ROBUST CONTROL OF ACTIVE SUSPENSION SYSTEM FOR A QUARTER CAR MODEL

Project leader

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ABSTRACT

The aims of this research are to establish the nonlinear mathematical model and the robust control technique of the hydraulically actuated active suspension system for a quarter car model. The purpose of a car suspension system is to improve riding quality while maintaining good handling characteristics subject to different road profile. A new nonlinear quarter car model, which incorporates the rotational motion of the wheel and the dynamics of the control arm, is used in this research . The proposed controller consist of two controller loops namely inner loop controller for force tracking control of the hydraulic actuator and outer loop controller to reject the effects of road induced disturbances. The outer loop controller utilized a proportional integral sliding mode control (PISMC) scheme. Whereas, proportional integral (PI) control is used in the inner loop controller to track the hydraulic actuator in such a way that it able to provide the actual force as close as possible with the optimum target force produced by the PISMC controller. A simulation study is performed to proof the effectiveness and robustness of the control approach. The performance of the controller is compared with the LQR controller and the passive suspension system. Force tracking performance of the hydraulic actuator is also investigated. The simulation is enhanced with 3-D animation of the car going on a road bump.

ABSTRAK

Kajian ini bertujuan mengenengahkan model matematik taklinar dan teknik kawalan yang baru dalam pemodelan dan kawalan ke atas sistem gantungan aktif dengan dinamik hidraulik untuk model kereta suku. Sistem gantungan kereta berfungsi untuk memperbaikan kualiti sistem pemanduan disamping mengekalkan ciri-ciri pemanduan yang baik dalam apa jua bentuk permukaan jalan. Model kereta suku taklinar yang baru, mengambil kira gerakan pusingan roda kereta dan dinamik lengan kawalan telah digunakan dalam kajian ini. Teknik kawalan yang dicadangkan terdiri daripada kawalan dua gelung iaitu gelung dalam untuk kawalan jejak daya bagi aktuator hidraulik dan gelung kawalan luar untuk memperbaiki gangguan daripada permukaan jalan. Gelung kawalan luar menggunakan kaedah kawalan ragam gelincir berkadaran-kamiran. Manakala kaedah kawalan berkadaran-kamiran digunakan bagi gelung kawalan untuk menjejak aktuator hidraulik supaya memberikan daya menghampiri daya optimum yang diperlukan. Penyelakuan komputer telah dilakukan untuk menentukan keberkesanan dan kebolehupayaan teknik kawalan yang dihasilkan. Prestasi teknik kawalan ini telah dibandingkan dengan teknik kawalan LQR dan sistem gantungan pasif. Disamping itu, prestasi jejak daya oleh aktuator hidraulik turut dinilai. Animasi 3-dimensi bagi model kereta melalui bonggol jalan turut dilakukan.

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LIST OF ABBREVIATIONS

GA	Genetic Algorithm
LPV	Linear Parameter Varying
LQG	Linear Quadratic Gaussian
LQR	Linear Quadratic Regulator
LTR	Loop Transfer Discovery
MIMO	Multi-Input Multi-Output
PI	Proportional Integral
PID	Proportional Integral Derivative
PISMC	Proportional Integral Sliding Mode Control
SMC	Sliding Mode Control
VSC	Variable Structure Control

LIST OF SYMBOLS

SYMBOL	DESCRIPTION
A	4 x 4 system matrix in state space equation of quarter car with linear actuator input
A_p	piston area
C_{d_i}	discharge coefficient = 0.7
C_s	damping of damper (Ns/m)
C_{tm}	leakage coefficient = 15e-12
D	damping energy
F	matrix related to system input in motion equation of the quarter car model
P_L	pressure induced by load
P_s	supply pressure = 20684 kN/m^2
T	kinetic energy
V	potential energy
f_a	control input (N)
k	Voltage to position conversion factor = 1481V/m
k_s	stiffness of car body spring (N/m)
k_t	stiffness of car tyre (N/m)
l_A	distance from A to O (m)
l_B	distance from B to O (m)
l_C	length of the control arm (m)
m_u	wheel mass (kg)
m_s	car body mass (kg)
u_1	spool valve position
u_2	bypass valve area

w	spool valve width = 0.008 m
z_r	irregular excitation from the road surface
z_s	vertical displacement of the body
$\dot{z}_u$	vertical velocity of the wheel
$\dot{z}_s$	vertical velocity of the body
θ	rotational angle of the control arm
θ_0	angular displacement of the control arm at a static equilibrium point (-2 deg)
$\sigma(t)$	Proportional-Integral (PI) sliding surface for quarter car model
$\lambda(*)$	eigenvalue of (*)
α	angle between y-axis and $\overline{OA}$
α_α	hydraulic coefficient = 2.273e9N/m^5
ρ	specific gravity of hydraulic fluid
τ	time constant
δ	boundary layer thickness
β	upper bound of function $f(*)$
$\Re(*)$	range space of matrix (*)

CHAPTER 1

INTRODUCTION

1.1 Background

A car suspension system is the mechanism that physically separates the car body from the wheels of the car. The performance of the suspension system has been greatly increased due to increasing vehicle capabilities. Appleyard and Wellstead (1995) have proposed several performance characteristics to be considered in order to achieve a good suspension system. These characteristics deal with the regulation of body movement, the regulation of suspension movement and the force distribution. Ideally the suspension should isolate the body from road disturbances and inertial disturbances associated with cornering and braking or acceleration. The suspension must also be able to minimize the vertical force transmitted to the passengers for their comfort. This objective can be achieved by minimizing the vertical car body acceleration. An excessive wheel travel will result in non-optimum attitude of tire relative to the road that will cause poor handling and adhesion. Furthermore, to maintain good handling characteristic, the optimum tire-to-road contact must be maintained on four wheels. In conventional suspension system, these characteristics are conflicting and do not meet all conditions. Automotive researchers have studied the suspension on the system extensively through both analysis and experiments. The main goal of the study is to improve the traditional design trade-off between ride and road handling by directly controlling the suspension forces to suit with the performance characteristics.

The suspension system can be categorized into passive, semi-active and active suspension system according to external power input to the system. A passive suspension system is a conventional suspension system consists of a non-controlled spring and shock-absorbing damper. The commercial vehicles today use passive suspension system as means to control the dynamics of a vehicle's vertical motion as well as pitch and roll. Passive indicates that the suspension elements cannot supply energy to the suspension system. The suspension spring and damper do not provide energy to the suspension system and control only the motion of the car body and wheel by limiting the suspension velocity according to the rate determined by the designer. Hence, the performance of a passive suspension system is variable subject to the road profiles.

The semi-active suspension has the same elements but the damper has two or more selectable damping rate. In early semi-active suspension system, the regulating of the damping force can be achieved by utilizing the controlled dampers under closed loop control, and such is only capable of dissipating energy (Williams, 1994). Two types of dampers are used in the semi- active suspension namely the two state dampers and the continuous variable dampers. The disadvantage of these dampers is difficulties to find devices that are capable in generating a high force at low velocities and a low force at high velocities, and be able to move rapidly between the two.

An active suspension is one in which the passive components are augmented by hydraulic actuators that supply additional force. Active suspensions differ from the conventional passive suspensions in their ability to inject energy into the system, as well as store and dissipate it. The active suspension is characterized by the hydraulic actuator that placed in parallel with the damper and the spring. Since the hydraulic actuator connects the unsprung mass to the body, it can control both the wheel hop motion as well as the body motion. Thus, the active suspension now can improve both the ride comfort and ride handling simultaneously.

Although various control laws such as adaptive control (Hac, 1987), optimal state-feedback (Hrovat, 1997), fuzzy control (Ting, 1995) and robust sliding mode control (Sam, 2004) have been proposed to control the active suspension system, the methods were successful applied in computer simulations based only but not in real applications.

Therefore, a real active suspension system is needed to implement and test the developed control strategy. A quarter car model is chosen as an initial model of controlling the active suspension system due to the simplicity of the model. Modeling of the quarter car suspension as well as the non-linear hydraulic actuator including its force tracking controller for an active suspension system is investigated in this study.

However due to budget constraint, a real active suspension system cannot be realized. Instead the dynamics of the hydraulically actuated active suspension system will be visualized in 3-D format for the initial stage. The dynamic characteristics of the half and full car active suspension models can be observed by using the quarter car model approach.

1.2 Problem Statement

The statement of the problem of this research is expressed as follow:

“to develop a robust controller that can improve the performances of the nonlinear active suspension system and its verifications using graphical and animation output”.

1.3 Research Objectives

The objectives of this research are as follows:

i) To develop a nonlinear mathematical model of the hydraulically actuated active suspension system for a quarter car model.

ii) To develop the control algorithm that based on a robust control scheme for the active suspension system.

iii) To visualise in 3-D format the dynamics of the hydraulically actuated active suspension system.

1.4 Structure and Layout of Report

This report is organized into five chapters. Chapter 1 gives the background of the suspension system and the objectives of the project. Chapter 2 discusses the literature review of the suspension system. Various model and controlling techniques related to the suspension system are outline.

In Chapter 3, the methodology of the research is presented. The research starts with the modeling of a quarter car nonlinear hydraulic actuator active suspension system. Firstly, the state space representation of the dynamic model of the nonlinear suspension system is outlined. Secondly, the dynamic of the hydraulic actuator is presented. Next the control strategy for hydraulically actuated active suspension for quarter car model is proposed. The controller structure utilizes two controller loops namely the outer loop and inner loop controllers which corresponds to vehicle controller and actuator controller. Subsequently, the procedure of creating the animation scene is outline.

Chapter 4 discusses the performance evaluation of the proposed Proportional-Integral Sliding Mode Controller by means of computer simulation in MATLAB-

SIMULINK. The passive and LQR are used as a mean of comparison. The force tracking performance of the hydraulic actuator is also investigated. Next, the virtual reality animation of the hydraulic actuated active suspension system is presented. Conclusion on the effectiveness of the approach are made and discussed based on the results obtained in this chapter.

The summary of the results and future research based on this study will be presented in Chapter 5.

CHAPTER 2

LITERATURE REVIEW

2.1 Suspension System

The suspension system can be categorized into passive, semi-active and active suspension system according to external power input to the system and/or a control bandwidth (Appleyard and Wellstead, 1995). A passive suspension system is a conventional suspension system consists of a non-controlled spring and shock-absorbing damper as shown in figure 2.1. The semi-active suspension as shown in figure 2.2 has the same elements but the damper has two or more selectable damping rate. An active suspension is one in which the passive components are augmented by actuators that supply additional force. Besides these three types of suspension systems, a skyhook type damper has been considered in the early design of the active suspension system. In the skyhook damper suspension system, an imaginary damper is placed between the sprung mass and the sky as shown in figure 2.3. The imaginary damper provides a force on the vehicle body proportional to the sprung mass absolute velocity. As a result, the sprung mass movements can be reduced without improving the tire deflections. However, this design concept is not feasible to be realized (Hrovat, 1988). Therefore, the actuator has to be placed between the sprung mass and the unsprung mass instead of the sky.

