CONTINUOUS-TIME NON-LINEAR NON-GAUSSIAN STATE-SPACE MODELING OF ELECTROENCEPHALOGRAPHY WITH SEQUENTIAL MONTE CARLO BASED ESTIMATION

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Dedicated to *Buddha*, *Dharmma*, *Sangha*, and my
Beloved Daddy, Mummy, Sisters & friends.

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ABSTRACT

Biomedical time series are non-stationary stochastic processes with hidden dynamics that can be modeled by state-space models (SSMs), and processing of which can be cast into optimal filtering problems for SSMs. The existing studies assume discrete-time linear Gaussian SSMs with estimation solved analytically by Kalman filtering for biomedical signals which are continuous, non-Gaussian and non-linear. However, general non-linear non-Gaussian models admit no closed form filtering solutions. This research investigates the general framework of continuous-time non-linear and non-Gaussian SSMs with sequential Monte Carlo (SMC) estimation for biomedical signals generally, electroencephalography (EEG) signal in particular, to solve two of its analysis problems. Firstly, this study proposes time-varying autoregressive (TVAR) SSMs with non-Gaussian state noise to capture abrupt and smooth parameter changes that are inappropriately modeled by Gaussian models, for parametric time-varying spectral estimation of event-related desynchronization (ERD). Evaluation results show superior parameter tracking performance and hence accurate ERD estimation by the proposed model. Secondly, a partially observed diffusion model is proposed for more natural modeling the continuous dynamics and irregularly spaced data in single-trial event-related potentials (ERPs) for single-trial estimation of ERPs in noise. More efficient Rao-Blackwellized particle filter (RBPF) is used. Evaluation on simulated and real auditory brainstem response (ABR) data shows significant reduction in noise with the underlying ERP dynamics clearly extracted. In addition, two non-linear non-Gaussian stochastic volatility (SV) models are proposed for better modeling of non-Gaussian dynamics of volatility in EEG noise especially of impulsive type. Application to denoising of simulated ABRs with artifacts shows well estimated volatility pattern and better elimination of impulsive noise with SNR improvement of 12.46dB by the best performing non-linear Cox-Ingersoll-Ross process.

ABSTRAK

Siri masa bioperubatan, stokastik process bukan pegun dengan dinamik tersembunyi, boleh dimodel oleh model ruang keadaan (SSM), dengan pemprosesannya boleh dijadikan masalah penurasan. Kajian kini menganggap SSMs sebagai diskrit, linear Gaussian dengan anggaran secara analitis oleh penapisan Kalman untuk isyarat bioperubatan yang biasanya berterusan, tidak linear dan tidak Gaussian. Tetapi, model tidak linear tidak Gaussian tiada penyelesaian tertutup. Kajian ini menyelidik rangka kerja umum model tidak linear dan tidak Gauss dengan penapisan *Monte Carlo* berjujukan (SMC) untuk isyarat bioperubatan secara umum dan *electroencephalography* (EEG) khasnya, untuk menyelesaikan dua masalah analisis khusus. Pertama, kajian ini mencadangkan model *time-varying autoregressive (TVAR)* dengan bunyi keadaan tidak Gaussian untuk menangkap perubahan parameter yang mendadak dan lancar yang tidak sesuai dimodelkan oleh model Gaussian, untuk anggaran spektrum *event-related desynchronization* (ERD) yang berubah masa. Penilaian menunjukkan model ini memberikan prestasi penjejakan yang lebih baik dan anggaran ERD yang tepat. Kedua, kajian ini mencadangkan *partially observed diffusion model* untuk memodelkan dinamik berterusan dan data berjarakan tak seragam dalam *single-trial event-related potentials* (ERPs). Penapis *Rao-Blackwellized* (RBPF) yang lebih efektif digunakan. Penilaian ke atas data *auditory brainstem response* (ABR) simulasi dan benar menunjukkan pengurangan ketara dalam bunyi dengan dinamik ERP tersembuyi jelas diekstrak. Dua model volatiliti stokastik yang tidak linear tidak Gaussian dicadangkan untuk memodelkan dinamik tidak Gaussian dalam volatility bunyi EEG dengan lebih baik terutama yang jenis impulsif. Aplikasi dalam pembuangan bunyi untuk simulasi ABRs dengan artifak menunjukkan anggaran corak volitiliti yang bagus dan pembuangan bunyi impulsif yang lebih baik dengan peningkatan SNR 12.46dB oleh proses Cox-Ingersoll-Ross yang tidak linear yang mempunyai prestasi tertinggi.

