EXACT SOLUTIONS FOR UNIDIRECTIONAL MAGNETOHYDRODYNAMICS FLOW OF NON-NEWTONIAN FLUID IN A POROUS MEDIUM WITH AND WITHOUT ROTATION

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*To my **parents**, who have always given me love, care*

and cheer and whose prayers have always been a source of great inspiration for me

and

*my **wife***

who waited patiently for me to complete my studies.

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ABSTRACT

In this work, the physical problems dealing with unidirectional magnetohydrodynamic (MHD) flows of some viscoelastic fluids in a porous medium and rotating frame are investigated. By using modified Darcy's law, the corresponding equations governing the flow are modelled. Employing Fourier sine transform, new results in terms of the exact solutions of the modelled equations are generated for the problems of constantly accelerating and oscillatory MHD flows of second grade fluid in a porous space, accelerated MHD flows of an Oldroyd–B fluid in a porous medium and rotating frame, accelerating rotating MHD flow of second grade fluid in a porous space, accelerated MHD flow of Maxwell fluid in a porous medium and rotating frame, accelerated rotating MHD flow of generalized Burgers' fluid in a porous medium. The Fourier sine and Laplace transforms are then utilized to obtain new exact solutions by solving analytically the Stokes' first problem for two types of MHD fluids, namely the second grade fluid and Maxwell fluid in a porous medium and rotating frame. The new explicit solutions for the corresponding velocity fields are obtained for constant accelerated, variable accelerated and constant velocity flows for each problem mentioned above. The well – known solutions for Newtonian fluid in a porous medium in the cases mentioned above, are significantly shown to appear as the limiting cases of the present analysis. Finally, the effects of the material parameters (i.e. rotation, MHD and porous) on the velocity fields are demonstrated via graphical illustrations. These graphs generally show that: (i) by increasing the rotation parameter, this would lead to a decrease in the real part of the velocity profile; however for the magnitude of imaginary part of the velocity profile, it is found to be quite the opposite, (ii) when MHD parameter increases, the real and imaginary parts of the velocity profile decrease, and (iii) by increasing the porous parameter, both parts of the velocity profile increase.

ABSTRAK

Kajian ini adalah mengenai masalah fizikal berkaitan dengan aliran sehala magnetohidrodinamik (MHD) bagi cecair kenyal-likat dalam medium berliang dan kerangka berputar. Dengan menggunakan hukum Darcy terubahsuai, persamaan sepadan yang menerajui cecair itu dimodelkan. Melalui jelmaan sinus Fourier, keputusan-keputusan baru dalam bentuk penyelesaian tepat bagi persamaan termodel dijana bagi masalah-masalah berikut: pecutan secara malar dan berayunan bagi aliran MHD dengan cecair gred kedua dalam ruang berliang, aliran MHD berpecutan dan berputar bagi cecair Oldroyd-B dalam medium berliang dan kerangka berputar, cecair gred kedua beraliran MHD yang memecut dan berputar dalam suatu ruang berliang, aliran MHD berpecutan bagi cecair Maxwell dalam medium berliang dan kerangka berputar, dan bagi aliran MHD berputar dengan cecair Burgers teritlak dalam medium berliang. Seterusnya digunakan jelmaan sinus Fourier dan Laplace untuk memperoleh penyelesaian tepat dan baru dengan menyelesaikan secara analisis masalah pertama Stokes bagi dua jenis bendalir MHD iaitu bendalir gred kedua dan bendalir Maxwell dalam medium berliang dan berputar. Ungkapan-ungkapan eksplisit yang baru bagi medan halaju sepadan diperoleh bagi aliran dipecutkan secara malar, dipecutkan pembolehubah dan halaju malar bagi setiap masalah yang dinyatakan. Penyelesaian terkenal bagi cecair Newtonan dalam suatu medium berliang dalam kes-kes yang dinyatakan di atas, secara signifikannya ditunjukkan sebagai kes-kes pengehad bagi analisis semasa. Akhirnya, kesan-kesan parameter bahan (iaitu putaran, MHD dan berliang) terhadap medan halaju diperlihatkan melalui ilustrasi bergraf. Graf-graf ini umumnya menunjukkan: (i) menaikkan parameter putaran, didapati bahagian nyata profil halaju menurun; walau bagaimanapun bagi magnitud bahagian khayal profil halaju, didapati bertentangan, (ii) apabila parameter MHD bertambah, maka bahagian-bahagian nyata dan khayal profil halaju berkurang, dan (iii) menaikkan parameter berliang, kedua-dua bahagian profil halaju juga menaik.