2.1.1 Passive Suspension System

The commercial vehicles today use passive suspension system to control the dynamics of a vehicle's vertical motion as well as pitch and roll. Passive indicates that the suspension elements cannot supply energy to the suspension system. The passive suspension system controls the motion of the body and wheel by limiting their relative velocities to a rate that gives the desired ride characteristics. This is achieved by using some type of damping element placed between the body and the wheels of the vehicle, such as hydraulic shock absorber. Properties of the conventional shock absorber establish the tradeoff between minimizing the body vertical acceleration and maintaining good tire-road contact force. These parameters are coupled. That is, for a comfortable ride, it is desirable to limit the body acceleration by using a soft absorber, but this allows more variation in the tire-road contact force that in turn reduces the handling performance. Also, the suspension travel, commonly called the suspension displacement, limits allowable deflection, which in turn limits the amount of relative velocity of the absorber that can be permitted. By comparison, it is desirable to reduce the relative velocity to improve handling by designing a stiffer or higher rate shock absorber. This stiffness decreases the ride quality performance at the same time increases the body acceleration, detract what is considered being good ride characteristics.

An early design for automobile suspension systems focused on unconstrained optimizations for passive suspension system which indicate the desirability of low suspension stiffness, reduced unsprung mass, and an optimum damping ratio for the best controllability (Thompson, 1971). Thus the passive suspension systems, which approach optimal characteristics, had offered an attractive choice for a vehicle suspension systems and had been widely used for car. However, the suspension spring and damper do not provide energy to the suspension system and control only the motion of the car body and wheel by limiting the suspension velocity according to the rate determined by the designer. Hence, the performance of a passive suspension system is variable subject to the road profiles.

2.1.2 Semi-Active Suspension System

In early semi-active suspension system, the regulating of the damping force can be achieved by utilizing the controlled dampers under closed loop control, and such is only capable of dissipating energy (Williams, 1994). Two types of dampers are used in the semi- active suspension namely the two state dampers and the continuous variable dampers.

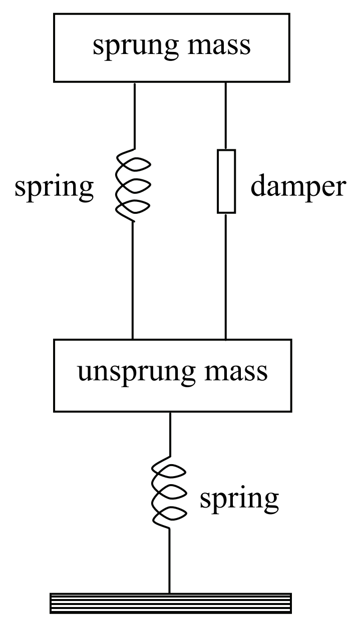

Figure 2.1 The passive suspension system

The two state dampers switched rapidly between states under closed-loop control. In order to damp the body motion, it is necessary to apply a force that is proportional to the body velocity. Therefore, when the body velocity is in the same direction as the damper velocity, the damper is switched to the high state. When the body velocity is in the opposite direction to the damper velocity, it is switched to the low state as the damper is transmitting the input force rather than dissipating energy. The disadvantage of this system is that while it controls the body frequencies effectively, the rapid switching, particularly when there are high velocities across the dampers, generates high-frequency harmonics which makes the suspension feel harsh, and leads to the generation of unacceptable noise.

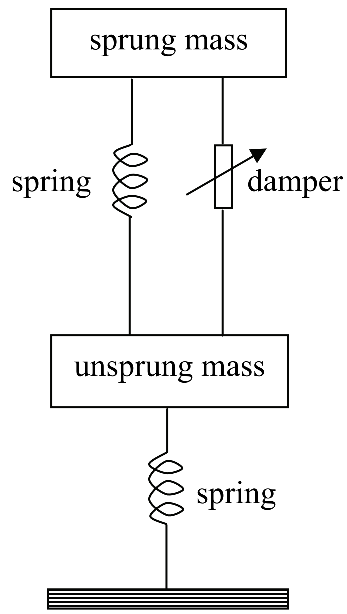

Figure 2.2 The semi-active suspension system

The continuous variable dampers have a characteristic that can be rapidly varied over a wide range. When the body velocity and damper velocity are in the same direction, the damper force is controlled to emulate the skyhook damper. When they are in the opposite directions, the damper is switched to its lower rate, this being the closest it can get to the ideal skyhook force. The disadvantage of the continuous variable damper is that it is difficult to find devices that are capable in generating a high force at low velocities and a low force at high velocities, and be able to move rapidly between the two. Karnopp (1990) has introduced the control strategy to control the skyhook damper. The control strategy utilized a fictitious damper that is inserted between the sprung mass and the stationary sky as a way to suppress the vibration motion of the spring mass and as a tool to compute the desired skyhook force. The skyhook damper can reduce the resonant peak of the spring mass quite significantly and thus achieves a good ride quality. But, in order to improve both the ride quality and handling performance of a vehicle, both resonant peaks of the spring mass and the unsprung mass need to be reduced. It is known, however, that the skyhook damper alone cannot reduce both resonant peaks at the same time (Hong *et al*., 2002).

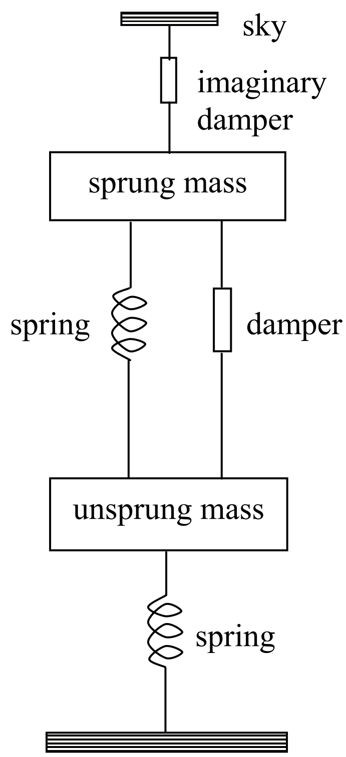

Figure 2.3 A skyhook damper

More recently, the possible applications of electrorheological (ER) and magnetorheological (MR) fluids in the controllable dampers were investigated by Yao *et al.* (2002) and Choi and Kim (2000). However, since MR damper cannot be treated as a viscous damper under high electric current, a suitable mathematical model is needed to be developed to describe the MR damper.

2.1.3 Active Suspension System

Active suspensions differ from the conventional passive suspensions in their ability to inject energy into the system, as well as store and dissipate it. Crolla (1988) has divided the active suspensions into two categories; the low-bandwidth or soft active suspension and the high-bandwidth or stiff active suspension. Low bandwidth or soft active suspensions are characterized by an actuator that is in series with a damper and the spring as shown in figure 2.4. Wheel hop motion is controlled passively by the damper, so that the active function of the suspension can be restricted to body motion. Therefore, such type of suspension can only improve the ride comfort. A high-bandwidth or stiff active suspension is characterized by an actuator placed in parallel with the damper and the spring as illustrated in figure 2.5. Since the actuator connects the unsprung mass to the body, it can control both the wheel hop motion as well as the body motion. The high-bandwidth active suspension now can improve both the ride comfort and ride handling simultaneously. Therefore, almost all studies on the active suspension system utilized the high-bandwidth type.

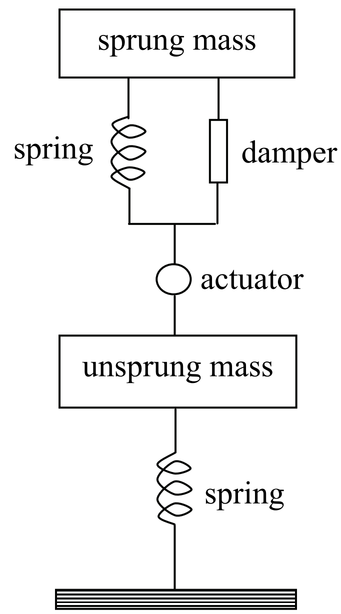

Figure 2.4 A low bandwidth or soft active suspension system

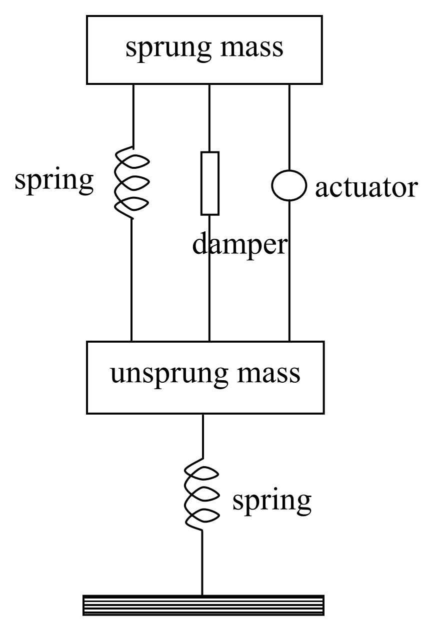

Figure 2.5 A high bandwidth or stiff active suspension system

Various types of active suspension model are reported in the literature either modeled linearly (used most) or non-linear; examples are Macpherson strut suspension system (Al-Holou *et al.*, 1999, Hong *et al*., 2002).

2.1.3.1 Macpherson Type Suspension System vs Conventional Model

In the conventional model, only the up-down movements of the sprung and the unsprung masses are incorporated. In Macpherson type (Figure 3.1) however, the sprung mass, which includes the axle and the wheel, is also linked to the car body by a control arm. Therefore, the unsprung mass can rotate besides moving up and down. Considering that better control performance is being demanded by the automotive industry, investigation of a new model that includes the rotational motion of the unsprung mass and allows for the variance of suspension types is justified (Hong *et al.,* 1999).

The Macpherson type suspension system has many merits, such as an independent usage as a shock absorber and the capability of maintaining the wheel in the camber direction. The control arm plays several important roles: it supports the suspension system as an additional link to the body, completes the suspension structure, and allows the rotational motion of the unsprung mass. However, the function of the control arm is completely ignored in the conventional model.

2. 2 Vehicle Suspension System Control Strategies

In the past years, various control strategies have been proposed by numerous researchers to improve the trade-off between ride comfort and road handling. These control strategies may be grouped into techniques based on linear, nonlinear and intelligent control approaches. In the following, some of these control approaches that have been reported in the literature will be briefly presented.