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ABR	-	Auditory brainstem response
AIC	-	Akaike information criterion
APF	-	Auxiliary particle filter
AR	-	Autoregressive
ARMA	-	Autoregressive moving-average
BCI	-	Brain computer interface
BIC	-	Bayesian information criterion
BS	-	Bowman-Shenton
CIR	-	Cox-Ingersoll-Ross
dB	-	decibel
EEG	-	Electroencephalography
EKF	-	Extended Kalman filter
EM	-	Expectation-maximization
ERD	-	Event-related desynchronization
ERP	-	Event-related potential
ERS	-	Event-related synchronization
ESS	-	Effective sample size
HMM	-	Hidden Markov model
i.i.d.	-	Independent identically distributed
IS	-	Importance sampling

KF	-	Kalman filter
LB	-	Ljung-Box
LMSE	-	log mean square error
MC	-	Monte Carlo
ML	-	Maximum likelihood
MLE	-	Maximum likelihood estimation
MMSE	-	Minimum mean-squared error
MSE	-	Mean square error
OU	-	Ornstein-Uhlenbeck
PF	-	Particle filter
RBPF	-	Rao-Blackwellized particle filter
RLS	-	Recursive least square
SDE	-	Stochastic differential equation
SIR	-	Sequential importance sampling-resampling
SIS	-	Sequential importance sampling
SMC	-	Sequential Monte Carlo
SNR	-	Signal-to-noise ratio
SSM	-	State-space model
STFT	-	Short-time Fourier transform
SV	-	Stochastic volatility
TVAR	-	Time-varying autoregressive
TVARMA	-	Time-varying autoregressive moving-average
$\{\mathbf{x}_t\}$	-	Hidden states
$\{\mathbf{y}_t\}$	-	Observations
$\mathbf{x}_0$	-	Initial state

$\mathbf{x}_{0:t} = \{\mathbf{x}_0, \ldots, \mathbf{x}_t\}$	-	Sequence of states until time *t*
$\mathbf{y}_{0:t} = \{\mathbf{y}_0, \ldots, \mathbf{y}_t\}$	-	Sequence of observations until time *t*
$\mathbf{v}_t$	-	Observation noise
$\mathbf{w}_t$	-	State noise
θ	-	Model parameters
$\hat{\theta}$	-	Estimated model parameters
σ_v^2	-	Variance of observation noise
σ_w^2	-	Variance of state noise
σ_g^2	-	Variance of driving noise for moving-average (MA) coefficient
q_c^2	-	Dispersion parameter of Cauchy distribution
$q(x)$ and $\pi(y \mid x)$	-	Importance density
$f_\theta(x' \mid x)$	-	State transition density
$g_\theta(y \mid x)$	-	Observation density
$p(\mathbf{x}_{0:t} \mid \mathbf{y}_{0:t})$	-	Posterior density
$p(\mathbf{x}_t \mid \mathbf{y}_{0:t})$	-	Filtering density
$p(\mathbf{x}_t \mid \mathbf{y}_{0:t-1})$	-	One-step ahead prediction density
$p_\theta(\mathbf{y}_{0:t})$	-	Marginal likelihood of $\mathbf{y}_{0:t}$ given θ
$p(\mathbf{y}_t \mid \mathbf{y}_{0:t-1})$	-	Predictive likelihood
$\ell_T(\theta)$	-	Log-likelihood of $\mathbf{y}_{0:T}$ given θ
$\hat{\mathbf{x}}_{t\mid t-1}$	-	Mean of $p(\mathbf{x}_t \mid \mathbf{y}_{0:t-1})$
$\mathbf{P}_{t\mid t-1}$	-	Covariance of $p(\mathbf{x}_t \mid \mathbf{y}_{0:t-1})$

$\hat{\mathbf{x}}_{t\|t}$	-	Mean of $p(\mathbf{x}_t \mid \mathbf{y}_{0:t})$
$\mathbf{P}_{t\|t}$	-	Covariance of $p(\mathbf{x}_t \mid \mathbf{y}_{0:t})$
$\bar{\theta}_t$	-	Mean of $p(\theta_t \mid \mathbf{y}_{1:t})$
$\mathbf{V}_t$	-	Covariance of $p(\theta_t \mid \mathbf{y}_{1:t})$
$\mathbf{x}_t^{(i)}$	-	Particle or sample i at time t
N	-	Number of samples
N_{eff}	-	Effective sample size
$\hat{N}_{eff}$	-	Estimated effective sample size
$W^{(i)}$	-	Normalized weights
$\tilde{w}^{(i)}$	-	Unnormalized weights
p	-	Order of AR model
q	-	Order of MA model
δ	-	Discounting factor
$b(\mathbf{X}_t, \boldsymbol{\theta})$	-	drift
$\sigma(\mathbf{X}_t, \boldsymbol{\theta})$	-	diffusion coefficient
$\mathbf{W}_t$	-	Standard Brownian motion
β	-	Rate of reversion
$U[a,b]$	-	Uniform distribution over interval $[a,b]$
$N(\mu, \sigma^2)$	-	Univariate Gaussian distribution with mean μ and variance σ^2
$N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$	-	Univariate Gaussian distribution with mean vector $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$

