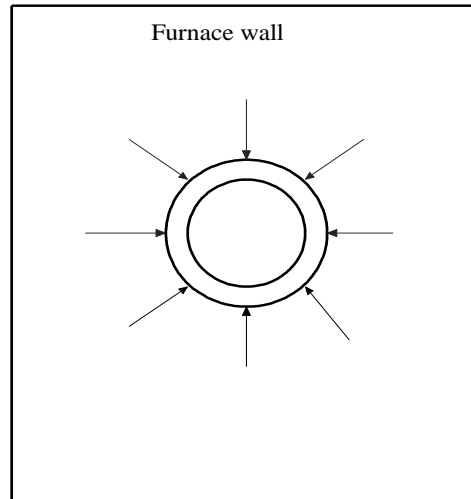

Figure 3. Horizontal cross-sectional diagram of a coil showing the hot flue gas radiation.

Figure 2 also shows the two-dimensional mesh network on the vertical cross-section of the furnace created using GAMBIT. The network consists of triangles with the sensor nodes forming the vertices along the vertical boundaries of the furnace. It must be noted that the element of the mesh start from the point of sensor node placement and spreads towards the face of the reactor tube. The mesh fulfills the space between the furnace wall and the reactor tube only.

Figure 3 shows the radiation heat transfer effect from the furnace wall in the form of hot flue gas on the two-dimensional cross-section of a single-coil tube. The radiation intensity on the external reactor tube skin originates from the hot furnace wall and hot flue gases. This creates a heat flux in the direction from the furnace wall to the coil, as shown by arrows in the figure.

3.1 Unisensor Model

Our unisensor model consists of sensors from the same type that sense only one parameter, for example, temperature and their input values contribute in calculating the heat flux using Equations (1), (2), (3) and (4). In this model, the triangular elements are drawn with their vertices along the inner boundaries of the furnace. The vertices coincide with the location of the sensors.
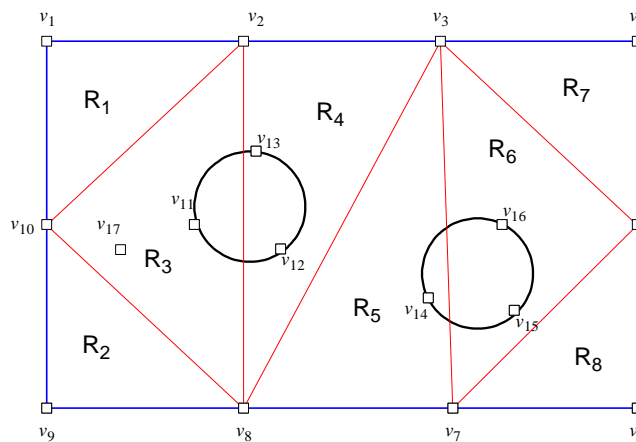

Figure 4. Unisensor model with sensors along the boundaries of the furnace and the triangular partitions.

4

Our computational model has sensors reading their values along the inner boundaries of the cracker furnace, and these values are used to compute values at other points inside the furnace. The model is borrowed from finite element modeling of using three shape functions at the vertices of triangular elements. The approximation at point (x, y) inside a triangle with vertices $[v_i, v_j, v_k]$ whose values are given as $[\varphi_i, \varphi_j, \varphi_k]$ is

$$\varphi(x, y) = c_1 + c_2 x + c_3 y = \begin{bmatrix} N_i^e & N_j^e & N_k^e \end{bmatrix} \begin{bmatrix} \varphi_i \\ \varphi_j \\ \varphi_k \end{bmatrix}. \tag{5}$$

In the above equation, N_i^e is the shape function of element e at vertex i, while c_i for $i = 1, 2, 3$ are the constants to be determined. The shape functions are determined through interpolation, as follows:

$$N_i^e = \frac{1}{2A^e} \left[x_j y_k - x_k y_j + x\,(y_j - y_k) + y\,(x_k - x_j) \right], \tag{6a}$$

$$N_j^e = \frac{1}{2A^e} \left[x_k y_i - x_i y_k + x\,(y_k - y_i) + y\,(x_i - x_k) \right], \tag{6b}$$

$$N_k^e = \frac{1}{2A^e} \left[x_i y_j - x_j y_i + x\,(y_i - y_j) + y\,(x_j - x_i) \right], \tag{6c}$$

where A^e is the area of the triangle e given by

$$A^e = \frac{1}{2} \begin{vmatrix} 1 & x_i & y_i \\ 1 & x_j & y_j \\ 1 & x_k & y_k \end{vmatrix} = \frac{1}{2} \left[(x_j y_k - x_k y_j) - (x_i y_k - x_k y_i) + (x_j y_i - x_i y_j) \right]. \tag{6d}$$