The most popular linear control strategy that has been used by researchers in the design of the active suspension system is based on the optimal control concept (Hrovat, 1997). Amongst the optimal control concepts used are the Linear Quadratic Regulator (LQR) approach, the Linear Quadratic Gaussian (LQG) approach and the Loop Transfer Recovery (LTR) approach. These methods are based on the minimization of a linear quadratic cost function where the performance measure is a function of the states and inputs to the system.

Application of the LQR method to the active suspension system has been proposed by Hrovat (1988), Tseng and Hrovat (1990) and Esmailzadeh and Taghirad (1996). Hrovat (1988) has studied the effects of the unsprung mass on the active suspension system. The carpet plots were introduced to give a clear global view of the effect of various parameters on the system performances. The carpet plots are the plots of the root mean square (r.m.s) values of the sprung mass acceleration and unsprung mass acceleration versus the suspension travel. The r.m.s. values of all parameters are obtained from a series of simulations on different weights of the performance index. Esmailzadeh and Taghirad (1996) included the passenger's dynamics in the suspension system and the input to the system is considered as a linear force. The study utilized two approaches in selecting the performances index.

Since it is desirable to measure all the system states in the actual implementations, observers are usually used. Ulsoy *et al.* (1994) used Kalman filter to reconstruct the states and to address the problem of the sensors noise and road disturbances. In the study, the robustness margin of the LQG controllers with respect to the parameter uncertainties and actuator dynamics were investigated. The results showed that the LQG controllers, using some measurements, such as the suspension stroke, should be avoided since the controllers did not produce the satisfactory robustness. LQG with loop transfer recovery (LQG/LTR) was studied by Ray (1993) as a solution to increase the robustness of the LQG controllers. However, when applying LTR, uncertain system parameters must be identified, and the magnitude of uncertainty expected should be known or estimated, otherwise, LQR/LTR system may exhibit robustness qualities that are not better than the original LQG system.

Most researchers that utilized the linear control approach did not consider the dynamics of the actuators in their study. Thus, the control strategies that have been developed did not represent the actual system which is highly nonlinear due to the hydraulic actuator properties and the presence of uncertainties in the system. Furthermore, when applying the linear control theory to the system, it may not give an acceptable performance due to the presence of uncertainties and nonlinearities in the system.

Suspension systems are intrinsically nonlinear and uncertain and are subjected to a variety of road profiles and suspension dynamics. The nonlinearities of a road profile are due to the roughness and smoothness of the road surfaces while the suspension dynamics are affected by the actuator nonlinearities. Yamashita *et al.* (1994) presented a control law for a full car model using the actual characteristics of hydraulic actuators based on the H-infinity control theory. The proposed controller has been implemented in an experimental vehicle, and evaluated for robust performance in a four-wheel shaker and during actual driving. The results showed that the system is robust even when the closed-loop system is perturbed by limited uncertainty. Instead of using the state feedback, Hayakawa *et al.* (1999) utilized the robust H-infinity output feedback control to a full car active suspension model. In the study, the linear dynamical model of a full car model is intrinsically decoupled into two parts to make the implementation of the output feedback control simpler and realizable.

The combination of the H-infinity and adaptive nonlinear control technique on active suspension system has been reported by Fukao *et al.* (1999). The study divided the active suspension structure into two parts. The car's body part utilized the H-infinity control design and the actuator part used the adaptive nonlinear control design technique. On the ride quality, there exists a range of frequency where passengers strongly feel the body acceleration caused by the disturbance from road surface.

Therefore, the H-infinity controller through frequency shaping performs improvement of the frequency property. The nonlinearities and the uncertainties of the actuator are overcome by the adaptive nonlinear controller based on the backstepping technique.

Lin and Kanellapoulos (1997a) presented a nonlinear backstepping design for the control of quarter car active suspension model. The intentional introduction of nonlinearity into the control objective allows the controller to react differently in different operating regimes. They improved further their works on nonlinear control design for active suspension system by augmenting such controller with the road adaptive algorithm as reported in Lin and Kanellakopoulos (1997b).

Alleyne and Hedrick (1995) presented a nonlinear adaptive control to active suspension system. The study introduced a standard parameter adaptation scheme based on Lyapunov analysis to reduce the error in the model. Then a modified adaptation scheme that enables the identification of parameters whose values change with regions of the state space is developed. The adaptation algorithms are coupled with the nonlinear control law that produce a nonlinear adaptive controller. The performance of the proposed controller is evaluated by observing the ability of the electro-hydraulic actuator to track a desired force. The results showed that the controller improved the performance of the active suspension system as compared to the nonlinear control law alone.

The road adaptive approach is also reported in Fialho and Balas (2002). In the study, combination of the linear parameter varying (LPV) control with a nonlinear backstepping technique that forms the road adaptive active suspension system is proposed. Two level of adaptation is considered with the lower level control to shape the nonlinear characteristic of the vehicle suspension and the higher level design involves adaptive switching between the different nonlinear characteristic based on the road condition.

On the other hand, Chantranuwathana and Peng (1999), D'Amato and Viassolo (2000) decomposed the active suspension control design into two loops. The main loop calculated the desired actuation force. The inner loop controls the nonlinear hydraulic actuators to achieve tracking of the desired actuation force. The results showed that the proposed controller performed better.

In the active suspension systems, the SMC technique was first utilized by Alleyne *et al.* (1993). In his work, the SMC strategy is used to control the electro-hydraulic actuator in the active suspension system. The performance of the SMC is compared to the proportional integral derivative (PID) control. The objective of the control strategy is to improve the ride quality of the vehicle using the quarter car suspension model. The ride quality is determined by observing the car body acceleration. The results showed that the proposed sliding mode controller has performed better than the PID controller in improving the ride quality but not the trade-off between ride quality and road handling.

Kim and Ro (1998) used a sliding mode controller in active suspension systems with the presence of the nonlinearities factor such as the hardening spring, a quadratic damping force and the 'tire lift-off' phenomenon in a real suspension system. The study utilized the model following technique by choosing the sky-hook damping system as a reference model. Therefore, the sliding mode controller is derived from this reference model. Due to the difficulties to apply the road disturbance to the reference model, such model has been simplified by ignoring the road disturbance. The results showed that the SMC scheme is more robust as compared to the self-tuning control approach under the extreme changes of the suspension parameters due to parameter uncertainties.

Yoshimura *et al.* (2001) used a pneumatic actuator to generate the force input signal in the design of active suspension system. The switching control part in the sliding mode control is determined by using the linear quadratic control approach. The road profile is estimated by using the minimum order observer based on a linear system transformed from the exact nonlinear system. The result indicated that the proposed active suspension system is more effective in the vibration isolation of the car body than the linear active suspension based on the LQ control theory.

The SMC scheme with multi-input multi-output (MIMO) has been reported by Park and Kim (1998) to control the active suspension system of a full car model. The study used decentralized VSC to control each of the four suspensions. In order to apply the SMC on the suspension system, the original plant model is transformed into the regular form as presented in DeCarlo *et al.* (1988) by using a transformation matrix. They reported that the performance of the SMC is better as compared to the linear quadratic regulator (LQR) technique.

Recently, intelligent based techniques such as fuzzy logic, neural network and genetic algorithm have been applied to the active suspension system. Ting *et al.* (1995) presented a sliding mode fuzzy control technique for a quarter car model active suspension system. In this study, the controller is organized into two levels. At the basic level, the conventional fuzzy control rule sets and inference mechanism are constructed to generate a fuzzy control scheme. At the supervising level, the control performance is evaluated to modify system parameters. The controller input consists of the input from the sliding mode controller and fuzzy controller. The results showed that the fuzzy SMC attained superior performance in body acceleration and road handling ability but worst in the suspension travel as compared to the conventional sliding mode scheme.

Yoshimura *et al.* (1999) presented an active suspension system for passenger cars, using linear and fuzzy logic control technique. The studied utilize vertical acceleration of the vehicle body as the principle source of control, and the fuzzy logic control scheme as the complementary control of the active suspension system for passenger cars. The fuzzy control rules are determined by minimizing the mean squares of the time responses of the vehicle body under certain constraints on the acceptable relative displacements between vehicle body and suspension parts and tire deflections.

The simulation results showed that both the skyhook damper-fuzzy logic and the linear-fuzzy logic controls are effective in the vibration isolation of the vehicle body.

The combinations of fuzzy-proportional integral/proportional derivative (PI/PD) control and genetic algorithm (GA) was introduced by Kuo and Li (1999) to improve the performance of the active suspension system. The fuzzy PI controller was employed to reduce the sprung mass acceleration due to the nature of road surface. The rule table of the fuzzy logic control was tuned using the GA. It means that, the GA was used to construct the optimal decision-making logic (DML) of the fuzzy logic control for the active suspension system to obtain the best performance.

In the above review, various active suspension system models with either quarter or half car models have been used in the design of the controllers. The quarter car model with linear force input has been used by Hac (1987), Hrovat (1997 and 1998), Tseng and Hrovat (1990), Sunwoo *et al.* (1991), Ray (1993), Ting *et al.* (1995), Kim and Ro (1998), Huang and Chao (2000) and Yoshimura *et al*. (2001) in their study. Modeling of the active suspension system as a linear force input is the most simple but it does not give an accurate model of the system because the actuator's dynamics have been ignored in the design. Thus the controller developed and the result presented may have problem when applying to the active suspension system in the real world.

In order to overcome the problem, Rajamani and Hedrick (1995), Alleyne and Hedrick (1995), Lin and Kanellakopoulos (1997a and 1997b), Fukao *et al.* (1999), Chantranuwathana and Peng (1999) and Fialho and Balas (2002) have considered the hydraulic actuator dynamics in the design of active suspension system for the quarter car model. All these researchers have utilized the hydraulic actuator dynamics formulated by Merritt (1967). On the other hand, Yoshimura *et al.* (1997 and 1999) has proposed a hydraulic actuator of the actuating ram type in the development of active suspension systems for half car model. The mathematical derivation of this approach is much more simpler compared to the previous approach. However, in the mathematical derivation, Yoshimura *et al.* (1997 and 1999) have assumed that the derivative term in the rate of change of the pressure difference in the cylinder which is nonlinear is insignificant compared to the large value of the effective bulk modulus of the oil, thus can be replaced by a linear term. Therefore, the rate of change of the pressure difference in the cylinder has been assumed to be a linear parameter in their modeling. However, the actual rate of change of the pressure difference in the cylinder is nonlinear, and this nonlinearity cannot be ignored if a complete mathematical modeling of the hydraulic actuator dynamics is required.

2.2.1 Summary of Existing Control Methods and Active Suspension System

The control strategies been proposed to control the active suspension system may be loosely group into linear, nonlinear and intelligent control approaches.

