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Unsteady Free Convection Flow near the Stagnation Point of a Three-dimensional Body

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Abstract: This study deals with the study of unsteady free convection in the stagnation point region of a three-dimensional body placed in an ambient fluid under the assumption of a step change in the surface temperature of the body. The non-linear coupled partial differential equations governing the free convection flow are solved numerically using an implicit finite-difference method for different values of the governing parameters entering these equations. Results for the flow and heat characteristics show that the transition from the initial unsteady-state flow to the final steady-state flow takes place smoothly.

Key words: Stagnation-point, three-dimensional, boundary layer, numerical results

INTRODUCTION

Unsteady free and mixed convection laminar boundary layer flows in the neighborhood of a two-dimensional or axisymmetric stagnation point have been extensively studied in the literature by Sano and Wakitani (1984), Amin and Riley (1995, 1996), Seshadri *et al.* (2002) and Nazar *et al.* (2003). These studies were motivated by the fundamental nature of the boundary layer flows near such points and by their relevance to the leading edge and nose regions of bodies in high speed flight (Bhattacharyya and Gupta, 1998). However, these two cases of two- dimensional and axisymmetric flows can be regarded as special cases of a general three-dimensional stagnation point flow. Such a flow occurs, for example, when the fluid coming from upstream divides to pass around a finite body in the shape of a wavy cylinder.

Several authors, such as Poots (1964), Banks (1972, 1974) and Gorla *et al*. (1995) have studied the steady free convection boundary layer flow in the region of the stagnation point of a three-dimensional body. The work of Poots (1964) is particularly interesting because he was the first to derive the boundary layer equations governing the free convection flow at the lower stagnation point of a general three-dimensional body and has shown that the two-dimensional and axisymmetric flows are just two special cases from a more general point of view. He has, in fact, given exact numerical solutions to the three-dimensional boundary layer equations, where the Prandtl number was 0.72, for a number of blunt body shapes.

Then, Banks (1972, 1974) showed that other solutions exist over the whole range of stagnation points and Prandtl numbers. To our best knowledge, however, the unsteady free convection near the stagnation point on a heated regular three-dimensional body has been studied only by Ingham *et al.* (1984) and Slaouti *et al.* (1998). Ingham *et al.* (1984) considered an isothermal body surface, while Slaouti *et al*. (1998) considered the case when there is an initial steady state that is perturbed by a step-change in the wall temperature.

In this study, the behavior of unsteady free convection of a viscous and incompressible fluid in the stagnation point region of a heated three-dimensional body subjected to a step change in its surface temperature from that of the ambient fluid is theoretically studied. The transformed non-linear coupled partial differential equations governing the flow are solved numerically using an implicit finite-difference method. The results for the steady-state flow are compared with those of Poots (1964) and Banks (1974), and found to agree very favorably.

BASIC EQUATIONS

Consider the unsteady free convection flow near the stagnation point of a heated three-dimensional body placed in a viscous and incompressible fluid of

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uniform temperature T_∞. It is assumed that the uniform temperature of the body is suddenly changed from T_w to T_∞, where $T_w > T_\infty$. A locally Cartesian orthogonal system (x,y,z) is chosen with the origin N at the nodal stagnation point, where the *x*- and *y*-coordinates are measured along the body surface, while the *z*-coordinate is measured normal to the body surface. Under these assumptions, the boundary layer equations governing unsteady free convection flow are (Slaouti *et al*., 1998):

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \quad (1)$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=\nu\frac{\partial^2 u}{\partial z^2}+g\beta ax(T-T_\infty) \quad (2)$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=\nu\frac{\partial^2 v}{\partial z^2}+g\beta by(T-T_\infty) \quad (3)$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}=\alpha\frac{\partial^2 T}{\partial z^2} \quad (4)$$

subject to the initial and boundary conditions:

$$t<0:\quad u=v=w=0,\quad T=T_\infty \quad \text{for any} \quad x,y,z$$

$$\begin{aligned} t\geq 0:\quad & u=v=0,\quad T=T_W \quad \text{on} \quad z=0, x\geq 0, y\geq 0 \\ & u=v=0,\quad T=T_\infty \quad \text{on} \quad x=0, y\geq 0, z>0 \\ & u=v=0,\quad T=T_\infty \quad \text{on} \quad y=0, x\geq 0, z>0 \\ & u=v=0,\quad T=T_\infty \quad \text{as} \quad z\to\infty, x\geq 0, y\geq 0 \end{aligned} \quad (5)$$