It follows that

$$\begin{bmatrix} \varphi_i \\ \varphi_j \\ \varphi_k \end{bmatrix} = \begin{bmatrix} 1 & x_i & y_i \\ 1 & x_j & y_j \\ 1 & x_k & y_k \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} \tag{7}$$

We obtain the approximation:

$$\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 1 & x_i & y_i \\ 1 & x_j & y_j \\ 1 & x_k & y_k \end{bmatrix}^{-1} \begin{bmatrix} \varphi_i \\ \varphi_j \\ \varphi_k \end{bmatrix}. \tag{8}$$

Algorithm POINTS below show how the model is implemented. To approximate values inside the furnace, the rectangular region is partitioned into triangular elements e_k for $k = 1, 2, ..., n-2$ where the vertices of each triangle are the sensor nodes placed along the boundaries.

Algorithm POINTS
Input: n sensors $v_i = (x_i, y_i)$ for $i = 1, 2, ..., n$ along the rectangular boundaries;
Input: Read the sensor values φ_i at $v_i = (x_i, y_i)$ for $i = 1, 2, ..., n$;

5

Input: C circles denoted as C_j for $j = 1, 2, ..., C$;

Input: r points on each circle, located equidistant along the circumference;

Partition the rectangular region into $n-2$ regional triangles R_k for $k = 1, 2, ..., n-2$;

Generate and plot m random points v_i for $i = 1, 2, ..., m$, and $v_i \notin C_j$;

Apply Equation (8) to approximate φ_i, for $i = 1, 2, ..., m$;

3.2 Multi-sensor model

The unisensor model is capable of approximating values of a single variable only, and it may not be able to predict the real scenario inside the furnace. The presence of turbulence, for example, can only be determined from variations due to several different parameters, such as temperature, pressure and heat flux. In practice, several different types of sensors can be placed along the boundaries for sensing the values of different variables. Our approach is the multi-sensor model where two different sensors are placed along the boundaries to read values, such as temperature and heat flux. The values read by these sensors contribute as input values in the boundary value problem, and our approach here is the finite element method for solving the problem.

Heat distribution over a region inside a closed boundary L has its strong form defined according to the Poisson's equation given by [8]:

$$div(\,hD\,\nabla T) + hQ = 0, \tag{9}$$

where div is the divergence, $D = \begin{bmatrix} k_{xx} & k_{xy} \\ k_{yx} & k_{yy} \end{bmatrix}$ is the constitutive matrix, h is the thickness of the domain, Q is the rate of heat distribution per unit volume and per unit time, and k is the conductivity constant. The boundary conditions along α and β are given by $\varphi = \alpha$ on L_α and $q_n = \boldsymbol{q}^* \boldsymbol{n} = \beta$ on L_β, where q_n is the flux in the direction of unit normal vector n. The problem has its weak form expressed as [8]

$$\int_A (\nabla w)^* hD\,\nabla T\,dA = -\int_{L_\alpha} whq_n\,dL - \int_{L_\beta} wh\beta\,dL + \int_A whQ\,dA, \tag{10}$$

where w is the weight function. The finite element solution is given by [8]

$$\left(\int_A B^* DBh\,dA \right) \tau = -\int_{L_\alpha} N^* q_n h\,dL - \int_{L_\beta} N^* \beta h\,dL + \int_A N^* Qh\,dA, \tag{11}$$

where $\tau = \begin{bmatrix} \varphi_1 & \varphi_1 & ... & \varphi_n \end{bmatrix}^*$ consists of quantities such as the temperature values at the sensor nodes along the boundaries, N is the shape function matrix and B is its gradient. The quantity in brackets in the left-hand equation is the stiffness matrix, while the first two terms of the right-hand equation are the boundary vector and the third term is the load vector. The shape function of node i is a tetrahedron with a value of 1 at the node and 0 at the other two nodes j and k. At other points, the value is given by [8]

$$N_i^e = \frac{1}{2A}\left[x_j y_k - x_k y_j + x\,(y_j - y_k) + y\,(x_k - x_j) \right], \tag{12}$$

where A is the area of the triangle.