The linear control strategies is mainly based on the optimal control theory such as the LQR, LQG, LTR and H-infinity and is capable of minimizing a defined performance index, however, they do not have the capability to adapt to significant system parameter changes and variations in the road profiles.

The intelligent techniques have shown satisfactory performances on the ride comfort and road handling characteristics of the active suspension systems. However, these techniques have a potential problem on stability. Usually, discussions on the stability factor were ignored in the designs.

The nonlinear techniques such as adaptive control and sliding mode control as described in previous section have been used to control the active suspension systems. Usually, in their applications, the adaptive control technique was combined with the optimal control method to deliver an acceptable performance. The sliding mode control technique was shown to be capable of improving the trade off between ride comfort and road handling characteristics. Furthermore, it is shown to be highly robust to the uncertainties. However, in the previous application of the sliding mode control technique on active suspension the conventional sliding surface has been used and none of the researchers have used the proportional-integral sliding surface method.

The active suspension systems for the quarter car models may be modeled as the linear force input or the hydraulically actuated input. The active suspensions systems of the hydraulically actuated input may represent a much more detail of the system dynamics compared to the linear force input. Therefore, the analysis and design of the active suspension systems by using this approach is much closer to the actual systems.

However, most of the published works are focused on the outer-loop controller in computation of the desired control force as a function of vehicle states and the road disturbance (Shen and Peng, 2003). It is commonly assumed that the hydraulic actuator is an ideal force generator and able to carry out the commanded force accurately. Thus, simulations of these outer-loop controllers were frequently done without considering actuator dynamics, or with highly simplified hydraulic actuator dynamics.

In the real implementation, actuator dynamics can be quite complicated, and the interaction between the actuator and the vehicle suspension cannot be ignored. It is also difficult to produce the actuator force close to the target force without implementing inner-loop or force tracking controller. This is due to the fact that the hydraulic actuator exhibits non-linear behavior resulting from servo-valve dynamics, residual structural damping, and the unwanted effects of back-pressure due to the interaction between the hydraulic actuator and vehicle suspension system.

CHAPTER 3

METHODOLOGY

3.1 Introduction

The development of an active suspension system for the vehicle is of great interest for both academic and industrial fields. The studies of active suspension system have been performed using various suspension models. In the quarter car model, the model takes into account the interaction between the quarter car body and the single wheel. Motion of the car is only in the vertical direction. Modeling of active suspension system in the early days considered that input to the active suspension system is a linear force. However due the development of new control theory as discussed in chapter 2, the force input to the active suspension system has been replaced by an input to control the actuator. Therefore, the dynamic of the active suspension now consist of the dynamic of the suspension and the dynamic of the hydraulic actuator.

In addition, researchers have recognized that actuator dynamics can be quite complicated, and the interaction between the actuator and the vehicle suspension cannot be ignored. Thus, the controller design must include force tracking control to produce the actuator force close to the target force.

In this chapter, detail derivation on the modeling of a new type (Macpherson strut) active suspension system for a quarter car and the modeling of the hydraulic actuator are presented. This is followed by the controller design and the 3D animation design procedure.

3.2 Modeling of a Non-Linear Quarter Car Suspension System

The active suspension system can be divided into 2 parts: the quarter car suspension and the hydraulic actuators. In the following subsections, detail derivation on the modeling of the active suspension system for quarter car model and modeling of the hydraulic actuator are presented.

3.2.1 Dynamic Model of a Non-Linear Quarter Car Suspension System

A schematic diagram of the suspension system is shown in Figure 3.1 (a). This model includes the rotational motion of the wheel and the dynamics of the control arm. The quarter car model for the active suspension system is shown in Figure 3.1 (b). The assumptions for the quarter car modeling are as follows: the tire is modeled as a linear spring without damping, there is no rotational motion in the body, the behavior of spring and damper are linear, the tire is always in contact with the road surface and the effect of friction is neglected so that the residual structural damping is not considered into the vehicle modeling.

The vertical displacement z_s of the sprung mass (body) and the rotational angle θ of the control arm are chosen as the generalized coordinate. Following the method outline in Hong *et al.* (1999; 2002):

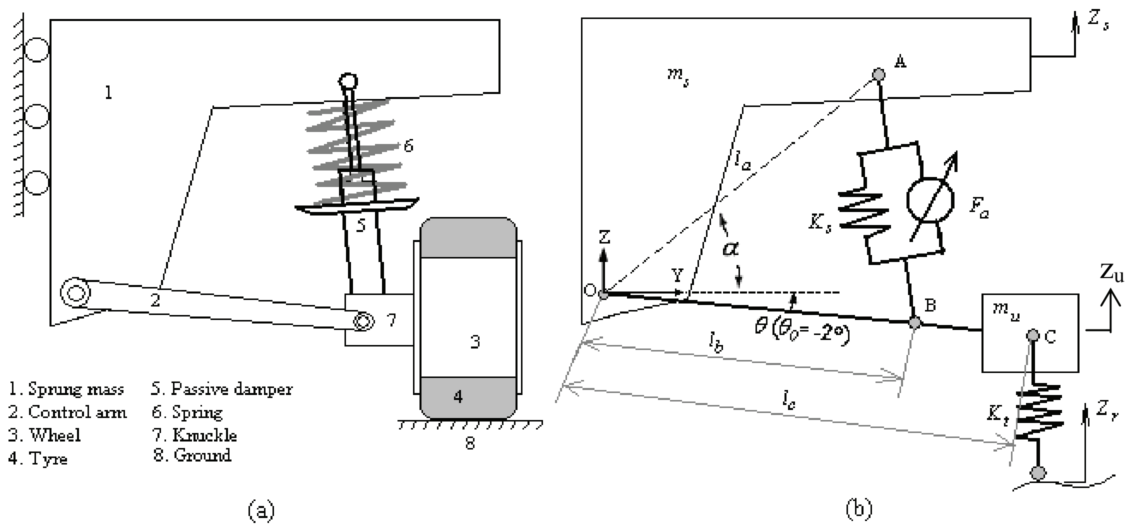

Figure 3.1 : Non-Linear quarter car model

Let T, V, and D denote the kinetic energy, the potential energy and the damping energy of the system.

$$T = \frac{1}{2}(m_s + m_u)\dot{z}_s^2 + \frac{1}{2}m_u l_C^2 \dot{\theta}^2 + m_u l_C \cos(\theta - \theta_0)\dot{\theta} \cdot \dot{z}_s \tag{3.1}$$

$$V = \frac{1}{2}k_s \left\{ 2a_l - b_l[\cos\alpha' + \cos(\alpha' - \theta)] - 2\begin{bmatrix} a_l^2 - a_l b_l(\cos\alpha') + \cos(\alpha' - \theta) \\ + b_l^2 \cos\alpha' \cos(\alpha' - \theta) \end{bmatrix}^{\frac{1}{2}} \right\} \tag{3.2}$$
$$+ \frac{1}{2}k_t\{z_s + l_C[\sin(\theta - \theta_0) - \sin(-\theta_0) - z_r]\}$$

$$D = \frac{1}{2}C_s\left(\dot{\Delta} l\right)^2 = \frac{C_s b_l^2 \sin^2(\alpha' - \theta)\dot{\theta}^2}{8[a_l - b_l \cos(\alpha' - \theta)]} \tag{3.3}$$

where

m_s	=	car body mass (kg)
m_u	=	wheel mass (kg)
z_s	=	vertical displacement of car body (m)
k_s	=	stiffness of car body spring (N/m)

k_t	=	stiffness of car tyre (N/m)
C_s	=	damping of damper (Ns/m)
z_r	=	irregular excitation from the road surface
l_A	=	distance from A to O (m)
l_B	=	distance from B to O (m)
l_C	=	length of the control arm (m)
α	=	angle between y-axis and $\overline{OA}$
θ_0	=	angular displacement of the control arm at a static equilibrium point (-2 deg)
f_a	=	control input (N)

and

$$a_l = l_A^2 + l_B^2$$

$$b_l = 2l_A l_B$$

$$\alpha' = \alpha + \theta_0$$

$$c_l = a_l^2 - a_l b_l \cos(\alpha')$$

$$d_l = a_l b_l - b_l^2 \cos(\alpha')$$

For the two generalized coordinates z_s and θ, the equations of motion are

$$\begin{aligned}&(m_s + m_u)\ddot{z}_s + m_u l_C \cos(\theta - \theta_0)\ddot{\theta} - m_u l_C \sin(\theta - \theta_0)\dot{\theta}^2 \\ &+ k_t\{z_s + l_C(\sin(\theta - \theta_0) - \sin(\theta_0)) - z_r\} = 0\end{aligned} \tag{3.4}$$

$$\begin{aligned}&m_u l_C^2 \ddot{\theta} + m_u l_C \cos(\theta - \theta_0)\ddot{z}_s + k_t l_C \cos(\theta - \theta_0)\{z_s + l_C(\sin(\theta - \theta_0) - \sin(-\theta_0)) - z_r\} \\ &- \frac{1}{2} k_s \sin(\alpha' - \theta) \times \left\{ b_l + \frac{d_l}{\left(c_l - d_l \cos(\alpha' - \theta)\right)^{\frac{1}{2}}} \right\} = -l_B f_a\end{aligned} \tag{3.5}$$

Define the state variables as

$$x(t) = \begin{bmatrix} x_1(t) & x_2(t) & x_3(t) & x_4(t) \end{bmatrix}^T = \begin{bmatrix} z_s & \dot{z}_s & \theta & \dot{\theta} \end{bmatrix}^T \tag{3.6}$$

Hence, the equations (3.4) and (3.5) can be rewritten as

$$\begin{aligned} \dot{x}_1 &= x_2, \qquad \dot{x}_2 = f_1(x_1, x_2, x_3, x_4, f_a, z_r) \\ \dot{x}_3 &= x_4, \qquad \dot{x}_4 = f_2(x_1, x_2, x_3, x_4, f_a, z_r) \end{aligned} \tag{3.7}$$

where

$$f_1 = \frac{1}{g_1(x_3)} \left\{ \begin{aligned} & m_u l_C^2 \sin(x_3 - \theta_0) x_4^2 - \frac{1}{2} k_s \times \sin(\alpha' - x_3) \cos(x_3 - \theta_0) g_3(x_3) \\ & - k_t l_C \sin^2(x_3 - \theta_0) z(\cdot) + l_B f_s \cos(x_3 - \theta_0) \end{aligned} \right\} \tag{3.8}$$

$$f_2 = -\frac{1}{g_2(x_3)} \left\{ \begin{aligned} & m_u^2 l_C \sin(x_3 - \theta_0) \cos(x_3 - \theta_0) x_4^2 - \frac{1}{2} (m_s + m_u) k_s \sin(\alpha' - x_3) g_3(x_3) \\ & + m_s k_t l_C \cos(x_3 - \theta_0) z(\cdot) + (m_s + m_u) l_B f_a \end{aligned} \right\} \tag{3.9}$$

$$g_1(x_3) = m_s l_C + m_u l_C \sin^2(x_3 - \theta_0) \tag{3.10}$$

$$g_2(x_3) = m_s m_u l_C^2 + m_u^2 l_C^2 \sin^2(x_3 - \theta_0) \tag{3.11}$$

$$g_3(x_3) = b_l + \frac{d_l}{\left(c_l - d_l \cos(\alpha' - x_3)\right)^{\frac{1}{2}}} \tag{3.12}$$

$$z(\cdot) = z(x_1, x_2, z_r) = x_1 + l_C \left(\sin(x_3 - \theta_0) - \sin(-\theta_0)\right) - z_r \tag{3.13}$$