$C(\boldsymbol{\mu}_c, \mathbf{Q}_c)$	-	Multivariate Cauchy distribution with location $\boldsymbol{\mu}_c$ and dispersion matrix $\mathbf{Q}_c$
$S\alpha S(\alpha, \gamma)$	-	Symmetric alpha-stable distribution with characteristic exponent α and scale parameter γ
R	-	Number of realizations
k	-	Number of estimated parameters
T	-	Observation length
Δt	-	Time step
$\delta(.)$	-	Delta function
∇	-	Gradient

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CHAPTER 1

INTRODUCTION

1.1 Introduction

Most physiological processes from human body are accompanied by or manifest themselves as *signals* that reflect their nature and activities. One type of such signals is electrical in the form of potential, among others are electromyogram (EMG), electrocardiogram (ECG), electroencephalography (EEG), phonocardiogram (PCG) (Rangayyan, 2002). The signal as a function of time is time series in mathematical sense. These bioelectrical signals, usually in digitized form, can be used for medical diagnostics purposes and human-computer interaction. EEG signals are studied in this thesis. EEG is bioelectrical activity of the brain recorded at the scalp using surface electrodes, which is an average of multifarious activities of many small zones of the cortical surface beneath the electrode. Clinically, several channels of EEG are recorded simultaneously from various locations on the scalp (Rangayyan, 2002). Figure 1.1 shows locations of the electrodes placement recommended by the International Federation of Societies for Electroencephalography and clinical Neurophysiology (After Rangayyan (2002)). This research proposes non-linear non-Gaussian modeling of EEG signals with estimation by sequential Monte Carlo (SMC) method, to solve two specific EEG processing problems, i.e. spectral estimation of event-related desynchronization (ERD) and single-trial estimation of event-related potentials (ERPs).

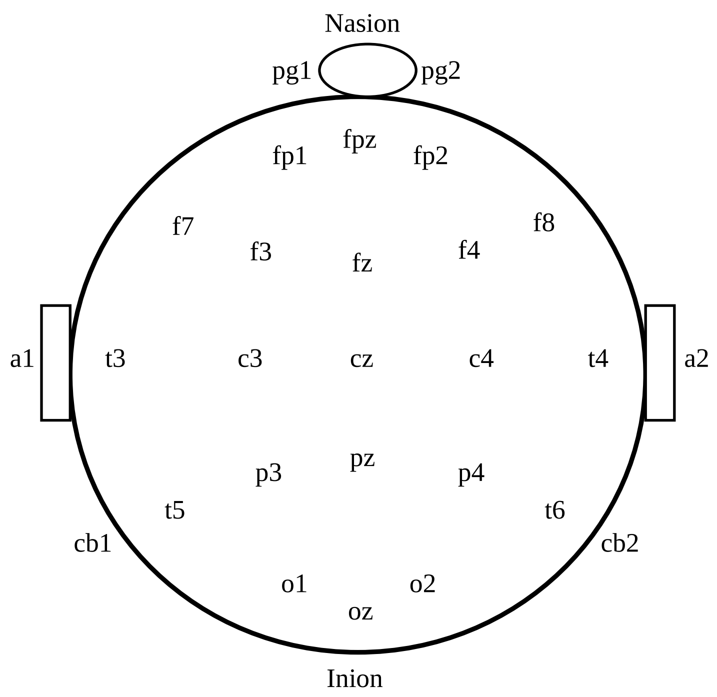

Figure 1.1: The 10 – 20 system of electrode placement for EEG recording (Copper *et al*., 1980). Notes regarding channel labels: pg- naso-pharyngeal, a- auricular (ear lobes), fp- pre-frontal, f- frontal, p- pareital, c- central, o- occipital, t- temporal, cb- cerebellar, z- midline, odd number on the left, even numbers on the right of the subject.

1.2 Background of Problems

Many digital signal processing (DSP) techniques have been adopted in the modern biomedical engineering field for the analysis of biomedical signals. Processing of biomedical signals such as recovering the clean signals from noises and artifacts (filtering) as well as extracting its features in time or frequency domain are of much importance to their uses as reliable tools for diagnostics purposes. Biomedical time series are complex real world processes which are highly non-stationary. The underlying dynamics behind biomedical signals contain important information for analysis. Many time series models and analysis techniques can be used for biomedical signal processing. Non-stationary processes (Kitagawa, 1987) with underlying hidden dynamics can be modeled by state-space models (SSMs).