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LIST OF SYMBOLS AND ABREVIATIONS

ρ	_	Fluid density
$\boldsymbol{V} = (u, \upsilon, w)$	_	Velocity filed
$\boldsymbol{J}$	_	Current density
$\boldsymbol{B} = B_\circ + b$	_	Total magnetic field
$B_\circ$	_	Applied magnetic field
b	_	Induced magnetic field
$\boldsymbol{E}$	_	Electronic field
μ_m	_	Magnetic permeability
$\boldsymbol{R}$	_	Darcy resistance
$\boldsymbol{S}$	_	Extra stress tensor
$\boldsymbol{T}$	_	Cauchy stress tensor
μ	_	Dynamic viscosity
$\alpha_i \ (i = 1, 2)$	_	Material constants
$\boldsymbol{A}_1$, $\boldsymbol{A}_2$	_	First two Rivlin – Eriksen tensors
t, τ, δ	_	Time
p	_	Pressure
ϕ	_	Porosity
ν	_	Kinematic viscosity
Re	_	Reynolds number
λ_1	_	Relaxation time
λ_2	_	Retardation time
λ_3, λ_4	_	Material constants
x, y, z	_	Three Cartesian coordinates

i	–	$\sqrt{-1}$
σ	–	Electrical conductivity of fluid
Ω	–	Solid body rotation angular
m	–	Hall parameter
k	–	Permeability of the porous medium
ω, ϖ	–	Frequency on the plate

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CHAPTER 1

INTRODUCTION

1.1 Research Background

The equations which govern the flows of Newtonian fluids are Navier–Stokes equations. It is a known fact that these equations in general are non–linear partial differential equations and few analytic solutions are reported to exist in the present literature. To obtain analytic solutions of such equations is still a current topic of research. Therefore mathematicians, physicists and engineers are involved in obtaining the analytic solutions of such equations by employing various techniques. Analytic solutions are very important not only because they serve as approximations to some specific problems but also serve a very important purpose, namely, they can be used as tests to verify numerical methods that are developed to study complex flow problems.

Recently the non Newtonian fluids are increasingly being considered as more important and appropriate in technological applications in comparison with Newtonian fluids.

A large class of real fluids does not exhibit the linear relationship between stress and the rate of strain. Because of the non linear dependence, the analysis of the behaviour of fluid motion of the non Newtonian fluids tends to be more complicated and subtle in comparison with that of the Newtonian fluid.

