

MULTI-OBJECTIVE OPTIMIZATION USING DIFFERENTIAL EVOLUTION  
FOR DYNAMIC MODEL STRUCTURE SELECTION

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Dedicated to my beloved mother and father  
for the love, care and moral support

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## **ABSTRACT**

Model structure selection is one of the procedures of system identification. The main objective of system identification is to select a parsimony model that best represents a dynamic system. Therefore, this problem needs two objective functions to be optimized at the same time i.e. minimum predictive error and model complexity. This research presents a new developed algorithm called multi-objective optimization using differential evolution. One of the main problems in identification of dynamic systems is to select a minimal model from huge possible models to be considered. The important concepts in selecting good and adequate model are used in the proposed algorithm are elaborated, including the implementation of the algorithm for modelling dynamic systems. The related issue such as parameter tuning on the proposed algorithm is discussed.

## **ABSTRAK**

Pemilihan struktur model adalah salah satu prosedur pengenalan sistem. Matlamat utama pengenalan system ialah memilih model termudah dan terbaik mewakili sistem dinamik. Oleh itu, masalah ini memerlukan dua fungsi objektif untuk dioptimum iaitu ralat ramalan yang minimum dan kerumitan model. Kajian ini membentangkan pembangunan algoritma baru dipanggil pengoptimuman berbilang objektif menggunakan evolusi kebezaan. Salah satu masalah utama dalam pengenalan sistem dinamik adalah untuk memilih model yang termudah daripada kemungkinan model yang banyak untuk dipertimbangkan. Konsep penting dalam memilih model yang baik dan memodai, digunakan oleh algoritma yang dicadangkan dihuraikan, termasuk pelaksanaan algoritma untuk pemodelan sistem dinamik. Isu yang berkaitan seperti penalaan parameter pada algoritma yang dicadangkan juga dibincangkan.

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## LIST OF ABBREVIATION

ARMAX	-	AutoRegressive Moving Average with eXogenous input
ARX	-	AutoRegressive with eXogenous input
DMA	-	Deterministic Mutation Algorithm
EA	-	Evolutionary Algorithm
ERR	-	Error Reduction Ratio
GA	-	Genetic Algorithm
LS	-	Least Square
MIMO	-	Multi Input-Multi output
MODE	-	Multi-Objective Differential Evolution
MOO	-	Multi-Objective Optimization
MOODE	-	Multi-Objective Optimization using Differential Evolution algorithm
MSE	-	Mean Square Error
OF <sub>n</sub>	-	Objective Function
OLS	-	Orthogonal Least Square
RLS	-	Recursive Least Square
SISO	-	Single Input-Single output
SOO	-	Single-Objective Optimization
NARMAX	-	Nonlinear ARMAX
NARX	-	Nonlinear ARX

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## **CHAPTER 1**

### **INTRODUCTION**

#### **1.1 Introduction**

Optimization refers to finding the values of decision (or free) variables, which correspond to and provide the maximum or minimum of one or more desired objectives. It is ubiquitous in daily life - people use optimization, often without actually realizing, for simple things such as traveling from one place to another and time management, as well as for major decisions such as finding the best combination of study, job and investment. Similarly, optimization finds many applications in engineering, science, business, economics, etc. except that, in these applications, quantitative models and methods are employed unlike qualitative assessment of choices in daily life. Without optimization of design and operations, manufacturing and engineering activities will not be as efficient as they are now. Even then, scope still exists for optimizing the current industrial operations, particularly with the ever changing economic, energy and environmental landscape.

Multi-objective optimization (MOO), also known as multi-criteria optimization, particularly outside engineering, refers to finding values of decision variables which correspond to and provide the optimum of more than one objective. Unlike in SOO (Single Objective Optimization) which gives a unique solution (or several multiple optima such as local and global optima in case of no convex problems), there will be many optimal solutions for a multi-objective problem; the exception is when the objectives are not conflicting in which case only one unique

solution is expected. Hence, MOO involves special methods for considering more than one objective and analyzing the results obtained.

In single-objective optimization, it is possible to determine between any given pair of solutions if one is better than the other. As a result, a single optimal solution is obtained. However, in multi-objective optimization a straightforward method to determine if a solution is better than other does not exist. The method most commonly adopted in multi-objective optimization to compare solutions is the one called Pareto dominance relation which, instead of a single optimal solution, leads to a set of alternatives with different trade-offs among the objectives. These solutions are called Pareto optimal solutions or non-dominated solutions.

Although there are multiple Pareto optimal solutions, in practice, only one solution has to be selected for implementation. Therefore, in multi-objective optimization process two tasks can be distinguished, namely: i) find a set of Pareto optimal solutions, and ii) choose the most preferred solution out of this set. Since Pareto optimal solutions are mathematically equivalent, the latter task requires a decision maker (DM) who can provide subjective preference information to choose the best solution in a particular instance of the multi-objective optimization problem.