The SSMs have become a powerful tool for modeling and forecasting dynamic systems. The SSM consists of two components: (1) state equation which models the dynamics of the hidden states $\{\mathbf{x}_t\}$ where t is the discrete time index, typically following a Markov process and (2) observation equation which describes the mapping of the hidden states to the observations $\{\mathbf{y}_t\}$. Besides, this formulation enables modeling of underlying hidden process behind the observations. The SSMs have been applied as statistical modeling framework for various kinds of time series such as speech signals, biomedical signals, DNA sequences, and financial time series. SSMs have been extensively used for modeling biomedical signals. However, the existing studies assume linear Gaussian model for biomedical signals, which is inappropriate for the complex real processes typically which exhibit non-linearity and non-Gaussianity. Besides, biomedical signals are mostly modeled by discrete-time model in the literature. However, biomedical signals are generated by continuous process for which continuous-time modeling may be appropriate choice, even though biomedical signals themselves are discrete-time samples. Continuous-time models are able to model conveniently and naturally irregularly spaced data which is also inherent in biomedical signal.

Formulation in state-space form enables online inference of the hidden states given the observations, which is known as Bayesian filtering or optimal filtering problem (Doucet *et al.*, 2000). Within the Bayesian framework, all the information about the system states $\mathbf{x}_{0:t} = \{\mathbf{x}_0, \ldots, \mathbf{x}_t\}$ given sequence of observations $\mathbf{y}_{0:t} = \{\mathbf{y}_0, \ldots, \mathbf{y}_t\}$ is reflected in the posterior density $p(\mathbf{x}_{0:t} \mid \mathbf{y}_{0:t})$. Since the observation often arrives sequentially in time, the objective is to perform online state inference which involves estimating recursively in time the posterior density $p(\mathbf{x}_{0:t} \mid \mathbf{y}_{0:t})$ and its marginals (including filtering density $p(\mathbf{x}_t \mid \mathbf{y}_{0:t})$). Many biomedical signal processing problems involve online inference of the underlying processes behind these observed non-stationary biomedical signals such as noise filtering and feature extraction, which can be considered as optimal filtering problems. Optimal filtering methods have been applied successfully to problems associated with biomedical signal processing.

However, the posterior distributions can be evaluated in closed form only in a few cases including the linear Gaussian state-space models using the well-known Kalman filter (KF) and hidden finite state-space Markov chain using hidden Markov model (HMM) filter. Analytical solution is intractable for more general non-linear non-Gaussian SSMs. KF has been used extensively to solve the optimal filtering problems related to signal processing based on linearity and Gaussianity assumptions of the models. Relaxing the assumptions to non-linearity and non-Gaussianity precludes analytical filtering solutions. This also poses a challenge in estimation of non-linear non-Gaussian modeling of biomedical signals. To solve this problem, many approximation schemes have been proposed such as the popular extended Kalman filter (EKF) which approximate the nonlinear model by local linearization using first order Taylor series expansion, however fails for substantial nonlinearity. Another example is the Gaussian sum filter which approximates the posterior distribution by a mixture of Gaussians. These approximation methods are still constrained by the assumption of linearity and Gaussianity. Refer to Cappe *et al.* (2007) for a review.

Alternative approaches are sequential Monte Carlo (SMC) methods or particle filtering (PF) methods which have significant advantages that allow inference of the full posterior densities in more general non-linear non-Gaussian SSMs. The SMC methods are simulation-based methods which recursively generate and update a set of weighted samples or *particles* to approximate the posterior density sequentially in time. (Refer to Doucet *et al.* (2000) for introduction and Cappe *et al.* (2007); Doucet and Johansen (2008) for survey of recent advances). The SMC filtering has been applied widely for discrete-time dynamical system and its extension to continuous-time diffusion models have been recently proposed (Fearnhead *et al*., 2008; Poyiadjis *et al*., 2006; Moral *et al*., 2001; Golightly and Wilkinson, 2006; Rimmer *et al*., 2005). Implementation of SMC methods is efficient, parallelizable and scalable. The flexibility of SMC methods is traded off with their expensive computation. However, the great increase of computational power enable their use in real-time applications in many areas including computer vision, signal processing, target tracking, control, financial econometrics, statistics, and robotics. However, there are limited studies of applying SMC methods in biomedical signal processing in the literature.