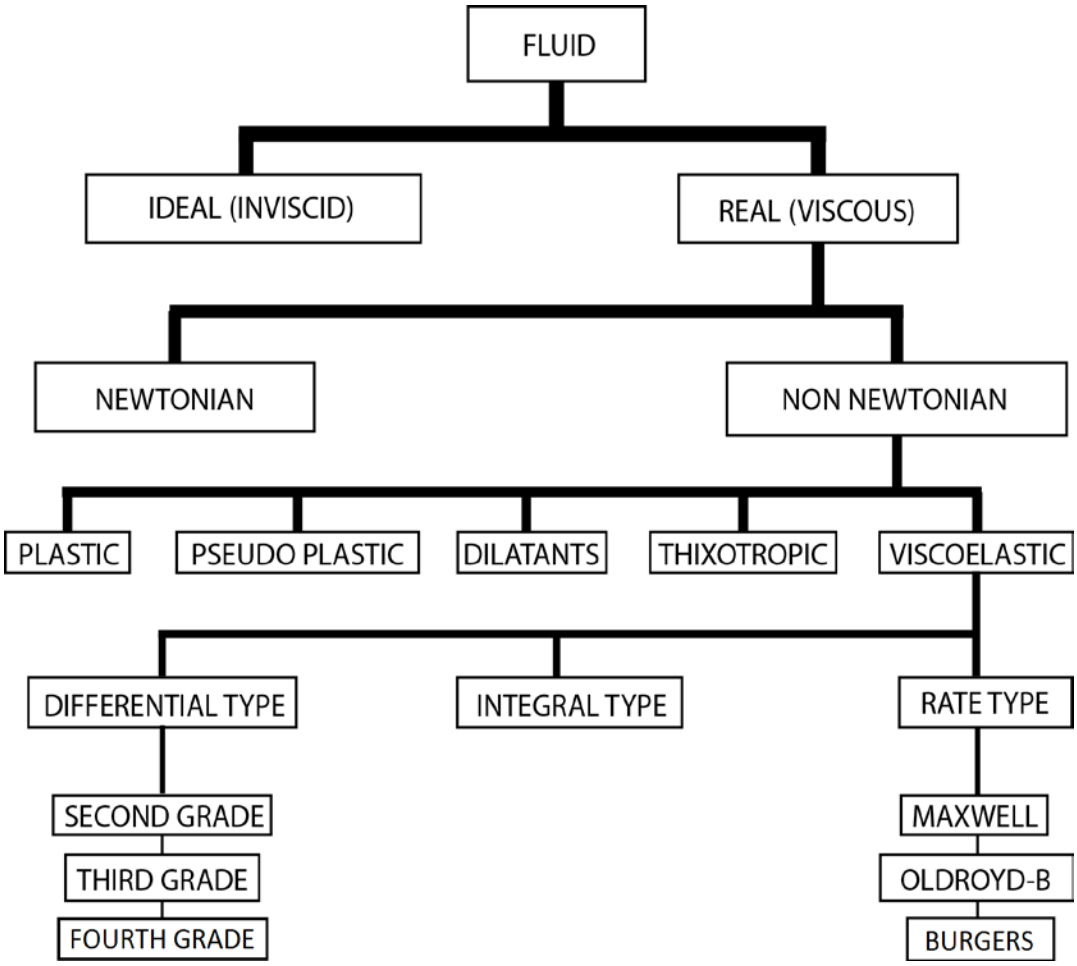

Figure 1.1: Classification of fluid.

The inadequacy of classical Navier-Stokes theory to describe rheological complex fluids such as polymer solutions, blood, paints, certain oils and greases, has led to the development of several theories of non Newtonian fluids. Because of the complexity of these fluids, several constitutive equations have been proposed. These constitutive equations are somewhat complicated and contain special cases of some of the previous fluids.

The constitutive equations of viscoelastic fluids are usually classified under the categories of differential type, rate type and integral type models (see Figure 1.1). The celebrated Navier-Stokes model describes a fluid of the differential type, and rate type models are used to describe the response of fluids that have slight memory such as diluted polymeric solutions while integral models are used to describe materials such as polymer melts that have considerable memory. Here by memory

one means the dependence of the stress on the history of the relative deformation gradient.

The first viscoelastic rate type, which is still used widely, is due to Maxwell. While Maxwell did not develop his model for polymeric liquids but instead for air, the methodology that he used can be generalized to produce a plethora of models. Maxwell recognized that the body had a means for storing energy and a means for dissipating energy, where the storing of energy characterizes the fluids elastic response and the dissipation of energy characterizes its viscous nature. Recently, Rajagopal and Srinivasa (2000) have built upon the seminal work of Maxwell and developed a systematic framework within which models for a variety of rate type viscoelastic fluids can be obtained.