Here u, v, w are the velocity components along x, y, z axes, t is the time, T is the fluid temperature, *g* is the magnitude of the gravity acceleration, α is the coefficient of thermal diffusivity, υ is the kinematic viscosity, β is the volumetric coefficient of thermal expansion and a and b are the parameters of the principal curvatures at N, of the body measured in the planes y and x, respectively. There is no loss of generality in requiring that $|a|\geq|b|$ with a>b.

Clearly b=0 corresponds to the plane stagnation flow case, while b=a is the axisymmetric case. We assume here that a and b are positive so that solutions of the resulting equations lead to stagnation points which are nodal points of attachment, i.e., $0\leq c\leq 1$, where c=b/a. However, a or b could also be negative which leads to saddle points of attachment, i.e., $-1\leq c\leq 0$. Since, most shapes of practical interest lie between cylinder (c=0) and sphere (c=1) we shall confine our analysis to nodal points of attachment only ($0\leq c\leq 1$).

A little inspection shows that Eq. 1 to 4 along with the boundary conditions (Eq. 5) admit a semi-similar solution of the form (Seshadri *et al*., 2002):

$$\begin{aligned} &\eta=Gr^{1/4}a\xi^{-1/2}z,\quad u=\nu a^2 xGr^{1/2}f'(\xi,\eta) \\ &v=\nu a^2 cyGr^{1/2}h'(\xi,\eta),\quad w=-\nu aGr^{1/4}\xi^{1/2}(f+ch) \\ &\theta(\xi,\eta)=(T-T_\infty)/(T_W-T_\infty),\quad \xi=1-e^{-\tau},\quad \tau=\nu a^2 Gr^{1/2}t \end{aligned} \quad (6)$$

where, $Gr=g_0\beta(T_w-T_\infty)/(a^3v^2)$ is the Grashof number and primes denote partial differentiation with respect to η. Substitution of Eq. 6 in Eq. 2 and 4 gives:

$$f'''+(1-\xi)\frac{\eta}{2}f''+\xi\left[(f+ch)f''-f'^2\right]+\xi\theta=\xi(1-\xi)\frac{\partial f'}{\partial\xi} \quad (7)$$

$$h'''+(1-\xi)\frac{\eta}{2}h''+\xi\left[(f+ch)h''-ch'^2\right]+\xi\theta=\xi(1-\xi)\frac{\partial h'}{\partial\xi} \quad (8)$$

$$\frac{1}{Pr}\theta''+(1-\xi)\frac{\eta}{2}\theta'+\xi(f+ch)\theta'=\xi(1-\xi)\frac{\partial\theta}{\partial\xi} \quad (9)$$

while the boundary conditions (Eq. 5) become:

$$\begin{aligned} &f(\xi,0)=f'(\xi,0)=0,\ h(\xi,0)=h'(\xi,0)=0,\ \theta(\xi,0)=1 \\ &f'\to 0,\ h'\to 0,\ \theta\to 0 \quad \text{as} \quad \eta\to\infty \end{aligned} \quad (10)$$

for $0\leq\xi\leq 1$. Here Pr is the Prandtl number and primes denote partial differentiation with respect to η.

The physical quantities of practical interest in this problem are the skin friction coefficients in the x and y directions, C_{fx} and C_{fy} and the Nusselt number, Nu, that are defined as:

$$\begin{aligned} &Cf_x=\mu\left(\frac{\partial u}{\partial z}\right)_{z=0}\Big/\left(\rho\nu^2 a^3 x\right),\qquad Cf_y=\mu\left(\frac{\partial v}{\partial z}\right)_{z=0}\Big/\left(\rho\nu^2 a^3 y\right) \\ &Nu=a^{-1}\left(\frac{\partial T}{\partial z}\right)_{z=0}\Big/\left(T_W-T_\infty\right) \end{aligned} \quad (11)$$

where, ρ and μ is the density and dynamic viscosity, respectively. In terms of the non-dimensional variables (Eq. 6), we have:

$$\begin{aligned} &C_{fx}\xi^{1/2}\big/Gr^{3/4}=f''(\xi,0),\ C_{fy}\xi^{1/2}\big/Gr^{3/4}=h''(\tau,0), \\ &Nu\xi^{1/2}\big/Gr^{3/4}=-\theta'(\tau,0) \end{aligned} \quad (12)$$