In the state space form, the suspension system is represented by

$$\dot{x}(t) = Ax(t) + B_1 f_a + B_2 z_r(t) \qquad x(0) = x_0 \tag{3.14}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_3} & \frac{\partial f_1}{\partial x_4} \\ 0 & 0 & 0 & 1 \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_3} & \frac{\partial f_2}{\partial x_4} \end{bmatrix} \tag{3.15}$$

and

$$B_1 = \begin{bmatrix} 0 \\ \frac{\partial f_1}{\partial f_a} \\ 0 \\ \frac{\partial f_2}{\partial f_a} \end{bmatrix} \tag{3.16}$$

$$B_2 = \begin{bmatrix} 0 \\ \frac{\partial f_1}{\partial z_r} \\ 0 \\ \frac{\partial f_2}{\partial z_r} \end{bmatrix} \tag{3.17}$$

The elements of matrix A are represented by,

$$\frac{\partial f_1}{\partial x_1} = \frac{-K_t l_C \sin^2(x_3 - \theta_0)}{m_s l_C + m_u l_C \sin^2(x_3 - \theta_0)}$$

$$\frac{\partial f_1}{\partial x_2} = 0$$

$$\frac{\partial f_1}{\partial x_3}=\frac{1}{\left(m_s l_C+m_u l_C\sin^2(x_3-\theta_0)\right)^2}\left(\begin{array}{l}\left\{\begin{array}{l}\frac{1}{2}K_s\left(b_1+\frac{d_l}{\left(c_l-d_l\cos(\alpha'-x_3)\right)^{\frac{1}{2}}}\right)\cos(\alpha'+\theta_0)+\\ \frac{1}{2}K_s\sin(\alpha'-x_3)\cos(x_3-\theta_0)\left(\frac{d_l^2\sin(\alpha'-x_3)}{2\left(c_l-d_l\cos(\alpha'-x_3)\right)^{\frac{3}{2}}}\right)-\\ K_t l_C^2\sin^2(x_3-\theta_0)\cos(x_3-\theta_0)\end{array}\right\}\times\left(m_s l_C+m_u l_C\sin^2(x_3-\theta_0)\right)-\\ m_s K_s l_C\sin(\alpha'-x_3)\sin(x_3-\theta_0)\cos^2(x_3-\theta_0)\times\left(b_1+\frac{d_l}{\left(c_l-d_l\cos(\alpha'-x_3)\right)^{\frac{1}{2}}}\right)\end{array}\right)$$

$$\frac{\partial f_1}{\partial x_4}=\frac{-2m_u l_C^2\sin(x_3-\theta_0)x_4}{m_s l_C+m_u l_C\sin^2(x_3-\theta_0)}$$

$$\frac{\partial f_2}{\partial x_1}=\frac{-m_s K_t l_C\cos(x_3-\theta_0)}{m_s m_u l_C^2+m_u^2 l_C^2\sin^2(x_3-\theta_0)}$$

$$\frac{\partial f_2}{\partial x_2}=0$$

$$\frac{\partial f_2}{\partial x_3}=\frac{1}{\left(m_s m_u l_C^2+m_u^2 l_C^2\sin^2(x_3-\theta_0)\right)^2}\left(\begin{array}{l}\left\{\begin{array}{l}\frac{1}{2}(m_s+m_u)K_s\left(b_1+\frac{d_l}{\left(c_l-d_l\cos(\alpha'-x_3)\right)^{\frac{1}{2}}}\right)\cos(\alpha'-x_3)+\\ -\frac{1}{2}(m_s+m_u)K_s\sin(\alpha'-x_3)\left(\frac{d_l^2\sin(\alpha'-x_3)}{2\left(c_l-d_l\cos(\alpha'-x_3)\right)^{\frac{3}{2}}}\right)-\\ m_s K_t l_C^2\sin^2(x_3-\theta_0)\cos(x_3-\theta_0)\end{array}\right\}\times\left(m_s l_C+m_u l_C\sin^2(x_3-\theta_0)\right)-\\ \frac{1}{2}(m_s+m_u)m_u^2 K_s l_C^2\sin(\alpha'-x_3)\sin(x_3-\theta_0)\times\left(b_1+\frac{d_l}{\left(c_l-d_l\cos(\alpha'-x_3)\right)^{\frac{1}{2}}}\right)\end{array}\right)$$

$$\frac{\partial f_2}{\partial x_4}=\frac{-2m_u^2 l_C^2\sin(x_3-\theta_0)\cos(x_3-\theta_0)x_4}{m_s m_u l_C^2+m_u^2 l_C^2\sin^2(x_3-\theta_0)}$$

And the elements of B1 and B2 are

$$\frac{\partial f_1}{\partial f_s} = \frac{l_B \cos(x_3 - \theta_0)}{m_s l_C + m_u l_C \sin^2(x_3 - \theta_0)}$$

$$\frac{\partial f_2}{\partial f_s} = \frac{-(m_s + m_u) l_B}{m_s m_u l_C^2 + m_u^2 l_C^2 \sin^2(x_3 - \theta_0)}$$

$$\frac{\partial f_1}{\partial z_r} = \frac{K_t l_C \sin^2(x_3 - \theta_0)}{m_s l_C + m_u l_C \sin^2(x_3 - \theta_0)}$$

$$\frac{\partial f_2}{\partial z_r} = \frac{m_s K_t l_C \cos(x_3 - \theta_0)}{m_s m_u l_C^2 + m_u^2 l_C^2 \sin^2(x_3 - \theta_0)}$$

In the case of passive suspension system, f_a in the equation (3.14) is represented by the force that produce by the conventional hydraulic damper. It is set to be equal to the multiplication between the damping constant of passive damper, C_s and the relative velocity between the car body and wheel. As the relative velocity between the car body and the wheel is represented by

$$\dot{\Delta l} = \frac{2 l_a l_b \sin(\alpha' - \theta)\dot{\theta}}{2\left(l_a^2 + l_b^2 - 2 l_a l_b \cos(\alpha' - \theta)\right)^{\frac{1}{2}}} \tag{3.18}$$

Therefore,

$$f_a = C_s \times \dot{\Delta l} = C_s \times \frac{2 l_a l_b \sin(\alpha' - \theta)\dot{\theta}}{2\left(l_a^2 + l_b^2 - 2 l_a l_b \cos(\alpha' - \theta)\right)^{\frac{1}{2}}} \tag{3.19}$$

3.2.2 Dynamic Model of Hydraulic Actuator

The hydraulic actuator consists of five main components namely the electro hydraulic powered spool valve, piston-cylinder, hydraulic pump, reservoir and piping system as shown in Figure 3.2. The power supply is needed to drive the hydraulic pump through the AC motor and to control the spool valve position. The hydraulic pump will keep the supply pressure at the optimum level of pressure. The spool valve position will control the flow of the fluid to the piston-cylinder that determines the amount of force produced by the hydraulic actuator.

The hydraulic actuator is governed by electro hydraulic servo valve which consists of an actuator, a primary power spool valve and a secondary bypass valve. As seen in Figure 3.3, the hydraulic actuator cylinder lies in a follower configuration to a critically centered electro hydraulic power spool valve with matched and symmetric orifices. Positioning of the spool u_1 directs the high pressure fluid to flow in either to one of the cylinder chambers and connects the other chamber to the pump reservoir. This flow creates a pressure difference P_L across the piston. This pressure difference multiplied by the piston area A_p is the active force F_a for the suspension system. The derivative of F_a is given by Eq. (3.20).

$$\dot{F}_a = A_p \alpha_\alpha \left[C_{d1} w u_1 \sqrt{\frac{P_s - \text{sgn}(u_1) P_L}{\rho}} - C_{d2} u_2 \,\text{sgn}(P_L) \sqrt{\frac{2P_L}{\rho}} - C_{tm} P_L - A_p (\dot{z}_s - \dot{z}_u) \right]$$

(3.20)

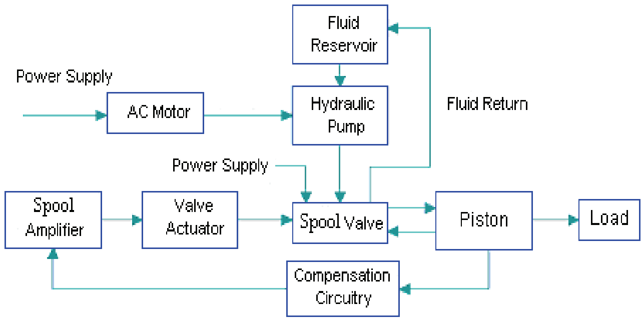

Figure 3.2: Diagram of a complete set of hydraulic actuator

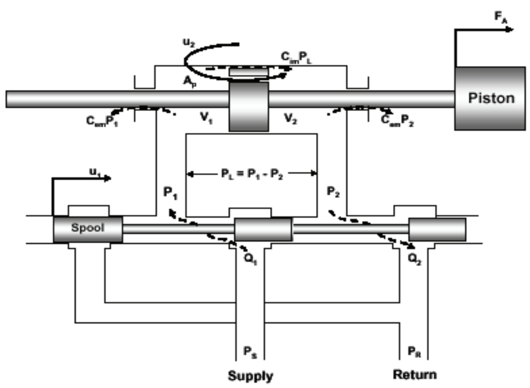

Figure 3.3: Physical schematic and variables for the hydraulic actuator

The dynamics for the hydraulic actuator valve are given as the following: the change in force is proportional to the position of the spool with respect to center, the relative velocity of the piston, and the leakage through the piston seals. A second input u_2 may be used to bypass the piston component by connecting the piston chambers. The bypass valve u_2 could be used to reduce the energy consumed by the system. If the spool position u_1 is set to zero, the bypass valve and actuator will behave similar to a variable orifice damper. Spool valve positions u_1 and bypass area u_2 are controlled by a current-position feedback loop. The essential dynamics of the spool have been shown to resemble a first order system (Donahue, 2001):

$$\tau\dot{u} + u = kv \tag{3.21}$$

The parameters of hydraulic actuator model are taken from Donahue (2001) as the followings: A_p= 0.0044 m^2 , α_α = 2.273e9 N/m^5 , C_{d1} = 0.7 , C_{d2} = 0.7 , w = 0.008 m , P_s = 20684 kN/m^2 , ρ =3500, C_{tm} = 15e-12 , τ = 0.001 sec^{-1} .