The study of viscoelastic fluids in a porous medium offers special challenges to mathematicians, engineers and numerical analysts, since such studies are important in enhanced oil recovery, paper and textile coating and composite manufacturing processes. Moreover, the study of the physics of the magneto hydrodynamic (MHD) flow of viscoelastic fluid through a porous medium in the presence of magnetic field has become the basis of many scientific and engineering applications. Specifically, the study of the interaction of the coriolis force with electromagnetic force is important with some geophysical as well as astrophysical problems.

The solid body rotation is defined as a fluid or object that rotates at a constant angular velocity so that it moves as a solid (see Figure 1.2).

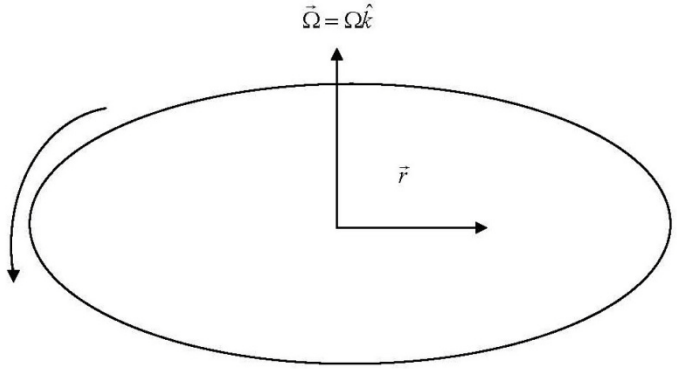

Figure 1.2: Rotating frame.

The influence of an external uniform magnetic field on the rotating flows studied by various workers Abelman *et al.* (2009) amongst the various models of non Newtonian fluids, the viscoelastic fluids of rate type models have acquired the special status.

This research project specifically considers and investigates the problems dealing with unsteady unidirectional flows of some non Newtonian fluids in a porous medium. By using modified Darcy's law and employing Fourier transformation technique and Fourier - Laplace integral transform method; the exact analytical solutions are developed for the MHD flows of the problems in a porous medium and rotating frame.

1.2 Statement of the Problem

This research specifically studies the MHD second grade and rate type fluids in a porous medium and rotating frame. The problems are related to

1. Constantly accelerating and oscillatory MHD flows of second grade fluid in a porous medium.
2. Accelerated MHD flows of an Oldroyd–B fluid in a porous medium and rotating frame.

   The limiting cases of this problem are considered as follows.

   2.1 Accelerated MHD flows of Second grade fluid in a porous medium and rotating frame.

   2.2 Accelerated MHD flows of Maxwell fluid in a porous medium and rotating frame.
3. Accelerated MHD flows of generalized Burgers' fluid in a porous medium and rotating frame.
4. New exact solution for Rayleigh–Stokes problem of Maxwell and second grade fluids in a porous medium and rotating frame.

1.3 Objectives of the Research

The objectives are

1. To derive exact solution of MHD second grade fluid induced by the constantly accelerating and oscillatory flows in a porous medium.
2. To establish exact solution for the accelerated MHD flow of an Oldroyd-B fluid in a porous medium and rotating frame.
3. To determine exact solution of accelerated MHD flow of Maxwell fluid in a porous medium and rotating frame.
4. To develop exact solution for the accelerated MHD flow of second grade fluid in a porous medium and rotating frame.
5. To examine the solution for large times for MHD flow of generalized Burgers' fluid in a porous medium.
6. To develop exact solutions for transient MHD flow of a second grade fluid in a porous medium and rotating frame.
7. To generate new exact solution for Rayleigh–Stokes problem of MHD Maxwell fluid in a porous medium and rotating frame.

1.4 Scope of the Research

This research will consider the problems of which the fluids are assumed to be incompressible and the flow is of two types namely MHD unidirectional flow and MHD rotating flow in porous medium.

By using modified Darcy's law, the equations governing the flows are modelled. The study will employ Fourier sine transform and Fourier–Laplace integral transform method, to derive the exact analytical solutions. The problems are simplified as linear ordinary differential equation, and following the works by Fetecau (2005), Hayat *et al.* (2008a) and Hayat *et al*. (2008b), Hussain *et al* . (2010), we are able to develop the exact analytical solutions.