For the initial-unsteady flow case, where ξ is small, that is, $\xi\approx 0$, Eq. 7 to 9 reduce to the following form:

$$f''' + \frac{\eta}{2} f'' = 0, \quad h''' + \frac{\eta}{2} h'' = 0, \quad \theta'' + \Pr \frac{\eta}{2} \theta' = 0 \qquad (13)$$

subject to the boundary conditions:

$$\begin{aligned} & f(0) = f'(0) = 0, \quad h(0) = h'(0) = 0, \quad \theta(0) = 1 \\ & f'(\infty) = 0, \quad h'(\infty) = 0, \quad \theta(\infty) = 0 \end{aligned} \qquad (14)$$

The solution of equations Eq. 14 in 15 is given by;

$$f = h = 0, \quad \theta(\eta) = \operatorname{erfc}\left(\sqrt{\Pr}\,\eta / 2\right) \qquad (15)$$

where, $\operatorname{erfc}\left(\sqrt{\Pr}\,\eta / 2\right)$ is the complimentary error function. For the final steady-state flow case, where ξ=1, Eq. 7 to 9 reduce to the following similar form

$$f''' + (f + ch)f'' - f'^2 + \theta = 0 \qquad (16)$$

$$h''' + (f + ch)h'' - ch'^2 + \theta = 0 \qquad (17)$$

$$\frac{1}{\Pr}\theta'' + (f + ch)\theta' = 0 \qquad (18)$$

Subject to the boundary conditions (Eq. 14). These equations are identical with those first found by Poots (1964).

RESULTS AND DISCUSSION

The two sets of Eq. 7 to 9 and 16 to 18 subject to the boundary conditions (10) and (14) were solved numerically using an implicit finite difference scheme, known as Keller-box method described in book by Cebeci and Bradshaw (1984). All the results quoted here were obtained using uniform grids in both ξ and η directions. We have used the step size of $\Delta\eta$=0.045 and $\Delta\xi$=0.002. In all cases we choose η_{max}=201 and ξ_{max}=501.

A solution is considered to converge when the maximum absolute pointwise change between successive iterates was 10^{-10}. Results are obtained for Pr = 0.72, 3 and 6.8 and c = 0 (plane stagnation point), 0.25, 0.5, 0.75 and 1.0 (axisymmetric stagnation point). Values for the reduced skin friction coefficients f"(0) and h"(0), and heat transfer from the surface of the body $-\theta$'(0) are given in Table 1. All these results are seen to be in excellent agreement. Therefore, we are confident that the present results are very accurate.

Figure 1 illustrates the variation of skin friction coefficients and the heat transfer from the surface of the body with ξ For some values of the parameter c with Pr = 0.72. It is seen that the skin friction coefficients decrease continuously with the increase of the curvature parameter c from 0 to 1. However, the heat transfer from the body surface increases with the increase of c and it has a minimum value for each value of c.

Finally, Fig. 2 shows the variation of the skin friction coefficients and the heat transfer from the surface of the body with ξ for some values of Pr when c = 0.5. As expected (Slaouti *et al*., 1998) both the skin friction coefficients decrease while the heat transfer from the body surface increases with the increase of the Prandtl number, Pr. The physical reason for this trend is that a higher Prandtl number fluid has a thinner thermal boundary layer which increases the gradient of the temperature.

Consequently, the surface heat transfer increases as Pr increases. The decrease of the skin friction coefficients

Table 1: Comparison of skin friction f "(0),h"(0) and heat flux rate $-\theta$'(0) for ξ=1 (steady-state flow), Pr = 0.72 and various values of c.