A system can be linear or nonlinear, time invariant or time varying, parametric or nonparametric, single input-single output (SISO) or multi input-multi output (MIMO), and so forth.

In practice, most control systems are non-linear and time varying. It is necessary to use a mathematical model to describe the relationships between system variables in the field of engineering and science. In some applications of system identification, a mathematical model is developed for controller design and to simulate an actual system.

Procedures involve in system identification are the acquisition of data, definition of model structure, parameter estimation and model validation.

The main task in model structure selection is to determine and select the significant terms to be included in the final model. Selecting a model with a large number of terms increases the complexity of the model and computation time. Meanwhile, it may cause over fitting of data. On the other hand, a model structure has too simple structure than required will not give good performance even for training data, thus the need for structural optimization.

Several research based on traditional approaches used various techniques of the non-linear mapping (Haber and Kevicsky, 1978). Since, the number of possible terms will be very large especially for complex system; simply expanding the system outputs in terms of its past input and output to non-linear model is impractical.

A more automated technique which will be more reliable is needed. Considering the requirements and the nature of modeling is to obtain an adequate predictive accuracy and optimum model structure, two objective functions namely minimization of mean square error (MSE) of the prediction and model complexity are proposed as the objective functions. To obtain a parsimonious structure the two objectives must be minimized simultaneously. This leads to the proposed multi-objective function optimization.

## **1.2 Statement of the Problem**

The major problem in system identification of a dynamic system is model structure selection so that the model predictive performance is good enough and the structure is simple with significant and effective terms. These conditions inspire



minimization in both performance and complexity of the model simultaneously. To obtain the above objectives a multi-objective differential evolution algorithm (MODE) is employed and investigated.

### **1.3 Objectives**

The main objective of this study is to develop an automated and effective algorithm for minimization of predictive error and model complexity in system identification of a dynamic system.

The primary objective of this study is to verify the principles of some parameter estimation methods such as least square and recursive least square methods and some evolutionary based algorithms like genetic algorithm and differential evolution algorithm.

The final objective would be the implementation of differential evolution algorithm in model structure selection of a dynamic system.

### **1.4 Scopes and Limitations**

Due to the wide development of study in the field of system identification and optimization of model structure, the research is limited to the following:

- Only time-discrete models were used.
- Data sets were generated using simulated system or/and published data.

- Only MODE method was used to estimate the best model structure.
- Algorithm implementations are carried out using MATLAB.
- The single input-single output systems were concerned.

### **1.5 Importance of the Research**

To start modeling, some parameters should be defined properly in advanced. Otherwise the structure will becomes complex and consequently causes excess computational load. This model will not guarantee to give good predictive accuracy due to loss of generalization. Although many researches resolve this problem using trial and error, but abundance of possible combination of parameters, make it impractical in many cases. For instance, to present a NARX model with maximum number of input and output lags of 3 and second-order of nonlinearity the maximum possible combination is  $2^{27} - 1 = 134,217,727$ , let alone when there is no information on lags and non-linearity order. However, many works have been done to find a parsimony model with minimum number of terms, but they still need to set some user defined parameter using trial and error.

The above reasons show the necessity of developing an algorithm to optimize both structure components and predictive error simultaneously.

### **1.6 Research Methodology**

The study starts with verification on the principles of system identification then some system parameter estimation methods such as LS and RLS will be studied. The next step is to be familiar with genetic algorithm and how it works to find an

optimal value for a function. Then differential evolution will be studied and its implementation will be investigated to find a minimum for Rosenbrock's function. Then the study will be continued on using DE to select an optimal model structure for a dynamic system.

## **1.7 Organization of the Report**

This report consists of five chapters. This chapter provides the research background, objectives, limitations and scope of the study, methodology and importance of research, summary of research contribution and the overall outline of this report.

Chapter 2 presents the literature review on related subjects concerning this research. In this second introductory chapter, the historical development of system identification, model structure selection, differential evolution algorithm and multi-objective optimization are presented.

Chapter 3 presents some features of system identification such as parameter estimation and model validation. Then principals of some parameter estimation methods such as least square and recursive least square are investigated and three case studies shows the application of these methods. Later the principals of genetic algorithm and differential evolution algorithm are studied.

Chapter 4 shows the implementation of differential evolution algorithm in multi-objective optimization. Then the effects of some DE's user defined parameters on dynamic model structure selection are studied. Three case studies show the result of this investigation.

Finally chapter 5 is the concluding chapter. This chapter summarizes the works done in entire study and provides recommendations for future works.

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