1.5 Significance of Research

The significance of the research are:

1. The research of viscoelastic fluids in a porous medium offers special challenges to mathematicians, engineers and numerical analysts.
2. The research of the physics of MHD flow of a viscoelastic fluid through a porous medium in the presence of a magnetic field has become the basis of many scientific and engineering applications.
3. More realistic mathematical models to interpret and enhance understanding of the flow of viscoelastic fluid in a porous space under appropriate conditions through mathematical behaviour.
4. The research of non–Newtonian fluid in a porous medium and rotating frame will be useful for many applications in meteorology, geophysics and turbo machinery.
5. Establish the analytical exact solutions to serve as :

    i. Approximations to some specific problems.
    ii. Useful as tests to verify numerical methods that are developed to study complex flow problems.

1.6 Methodology

To achieve these objectives, the methodology adopted is Fourier sine transforms and Fourier–Laplace integral transform methods. These methods are essentially mathematical techniques which can be used to solve several problems in mathematics and engineering. The main question as to why one should use the traditional Fourier sine transform and Fourier–Laplace integral transform methods when other methods are available, justifiably, these traditional methods have the following important features. They are a very powerful technique for solving these kinds of problems, which literally transforms the original linear differential equation into an elementary algebraic expression. More importantly, the transformation is

used to generate the solution of certain problems with less effort and in a simple routine way.

1.7 Thesis Outline

This thesis is divided into seven chapters including this introductory chapter. Chapter 2 begins with a review of previous studies on Newtonian fluids and non–Newtonian fluids. The discussion focuses on rate type and differential type fluids particularly, second grade fluid, Maxwell fluid, Oldroyd–B fluid and Generalized Burgers' fluid. These fluids occupy the porous space and are electrically conducting. In addition, the whole system is also rotating.

Chapter 3 deals with MHD flows of second grade fluid in a porous medium. We investigate the constant accelerated flows over an oscillating plate. In addition, the fluid is electrically conducting and the Hall effects are taken into account. Two flow problems are considered and exact solutions for velocity field are established. In the first problem, the fluid occupying the half space is bounded by an oscillating and accelerated rigid plate. The second problem deals with the flow between the two plates. The upper plate is taken stationary while the lower one is constantly accelerated and oscillating. In these problems, both sine and cosine oscillations are considered.

In Chapter 4 the rotating flow of an electrically conducting Oldroyd–B fluid due to an accelerated plate is examined. Modified Darcy's law is used to formulate the mathematical problem in a porous space. Two cases namely constant and variable accelerated flows are addressed. Fourier sine transform technique is adopted to solve the resulting problems. Many interesting results in are obtained as the special cases of the present analysis. Finally, the effects of emerging flow parameters on the velocity component are displayed and discussed.

The aim of Chapter 5 is to determine the exact steady-state solution of magnetohydrodynamic (MHD) and rotating flow of generalized Burgers' fluid induced by (1) constant accelerated plate (2) variable accelerated plate. This is attained by using the Fourier sine transform. This result is then presented in equivalent forms in terms of exponential, sine and cosine functions. Similar solutions for Burgers', Oldroyd–B, Maxwell, Second grade and Navier–Stokes fluids can be shown to appear as the limiting cases of the present exact solution. The graphical results illustrate the velocity profiles which have been determined for the flow due to the constant and variable acceleration of an infinite flat plate.

The aim of Chapter 6 is to determine new exact solution of a magnetohydrodynamic (MHD) and rotating flow of some non–Newtonian fluids over a suddenly moved flat plate. Two fluids namely Second grade and Maxwell fluids are addressed. The new exact solution is derived by using the Fourier sine and Laplace transforms. Based on the modified Darcy's law, the expression for the velocity field is obtained. Many interesting available results in the literature are obtained as limiting cases of our solution. Finally some graphical results are presented for different values of the material constants.

Finally, Chapter 7 concludes the thesis with a summary of the work presented together with suggestions for further research in the future.

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