	Present		Poots (1964)	
c	0	1	0	1
f"	0.855909	0.764685		0.764632
h"	1.080763	0.764685	0.856045	0.764632
-θ'	0.374102	0.462223	0.374105	0.462221

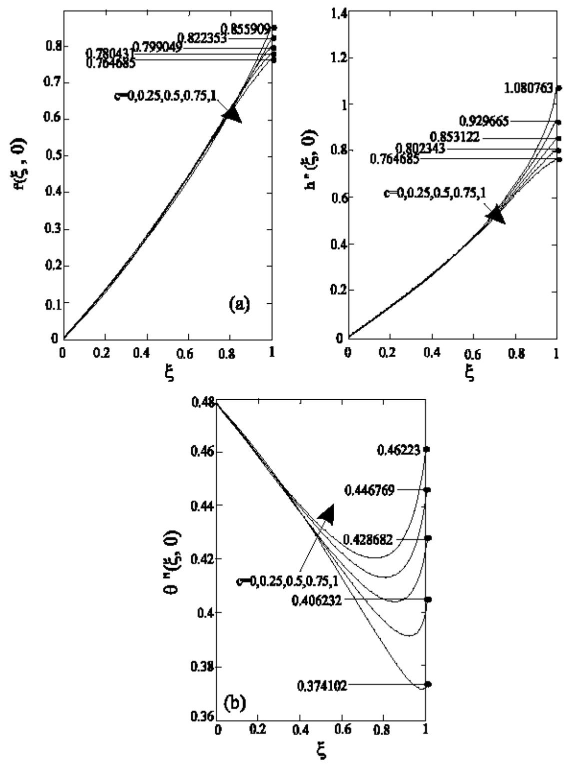

Fig. 1: (a) Variation of surface skin friction f"(ξ,0) and h"(ξ,0) with ξ; b) Variation of heat flux on the wall -θ'(ξ,0) with ξ for some values of c when Pr = 0.72

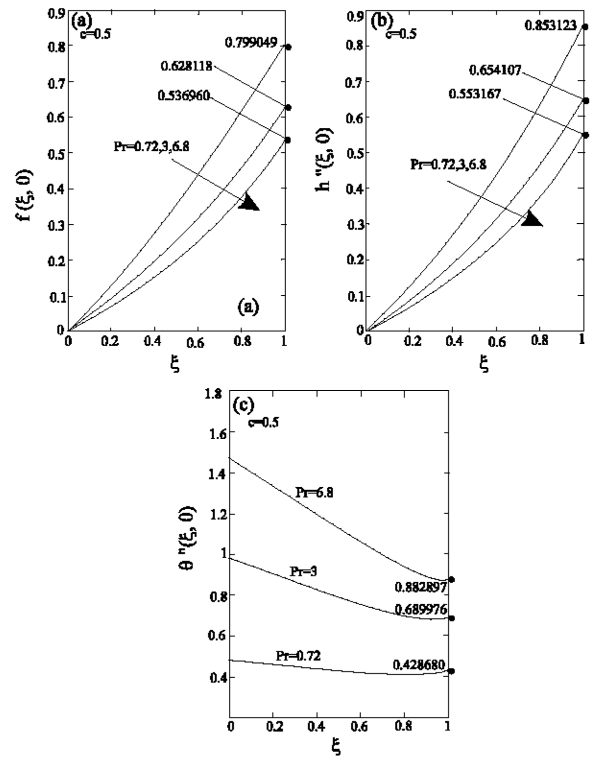

Fig. 2: (a) Variation of surface skin friction f”(ξ,0) and h”(ξ,0) with ξ; b) Variation of heat flux on the wall −θ’(ξ,0) with ξ for several values of Pr when c = 0.5. (• steady-state flow)

with the increase of Pr happens because a higher Pr implies more viscous fluids having a comparatively larger momentum boundary layer thickness.

CONCLUSION

The outcome of the analysis herein can be summarized as follows: (1) the initial unsteady free convection (ξ = 0) exhibits a structure similar to that of a body with a step change of its temperature. (2) The steady-state flow has a similar structure with that studied by Poots (1964) and Banks (1974). (3) there is a smooth transition from the initial unsteady-state flow to the final steady-state flow; (4) for a fixed value of c, the skin friction coefficients decrease while the heat transfer from the surface of the body increases with the increase of Pr.

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