3.3 Controller Design

The controller structure adopted in this study is shown in Figure 3.4. Basically, the controller structure of a suspension system utilizes two controller loops namely the outer loop and inner loop controllers which corresponds to vehicle controller and actuator controller. The similar terms, which are often used for outer and inner loop controllers, are global and local controllers. The controller structure was used for an active suspension system in (Chantranuwatana, 2001; Chantranuwatana and Peng, 2000; D'Amato and Viassolo, 2000). The similar controller structure was used for semi-active suspension control in Sims *et al*., 1999; Hudha *et al*., 2005).

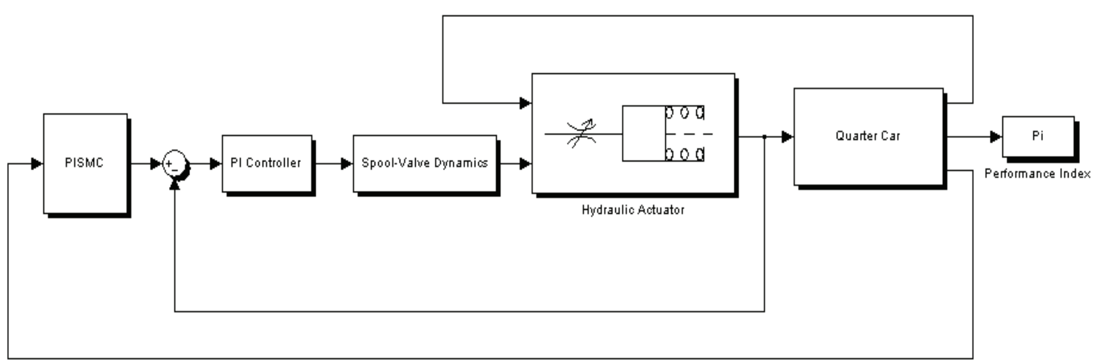

Figure 3.4: Controller structure of the active suspension system

Once the inner loop controller of hydraulic actuator is able to track well the target forces with acceptable error, the hydraulic actuator model and the inner loop controller are then integrated with the outer loop of active suspension control. In this configuration, the inner loop controller must be able to track the optimum target force of the hydraulic actuator calculated by the outer loop controller. The actual force of the hydraulic actuator is inserted to the vehicle model to reject the effects of road disturbance.

3.3.1 Inner Loop Controller Design

The structure of force tracking control of hydraulic actuator is shown in Figure 3.5. The hydraulic actuator model takes two inputs namely the spool valve position and the real time piston speed. Proportional Integral control is implemented which takes force tracking error as the input and delivers control voltage to drive the spool valve. The forcing functions are selected to represent real world situations which depending on the type of road disturbance, and may be represented by sinusoidal, saw-tooth, square, random functions and/or their combinations.

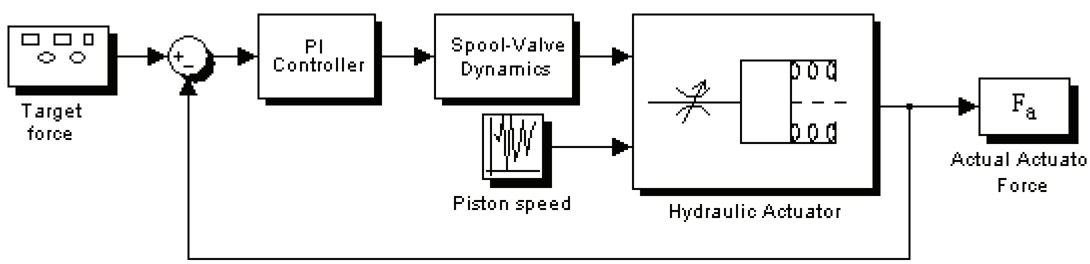

Figure 3.5: Force tracking control of hydraulic actuator

3.3.2 Outer Loop Controller Design

The outer loop controller is used for disturbance rejection control to reduce unwanted vehicle motion. The inputs of the outer loop controller are the body velocity and wheel velocity, whereas the output of the outer loop controller is the target force that must be tracked by the hydraulic actuator. On the other hand, the inner loop controller is used for force tracking control of the hydraulic actuator in such a way that the force produced by the hydraulic actuator is as close as possible to the target force produced by the disturbance rejection control. The disturbance rejection control utilized in this study is the proportional integral sliding mode controller where the PI sliding surface is defined as follows (Sam, 2004; Sam *et al.*, 2004; Sam *et al.*, 2005):

$$\sigma(t) = Cx(t) - \int_0^t (CA + CB_1K)x(\tau)d\tau \tag{3.22}$$

where $C \in \Re^{mxn}$ and $K \in \Re^{mxn}$ are constant matrices. The matrix K satisfies $\lambda(A + B_1K) < 0$ and C is chosen so that CB_1 is nonsingular. Suppose there exists a finite time t_s such that the solution to Equation (3.14) represented by $x(t)$ satisfy $\sigma(t) = 0$ for all $t \geq t_s$, then an ideal sliding motion is said to be taking place for all $t > t_s$. Therefore the equivalent control, $u_{eq}(t)$ can thus be obtained by letting $\dot{\sigma}(t) = 0$ (Itkis, 1976), i.e,

$$\dot{\sigma}(t) = C\dot{x}(t) - \{CA + CB_1K\}x(t) = 0 \tag{3.23}$$

If the matrix C is chosen such that CB_1 is nonsingular, this yields

$$u_{eq}(t) = Kx(t) - (CB_1)^{-1}CB_2z_r(t) \tag{3.24}$$

Substituting Eq. (3.24) into Eq. (3.14) gives the equivalent dynamic equation of the system in sliding mode as:

$$\dot{x}(t) = (A + B_1K)x(t) + \{I_n - B_1(CB_1)^{-1}C\}B_2z_r(t) \tag{3.25}$$

Theorem 1. If

$\left\|\tilde{F}(t)\right\| \leq \beta_1 = \left\|I_n - B_1(CB_1)^{-1}C\right\|\beta$, the uncertain system in Eq. (3.25) is boundedly stable on the sliding surface $\sigma(t) = 0$.

Proof. For simplicity, let

$$\tilde{A} = (A + B_1 K) \tag{3.25a}$$

$$\tilde{F}(t) = \{I_n - B_1(CB_1)^{-1}C\}B_2 z_r(t) \tag{3.25b}$$

Eq. (3.25) can be rewritten as

$$\dot{x}(t) = \tilde{A}x(t) + \tilde{F}(t) \tag{3.26}$$

Let the Lyapunov function candidate for the system chosen as

$$V(t) = x^T(t)Px(t) \tag{3.27}$$

Taking the derivative of $V(t)$ and substituting Eq. (3.26), gives

$$\begin{aligned} \dot{V}(t) &= x^T(t)[\tilde{A}^T P + P\tilde{A}]x(t) + \tilde{F}^T(t)Px(t) + x^T(t)P\tilde{F}(t) \\ &= -x^T(t)Qx(t) + \tilde{F}^T(t)Px(t) + x^T(t)P\tilde{F}^T(t) \end{aligned} \tag{3.28}$$

where P is the solution of $\tilde{A}^T P + P\tilde{A} = -Q$ for a given positive definite symmetric matrix Q. It can be shown that Eq. (3.28) can be simplified as

$$\dot{V}(t) = -\lambda_{\min}(Q)\|x(t)\|^2 + 2\beta_1\|P\|\|x(t)\| \tag{3.29}$$

Since $\lambda_{\min}(Q) > 0$, $\dot{V}(t) < 0$ for all t and $x \in \mathrm{B}^c(\eta)$, where $\mathrm{B}^c(\eta)$ is the complement of the closed ball $\mathrm{B}(\eta)$, centered at $x = 0$ with radius $\eta = \dfrac{2\beta_1\|P\|}{\lambda_{\min}(Q)}$. Hence, the system is boundedly stable.

Remark. For the system with uncertainties that satisfies the matching condition, i.e, $rank[B_1 \mid B_2 z_r(t)] = rank[B_1]$, then Eq. (3.25) can be reduced to $\dot{x}(t) = (A + B_1K)x(t)$ (Edward and Spurgeon, 2000). Thus asymptotic stability of the system during sliding mode is assured.

The proposed control scheme is designed in such a way that drives the state trajectories of the system in Eq. (3.14) onto the sliding surface $\sigma(t) = 0$ and the system remains in it thereafter. For the uncertain system in Eq.(3.14), the following control law is proposed:

$$u(t) = -(CB_1)^{-1}[CAx(t) + \phi\sigma(t)] - k(CB_1)^{-1}\frac{\sigma(t)}{\|\sigma(t)\| + \delta} \tag{3.30}$$

where $\phi \in \Re^{mxm}$ is a positive symmetric design matrix, k and δ are the positive constants.

Theorem 2. The hitting condition of the sliding surface in Eq. (3.22) is satisfied if

$$\|A + B_1K\|\|x(t)\| \geq \|B_2 z_r(t)\| \tag{3.31}$$

Proof: In the hitting phase $\sigma^T(t)\sigma(t) > 0$, by using the Lyapunov function candidate $V(t) = \frac{1}{2}\sigma^T(t)\sigma(t)$, the following function is obtained

$$\begin{aligned}\dot{V}(t) &= \sigma^T(t)\dot{\sigma}(t) \\ &= \sigma^T(t)[-(CA + CB_1K)x(t) - \phi\sigma(t) - \frac{k\sigma(t)}{\|\sigma(t)\| + \delta} + CB_2(t)] \qquad (3.32) \\ &\leq -[\{\|\phi\| + \left\|\frac{k}{\|\sigma(t)\| + \delta}\right\|\}\|\sigma(t)\|^2 + \{\|C\|\|A + B_1K\|\|x(t)\| - \|C\|\|B_2 z_r(t)\|\}\|\sigma(t)\|]\end{aligned}$$

It follows that $\dot{V}(t) < 0$ if condition in Eq. (3.31) is satisfied. Thus, the hitting condition is satisfied.