The likelihood evaluation and maximum likelihood estimation (MLE) of linear Gaussian SSMs can be obtained analytically by KF. The analytical derivation of the marginal likelihood for the non-linear and non-Gaussian SSMs is intractable. This thesis also considers model parameter estimation problems in general SSMs. Many SMC techniques have been proposed to solve unknown static parameter estimation for general SSMs (Kantas *et al.*, 2009). In MLE, the optimal estimates are obtained by maximizing the particle approximated (marginal) likelihood of the observations. Gradient methods and expectation-maximization (EM) algorithm have been proposed for maximizing the likelihood in the SMC approximation framework. These methods provide guaranteed convergence, however, tends to be easily trapped in a local maximum. We consider another approach i.e. Bayesian estimation where the unknown parameters are augmented with the hidden states and cast the problem to the filtering one. This method is simple and needs less computational effort than the approximation based maximum likelihood approach and thus more practical for real biomedical signal processing problems. Besides, the setting of prior distribution of parameters can be tailored by prior knowledge. This method, however, gives estimates optimal in minimum mean-squared error (MMSE) sense.

1.2.1 Filtering Problems in EEG Analysis

In this thesis, we focus on state-space modeling and sequential estimation of a particular type of biomedical signal i.e. EEG and consider two classes of problems related to EEG analysis which can be formulated into optimal filtering problems, i.e. (1) Parametric time-varying spectral estimation and (2) Single-trial event-related potential (ERP) estimation. Different variants of state-space models for EEG have been proposed respectively to solve these two problems, and will be reviewed in Section 2.7. However, the existing studies assume linear Gaussian modeling for EEG signals with parameter estimation solved analytically by KF. But, real EEGs are non-linear non-Gaussian processes, for which closed-form solution for the optimal filtering is not available. Besides, continuously evolving processes in EEG are typically modeled by discrete models.

1.2.1.1 Parametric Time-varying Spectral Estimation

Time-varying spectrum of non-stationary EEG signals can be obtained by parametric approach using time-varying autoregressive (TVAR) models and time-varying autoregressive moving-average (TVARMA) models. The parametric spectral estimates, which provide high time resolution, have been used for analysis of event-related desynchronization (ERD) and synchronization (ERS). ERD and ERS are used to represent frequency-specific changes of on-going EEG activity, induced by specific stimulus, which consist either of decrease or increase of power in specific frequency band. The objective is to estimate sequentially the TVAR coefficients which are subsequently used to compute the time-varying power spectral density. This can be formulated into optimal filtering problem, i.e. formulating TVAR model into state-space model and estimating sequentially in time the filtered density of TVAR coefficients given the EEG observations. The challenge is that the underlying TVAR process of EEG, especially in ERD and ERS, exhibit abrupt changes, which is kind of non-Gaussian behavior and cannot be tracked rapidly by Gaussian TVAR models used in the existing studies.

1.2.1.2 Single-trial ERP Estimation

ERPs are scalp-recorded bioelectrical potentials generated by brain activity in response to specific stimulation. ERPs provide useful information about various neurological disorders and cognitive processes. Besides, ERP waveforms vary from trial to trial due to different degrees of fatigue, habituation, or levels of attention of subjects (Georgiadis *et al*., 2005). The single-trial based ERP estimation involves extracting these inter-trial dynamics of ERPs hidden in various noises e.g. background EEG and non-neural artifacts, typically with poor signal-to-noise ratio (SNR). This can be considered as optimal filtering problem which aims to estimate sequentially in time the filtered density of ERP parameters given the noisy EEG observations. The underlying physiological process in ERP dynamics is continuous by nature, which is however modeled by the currently used discrete-time models. Besides, the irregularly spaced data problem inherent in ERP estimation cannot be solved implicitly by the discrete-time models. The variances in real EEG noise are time-varying with smooth and occasionally abrupt changes, especially in noises of impulsive type. This non-Gaussian characteristic of EEG noise volatility cannot be properly modeled by conventional linear Gaussian random-walk of log-variance.

To the best of author's knowledge, there are no studies on applying continuous-time non-linear non-Gaussian state-space models estimated using SMC filtering to address these two problems, and limited studies for biomedical signal processing in general.

1.3 Statement of Problems

The problems of the research are summarized as follows:

(1) Existing studies assume inappropriate linear Gaussian SSMs for biomedical signals. Relaxing this invalid assumption to non-linear non-Gaussian form in modeling biomedical signals is the main concern of this research.

(2) Existing studies use discrete-time models for biomedical signals which are typically continuous processes. Continuous-time models may be appropriate for continuous process of biomedical signals and are ideally suited for modeling irregularly spaced data in biomedical signals. Continuous-time state-space modeling of continuous transient process in real biomedical signals is considered in this research.

(3) Many biomedical signal processing problems are optimal filtering problems. This research attempts to investigate the application of optimal filtering methods to biomedical signal processing. The online state inference problem for linear Gaussian model can be solved analytically using KF. However, non-linear non-Gaussian state-space modeling of biomedical signals renders the closed form solution intractable. Inference problem for non-linear non-Gaussian SSMs of biomedical signals is addressed in this research.