Our computational model consists of the deployment of sensors along the inner boundaries of the furnace and the outer layer of the coils for two purposes: one set for sensing the temperature φ, while the other one for sensing the flux q_n. The model also generates randomly distributed points between the coils and the boundaries for forming an unstructured triangular mesh. The triangles in the mesh serve as elements in the boundary value problem, and finite element method is then applied to solve the problem.

Figures 5 and 6 show the triangular mesh results from our initial simulation program in cases of one and four coils, respectively (each coil has two tubes). The triangles are generated using Delaunay triangulation technique with randomly distributed points as their vertices. Simulation is performed using the C++ programming language.
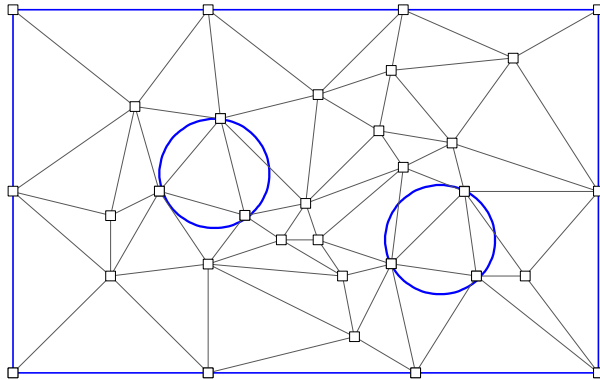

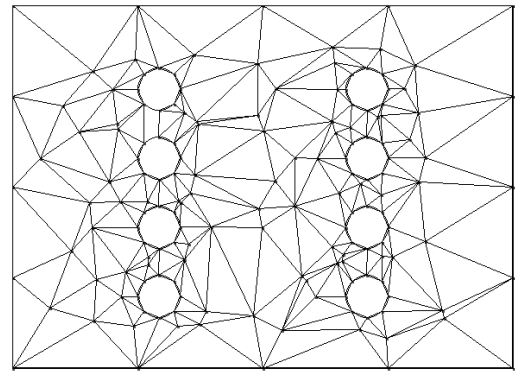Figure 5. Triangular mesh formation for a single coil, each with two circles.

Figure 6. Triangular mesh formation for four coils, each with two circles.

4.   Summary and conclusion

A conceptual model of integrating the deployment of sensor network and radiative heat transfer equation in the ethylene furnace to generate the heat flux profile has been proposed in this paper. The function of the sensor node is to supply the temperature value so that the boundary value of intensity can be calculated as an initial value in the iterative approach of solving the radiative equation. We propose two models, one is based on a single sensor type while the other is the multi-sensor model. This initial work will be followed by some massive simulations which will involve mesh generation on an arbitrary number coils, and the finite element method and simulation for modeling the radiative heat transfer in the furnace.

Acknowledgement

References

[1]   Gielen, D., Bennaceur, K., & Tam, C. (2006). IEA Petrochemical Scenarios for 2030-2050: Energy Technology Perspectives.   Retrieved 17 Dec, 2008, from http://www.iea.org/textbase/work/2006/petrochemicals/Discussion_Paper.pdf.

[2]   Lan, X., Gao, J., Xu, C., & Zhang, H. (2007). Numerical simulation of transfer and reaction process in ethylene furnaces. *Trans IChemE, 85*, 1565-1579.

[3]   Li, B. Q. (2006). *Discontinuous Finite Elements in Fluid Dynamics and Heat Transfer*. Germany: Springer-Verlag.

[4]   Modest, M. F. (2003). *Radiative heat transfer*. California: Academic Press.

[5]   Oprins, A. J. M., & Heynderickx, G. J. (2003). Calculation of three-dimensional flow and pressure fields in cracking furnaces. *Chemical Engineering Science*, 4883 – 4893.

[6]   Versteeg, H. K., & Malalasekera, W. (2007). *An introduction to computational fluid dynamics* (2 ed.). London: Pearson Prentice Hall.

[7]   Stefanidis, G. D., B.Merci, Heyndericks, G. J., & Marin, G. B. (2007). Gray/Nongray gas radiation modeling in steam cracker CFD calculations. *AIChE Journal, 53*, 1658-1669

[8]   Ottosen, N. and Petersson, H. (1992). *Introduction to finite element method,* Prentice-Hall.