3.4 Virtual Reality Animation

3.4.1 Introduction

The virtual reality animation of the active suspension system is realized using the Virtual Reality Toolbox. The Virtual Reality Toolbox becomes the interface to link between the car suspension system developed in the Simulink environment and the virtual reality model. The virtual model or world is created using the standard Virtual Reality Modeling Language (VRML). It is a text language used for describing 3-D shapes and interactive environments. The Virtual Reality Toolbox uses the VRML97 technology to deliver the open 3-D visualization.

3.4.2 Methodology

The followings are the steps taken in developing the model:

Passive and active nonlinear model of the quarter car suspension system is first developed in Simulink from the dynamic equations of motion of the non-linear quarter car model. These Simulink models are simulated and the outputs of each model are verified to produce satisfactory result. These Simulink models will later be used to generate signals data for the suspension system animation. This data will be used to control and manipulate the virtual reality objects.

Next, the virtual world or three-dimensional scenes of a car going on the road bump on a straight road is created using standard Virtual Reality Modeling Language (VRML) technology. The V-Realm Builder which comes together with the Virtual Reality Toolbox is used to create and edit the VRML code. Further detail adjustment of the coding is later performed using the notepad program.

The scene is kept simple to avoid the system slowing down during simulation. Thus, the IndexedFaceSet node and texture mapping is use to create some of the physical object in the scene such as trees, road, road bump sign, and the ground. The image texture is obtained from the internet and edited using Paint Shop Pro to obtained transparent background in *gif* format. The overall material characteristics and appearance of each object is specified in the Material node. Several view points or *camera positions* are also defined in the virtual reality model to enable the observer to view the motion of the car from many perspectives.

The car movement can be viewed from different perspective using the VRML FieldOfView. There are several viewpoint defined in this model, *static view* where the observer will observe the car moves further from the same viewpoint and *dynamic view* where the observer follow the movement of the car as it moves further. User can choose to view the car from the back right's corner, the side and from the back. Besides that, user can use the navigation panel to travel through the scene.

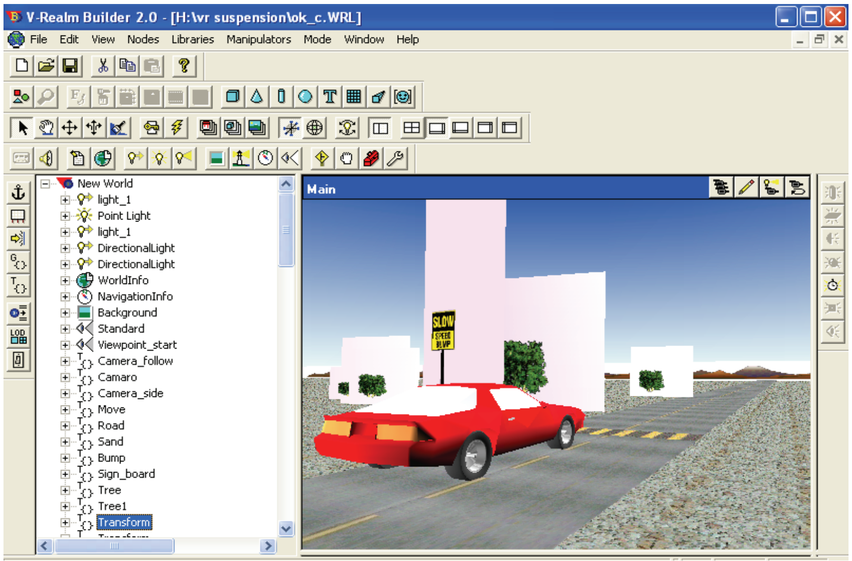

Figure 3.6: The scene constructed using V-Realm Builder

To interface the Simulink block diagram with the virtual world, VR Sink block from the Virtual Reality Toolbox is utilized. The VR sink block is copied and pasted in the Simulink model. To define the association between the Simulink model and the virtual world, the VR Sink block is double clicked to open the Block Parameters: VR Sink dialog box. The respective virtual world is then selected and opened. The Virtual Reality Toolbox automatically scans a virtual world for available VRML nodes that Simulink can drive. The available VRML nodes and its properties are then listed in a hierarchical tree-style viewer. The *car* and *move* transform node *translational field* are selected as the node to connect the model signal.

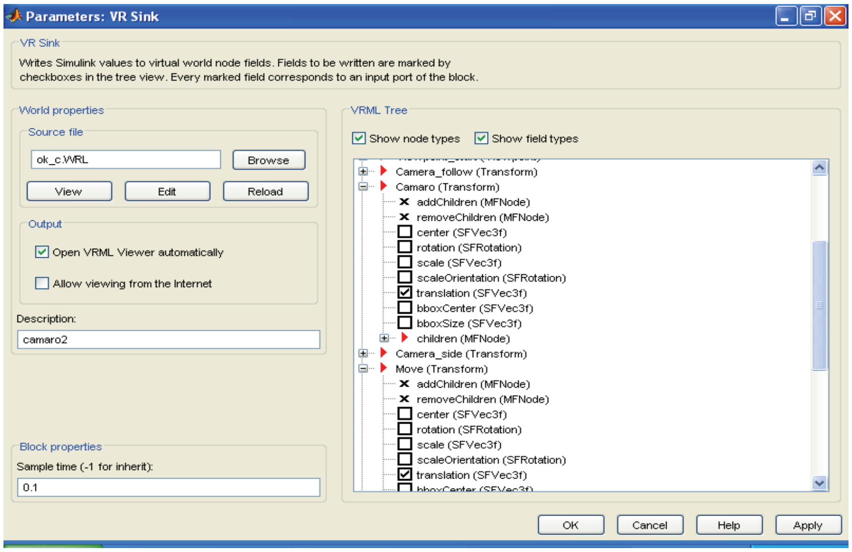

Figure 3.7: The VR Sink block parameters

As a result, the VR Sink block will appear with the corresponding input. These input lines are then connected to the matching signals in the Simulink model. These signals are used to control the motion of the car.

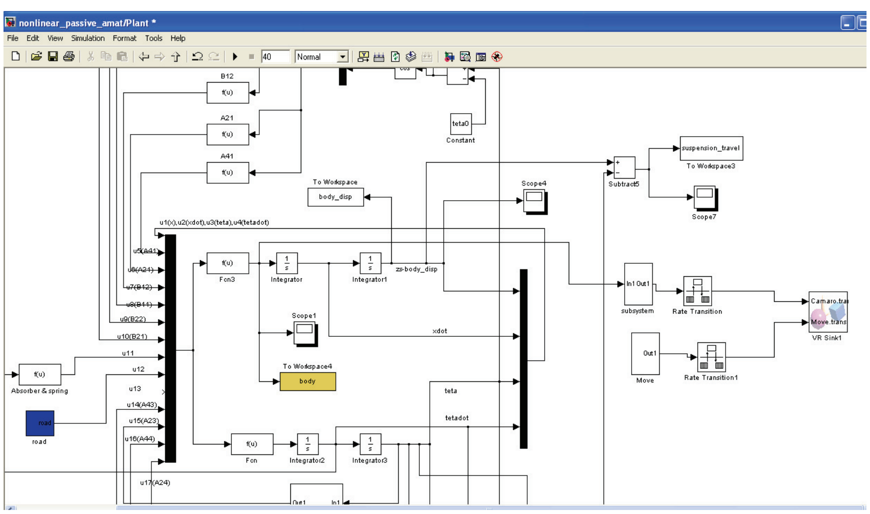

Figure 3.8: The link between Simulink model and virtual world

CHAPTER 4

RESULTS AND DISCUSSION

4.1 Simulation

This section contains the results of simulation studies in both inner loop and outer loop controllers. The parameters of inner loop controller must be optimized separately until the hydraulic actuator is able to provide the actual target force as close as possible with the predefined target force. Then, the inner loop controller is integrated with the outer loop controller. In this configuration, the inner loop controller is used to track the optimum target force produced by the outer loop controller. The performance of the inner loop controller is characterized by its ability in tracking the target force with small amount of force tracking error. Whereas, the performance of the outer loop controller is characterized by the four performance criteria namely body acceleration, body displacement, suspension travel, wheel displacement and the rotational angle of control arm.

4.1.1 Performance of Force Tracking Controller

The force tracking error of the hydraulic actuator model using Proportional Integral controller for sinusoidal, square, saw-tooth and random functions of the target force are shown in Figures 4.1, 4.2, 4.3 and 4.4 respectively. This is to check the controllability of the force tracking controller for a class of continuous and discontinuous functions. In this simulation study, the parameter of proportional gain *P* is set to 1.25 and for Integral gain *I* is set to 0.75. From these figures, it can be seen clearly that the hydraulic actuator model tracks the desired force well.