(4) Motivated by the abovementioned more appropriate modeling of biomedical signals in continuous-time non-linear non-Gaussian models and advantages of SMC methods for their estimation, studies of which are still limited in the literature, this research investigates the non-linear non-Gaussian SSMs with online inference problems solved by SMC methods for biomedical signal processing. In addition, we investigate continuous-time state-space modeling of biomedical signal with SMC estimation.

(5) This research focuses on the state-space modeling and estimation of a particular type of biomedical signal i.e. EEG, with application to two specific filtering problems as discussed in Section 1.2.1. EEG signal is inappropriately modeled by linear Gaussian models with estimation by KF in the existing studies. This is because EEGs are non-linear non-Gaussian processes, modeling of which however, renders filtering solution intractable. Besides, continuous process of EEG is modeled by discrete-time models. Development of continuous-time non-linear non-Gaussian models for EEG time series with their online parameter estimation solved by SMC filtering methods is the main interest of this thesis. Applications of these general SSMs of EEG with SMC estimation to the two important areas of EEG analysis, have not been studied in the literature, but are addressed in this research.

   i. Use of Gaussian state noise in TVAR state-space modeling of EEG signals is inappropriate due to its inability to model both abrupt and smooth changes of TVAR state parameters which are typically inherent in ERD/ERS in EEG process. Modeling this non-Gaussian behavior in TVAR parameter changes in state-space form is studied in this research.

   ii. The underlying physiological process behind the single-trial ERPs is continuous process which is however modeled by discrete-time models in existing studies. Besides, irregularly spaced ERP data cannot be modeled efficiently by discrete-time models. Continuous-time state-space modeling is investigated in this research. The use of continuous-time models is motivated by more appropriate modeling of the continuous physiological process generating ERP observations, even though the observations themselves are available only at discrete times, i.e. at each single-trial. Besides, continuous-time models are able to solve implicitly the irregularly spaced data problem in ERPs.

iii. The changing volatility in real noises in EEG is inappropriately modeled by fixed variance models. The variance of the observation noise can be allowed to be time-varying for better capturing the changing-variance characteristics in real EEG noises. Besides, volatilities in real noises especially of the impulsive type e.g. artifacts typically exhibit non-Gaussian dynamics which are inappropriately modeled by linear Gaussian stochastic volatility (SV) models. Modeling of the changing volatility in EEG noise and its non-Gaussian dynamics is addressed.

iv. Online state inference and parameter estimation for the use of non-linear non-Gaussian state-space modeling of EEG do not admit closed form solutions, and will be solved in this research.

v. Performance comparisons between linear Gaussian and non-linear non-Gaussian modeling of biomedical signals are limited. Comparisons are performed in term of performance in the two EEG analyses.

1.4 Objectives of the Research

The main objectives of the research are as follows:

(1) To develop the general framework of continuous-time non-linear non-Gaussian state-space models with SMC based estimation for biomedical signals.

(2) To propose continuous-time non-linear non-Gaussian state-space modeling of EEG signals, with parameter estimation solved by SMC methods.

   i. To propose non-Gaussian TVARMA SSM of EEG signals to capture non-Gaussian parameter changes.
   ii. To introduce continuous-time diffusion process in state-space form for more natural modeling of continuous dynamics and irregularly spaced data in ERPs.
   iii. To apply non-linear non-Gaussian SV models for modeling the non-Gaussian dynamics of volatility in EEG noise, and to incorporate them in the state-space framework of EEG for reduction of impulsive noise.
   iv. To apply SMC methods to solve online state inference problems and model parameter estimation in the proposed models.

(3) To solve two class of filtering problems in EEG analysis as special case investigation of this framework: (a) Parametric time-varying spectral estimation and (b) Single-trial ERP estimation.

1.5 Scope of the Research

The scope of this research is given as follows:

(1) We establish a general framework of applying the non-linear non-Gaussian and continuous-time SSMs with estimation by SMC methods for EEG signals in particular and biomedical signals in general. In this research, we develop mathematical models with general properties, which are not restricted for modeling EEG signals but also can be applied to other biomedical signals with similar characteristics as EEG, such as heart sound signals and ECG.

(2) To develop continuous-time non-linear non-Gaussian SSMs of EEG signals. The models developed with application to solve the two filtering problems are respectively:

Non-Gaussian TVARMA state-space models of EEG.

i. Non-Gaussian state noise i.e. heavy-tailed distribution (such as Cauchy distribution) is used to model the abrupt and smooth changes of TVARMA coefficients.
ii. This proposed model is used for modeling EEG signals and applied to parametric spectral estimation for ERD.

Partially observed diffusion model of single-trial ERP dynamics.

i. The ERP dynamics are modeled as continuous-time diffusion process discretely observed in background noises, formulated in state-space form.
ii. In observation equation, the ERP waveform at each trial is modeled as a mixture of shifted Gaussian functions observed in additive noise. The single-trial ERPs are assumed as discrete samples from an underlying continuous process.
iii. In state equation, the underlying ERP transients are modeled by an example of diffusion process, i.e. mean-reverting Ornstein-Uhlenbeck (OU) process, to model both the inter-trial dynamic changes in ERP parameters and their stationary trends.
iv. The SV of observation noise in the SSM of ERPs is modeled as follows
    (a) Log-variance follows a random walk model with Gaussian noise. (discrete-time linear Gaussian SV model)

(b) Log-variance follows a random walk model with non-Gaussian heavy-tailed noise. (discrete-time linear non-Gaussian SV model)

(c) Cox-Ingersoll-Ross (CIR) process (continuous-time non-linear SV model).

v. The proposed model is applied to modeling and dynamical estimation of single-trial chirp-evoked auditory brainstem responses (ABRs).

(3) SMC methods are applied to online state inference problems in the proposed SSMs to solve the two EEG analysis problems.

i. Online inference of the state of TVARMA coefficients are performed by a generic SMC method i.e. sequential importance sampling Resampling (SIR).

ii. The ERP SSM are estimated by more efficient Rao-Blackwellized particle filtering (RBPF) based on variance reduction techniques.

(4) SMC methods are applied for model parameter estimation.

i. The unknown parameters of the non-Gaussian TVARMA model, such as the variance of the Cauchy state noise, need to be estimated.

ii. The unknown model parameters of the proposed partially observed OU SSM includes stationary trend components and the time-varying variance of observation noise.

iii. Bayesian estimation is used, where the model parameters are augmented to the state and jointly estimated using PF.

(5) To perform comparison

i. Gaussian and non-Gaussian TVARMA modeling of EEG signals for ERD estimation, in term of spectrum resolution, ERD tracking performance, and goodness of fit of the models.

ii. Linear Gaussian and non-linear non-Gaussian SV models in estimating the volatility changes on impulsive type of EEG noise for noise reduction in ERPs.

1.6 Contribution of the Research

The research contributes in developing continuous-time non-linear non-Gaussian SSMs of EEG with SMC based estimation with application to solve two classes of optimal filtering problems in EEG analysis.

Firstly, this research proposes non-Gaussian TVAR SSM which allows the state noise to be non-Gaussian heavy-tailed distributed to simultaneously capture smooth and abrupt parameter changes. The heavy-tailed distribution has larger spread out at tails to predict rare large parameter changes. We extend to TVARMA model with the MA coefficients to smooth the spurious spectral pole by the heavy-tailed AR model and formulate it in state-space form. We apply SMC methods for parameter estimation in the proposed model. The model is used for modeling EEG signals with application to solve spectral estimation of ERP.

Secondly, we develop a partially observed mean reverting OU process where the continuous-time OU process is discretely observed in noise. The process is used for modeling time-varying Gaussian mixture model parameters. We allow the model parameters i.e. the variance of observation noise and process asymptotic mean, to be time-varying. We use SV models to model the stochastic observational noise variance. Thus, a hybrid model based on combination of partially observed diffusion process and SV model is introduced. We formulate it into state-space form and apply combined state and model parameter estimation using SMC methods. A more efficient RBPF is used taking advantage of the formulated conditionally linear Gaussian state-space form. We adopt the model to better describe the continuous dynamics of single-trial ERPs hidden in noise, where continuous dynamics of the ERP Gaussian mixture parameters and their trends are defined by the discretely observed OU process and the background EEG noises are modeled by the SV models.

The OU process which is a continuous-time model described by SDE is able to describe the continuous transient underlying ERPs. Besides, the continuous-time model can implicitly define arbitrary time-intervals between observations, and thus is convenient and flexible to handle irregularly spaced data in ERPs. The asymptotic mean of mean-reverting OU process can also model the trends of ERP dynamics. The approach is applied for dynamical estimation of single-trial ABRs hidden in noises and to solve missing data problem in ERP estimation.

Finally, two non-linear non-Gaussian SV models for better modeling the non-Gaussian dynamics of volatility in EEG noise especially of impulsive type are introduced. We propose random-walk model with non-Gaussian noise and non-linear CIR process with adjustable heavy-tailed conditional distribution to better capture both smooth and abrupt volatility changes in impulsive EEG noise. The models are applied for denoising of single-trial ABRs corrupted by artifacts.

The contributions of the research are summarized in Figure 1.2.

1.7 Outline of the Thesis

The structure of the thesis is summarized as follows. The thesis consists of introductory material (motivation, objectives, scope and contributions of the research – Chapter 1), review on SMC methods (Chapter 2), and our novel contributions and methodology (Chapter 3, 4 and 5) and conclusion and future works (Chapter 6).

Chapter 2 presents literature review on SMC methods for state and model parameter estimation in general state-space models. The mathematical formulation of general SSMs is presented and their related filtering objectives are defined. Analytical solution for linear Gaussian models i.e. Kalman filtering is described. SMC approaches for filtering of non-linear non-Gaussian models are investigated in details: the basic ideas, algorithm, and implementation such as resampling, choice of importance function, variance reduction techniques. SMC methods for model parameter estimation are also introduced.

Chapter 3 proposes non-Gaussian TVARMA state-space models for parametric spectral estimation with application to event-related desynchronization (ERD) estimation of non-stationary EEG. We firstly introduce non-Gaussian state-noise to capture the abrupt and smooth changes in TVAR coefficients. We extend the non-Gaussian TVAR model to TVARMA to further smooth spurious spectral peaks and illustrate its formulation into state-space form. We show how to apply PF methods for estimation of TVARMA coefficients and static model parameters and the subsequent spectral estimation of ERD. Simulation results and comparisons of the Gaussian and non-Gaussian models on ERD estimation and model fitness evaluation are presented and discussed.

Chapter 4 proposes partially observed diffusion model of ERP dynamics with RBPF estimation for single-trial estimation of ERPs. We propose the use of partially observed OU process for modeling the continuous process underlying ERP dynamics. We illustrate how the proposed model is formulated into conditionally linear Gaussian state-space model with its joint state and model parameters estimation efficiently solved by the RBPF. Single-trial dynamical estimation results for simulated and real ABR data are presented and discussed. We also demonstrate the proposed continuous-time model in solving irregularly spaced data problem in ERPs.

Chapter 5 proposes non-linear non-Gaussian SV models for modeling stochastic volatility of impulsive EEG noise. We discuss two types of models for SV i.e. non-Gaussian random walk model and non-linear CIR process for modeling the non-Gaussian volatility changes in impulsive noise. Comparisons of different SV models of EEG noise for denoising of ABRs on simulated data with artifacts are presented and discussed.

Chapter 6, the final chapter summaries the research findings. Some suggestions for future works which might be useful for further development and improvement of the proposed models and their SMC estimation are discussed.

Analysis of EEG Signals

Parametric Time-varying Spectral Estimation
(Frequency domain analysis)

Application:
Estimation of event-related Desynchronization (ERD) of motor imagery.

Objective:
To estimate the TVAR coefficients to obtain the time-frequency representation of EEG.

Filtering Problem:
To estimate sequentially in time the filtered density of TVAR coefficients given the EEG observations.

Motivation:
The underlying TVAR coefficients of EEG process in ERD exhibit abrupt changes which cannot be tracked rapidly by Gaussian TVAR models.

Single-Trial ERP Estimation
(Time domain analysis)

Application:
De-noising of ABRs for analysis of inter-trial variability and detection of wave V.

Objective:
To extract the inter-trial dynamics of ERPs hidden in various noises.

Filtering Problem:
To estimate sequentially in time the filtered density of ERP parameters given the noisy EEG observations.

Motivation:

- The real noises in EEG exhibit changing volatility with non-Gaussian dynamics, for which linear Gaussian SV models are inappropriate.
- The physiological processes underlying ERP dynamics are continuous processes which are unsatisfactorily modeled by discrete-time models.
- Irregularly spaced data problem in ERP estimation cannot be solved implicitly by discrete-time models.

Figure 1.2(a): Two classes of filtering problems in EEG analysis.

Proposed Models:
Non-Gaussian TVARMA state-space model of EEG

Observation Eq.:
TVARMA with Gaussian noise (fixed variance)

State Eq. for AR Changes:
Random-walk model with non-Gaussian noise (heavy-tailed distribution)

Filtering Methods:
Particle filter

Model Parameter Estimation:
Bayesian approach

Proposed Models:
Partially observed diffusion model of single-trial ERP dynamics

Observation Eq.:
Gaussian mixture modeling of ERP components, with additive Gaussian noise with time-varying variance modeled by:

- Log-variance follows random-walk with Gaussian noise (discrete-time linear Gaussian SV model)
- Log-variance follows random-walk with non-Gaussian noise (discrete-time linear non-Gaussian SV model)
- Cox-Ingersoll-Ross (CIR) process (continuous-time non-linear SV model)

State Eq. for ERP Dynamics:
Continuous-time diffusion model (Mean-reverting Ornstein-Uhlenbeck process)

Filtering Methods:
Rao-Blackwellized particle filter

Model Parameter Estimation:
Bayesian approach

Figure 1.2(b): Contributions of the research for solving the filtering problems in EEG analysis.

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