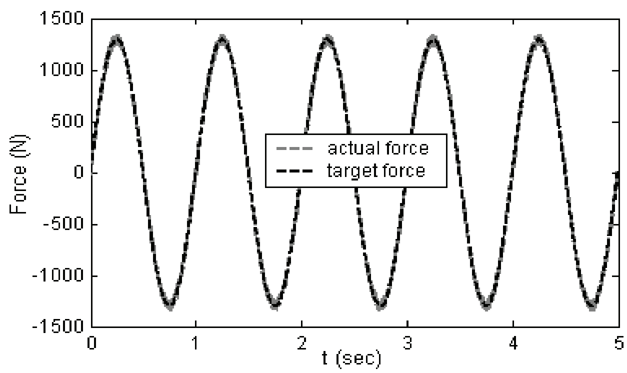

Figure 4.1: Force tracking performance for sinusoidal function of target force

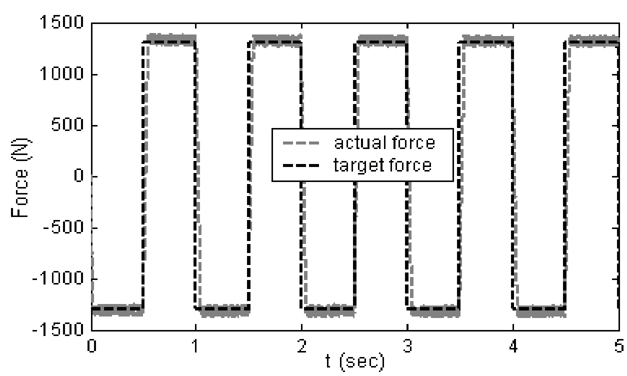

Figure 4.2: Force tracking performance for square function of target force

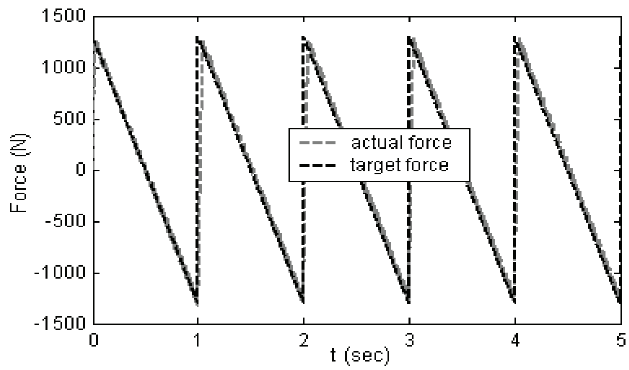

Figure 4.3: Force tracking performance for saw-tooth function of target force

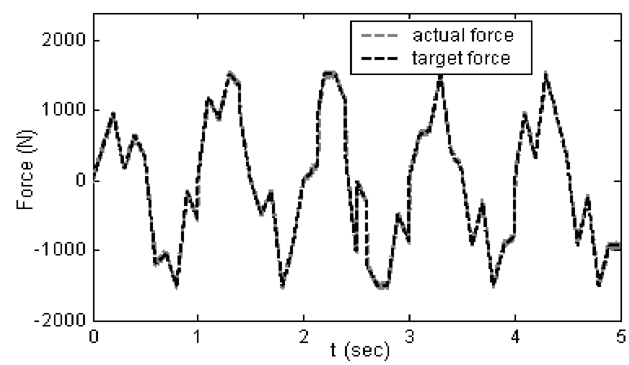

Figure 4.4: Force tracking performance for random function of target force

It is also noted that due to the rapid changes of force magnitude in the case of discontinuous functions of target force such as saw-tooth and square functions, the performance of the force tracking controller is slightly worse than that of the continuous function of target force. This is caused by the response of the spool valve that fails to follow the target force without time delay particularly when rapid change of force magnitude occurred.

4.1.2 Performance of Disturbance Rejection Control

The typical road disturbance considered in this simulation study is a step road bump. This type of road disturbance has been used in (Smith and Wang, 2001; 2002) and set in the following form where, *a* denote the bump amplitude which is set to be ± 10 cm.

$$z_r = \begin{cases} 0 \quad if \quad t < 0.5 \\ a \quad if \quad 0.5 \le t \le 3 \end{cases} \tag{4.1}$$

The simulation was performed for a period of 3 seconds using Heun solver with a step size of 0.001 second. The numerical values of quarter car model parameters are set based on the quarter car characteristics used in Hong *et al.* (2002). The parameters are:

m_s = 453 kg	l_a = 0.66 m
m_u = 36 kg	l_b = 0.34 m
K_s = 17658 N/m	l_c = 0.37 m
C_s = 1500 Nsec/m	α = 74 deg
K_t =183887 N/m	θ_0 = -2 deg

The performance of the PISMC is compared to the linear quadratic regulator (LQR) control approach. Assumes a quadratic performance index in the form of

$$J = \frac{1}{2}\int_0^t (x^T Qx + u^T Ru)dt \tag{4.2}$$

Where the matrix Q is symmetric positive semi-definite and R is positive symmetric definite. Then, the optimal linear feedback control law is obtained as

$$u(t) = -Kx(t) \tag{4.3}$$

Where K is the designed matrix gain.

In the design of the LQR controller, weighting matrices Q and R are selected as Q = diag($q1,q2,q3,q4$) in which $q_1 = q_2 = q_3 = q_4 = 200$ and R = [0.15] respectively. The weights for the LQR are determined by using trial and error method with the series of sensitivity test. In the testing, the weighted matrix Q is positive semidefinite and the weighted matrix R is positive definite. It is widely accepted in comparing the advanced controller to the basic controller such as LQR. The designed gains of the LQR controller are K = [5 30 150 -900]. The pole-zero map of the LQR controller is shown in Figure 4.5. The value of the matrix K for PISMC is similar to the value of the designed gains in the LQR controller. In this simulation study, the following values namely $C = [400 \quad 10 \quad 7500 \quad 15000], \phi = 100$, k=1 and $\delta = 10$ are also selected for the PISMC controller parameters. The sliding surface obtained from the simulation is shown in Figure 4.6.

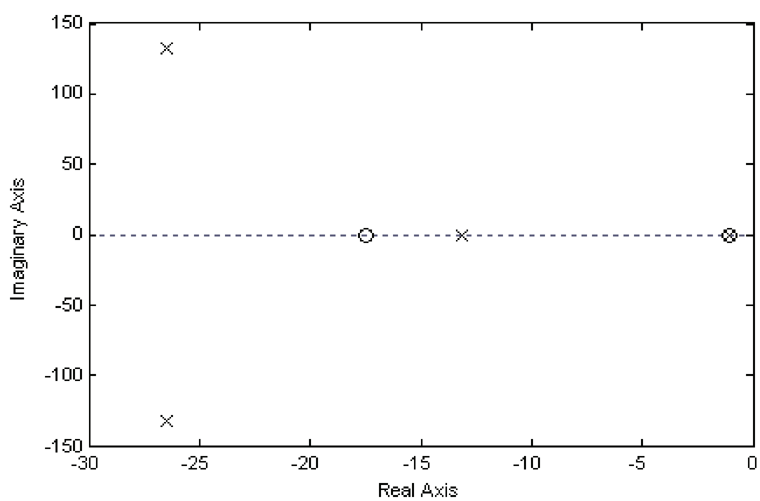

Figure 4.5: Pole-zero map of LQR controller

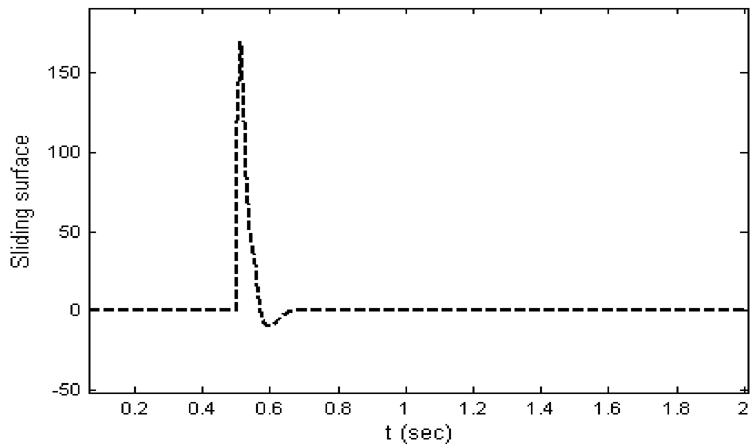

Figure 4.6: Sliding surface of PISMC

From the simulation results, the body acceleration and body displacement performances of PISMC compared to LQR controller along with passive system are shown in Figures 4.7 and 4.8. From the figures, it is clear that the active system with PISMC is able to significantly reduce both amplitude and the settling time of unwanted body motions in the forms of body acceleration and body displacement as compared with the counterparts. It is noted that the root mean square (RMS) values of body acceleration for PISMC, LQR and passive system are 2.994 m/sec^2, 4.135 m/sec^2 and 4.544 m/sec^2 respectively. Similarly, the root mean square (RMS) values of body displacement for PISMC, LQR and passive system are 0.088 m, 0.091 m and 0.094 m respectively.

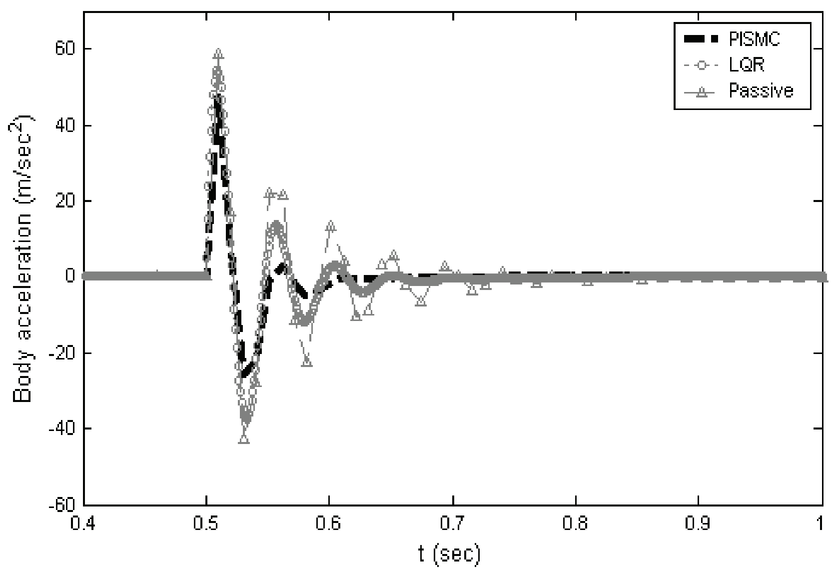

Figure 4.7: Body acceleration

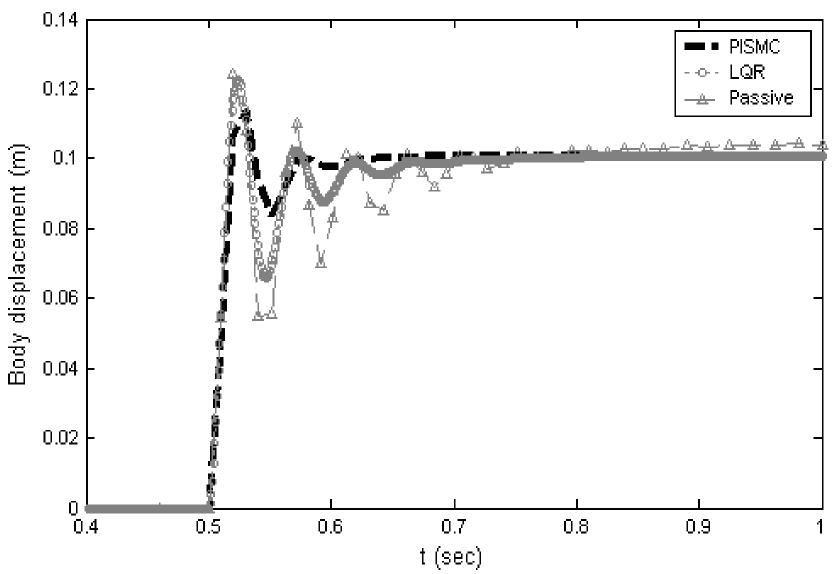

Figure 4.8: Body displacement

Similar trend was found on the suspension deflection performance as shown in Figure 4.9, in which the active system with PISMC shows significant performances in reducing both amplitude and the settling time compared with the LQR controller and the passive system. From the figure, it is clear that the active system with PISMC is able to improve the rattle-space dynamics of the suspension system. It is noted that the root mean square (RMS) values of suspension deflection for PISMC, LQR and passive system are 2.437e-3 m, 3.091e-3 m and 4.032e-3 m, respectively.

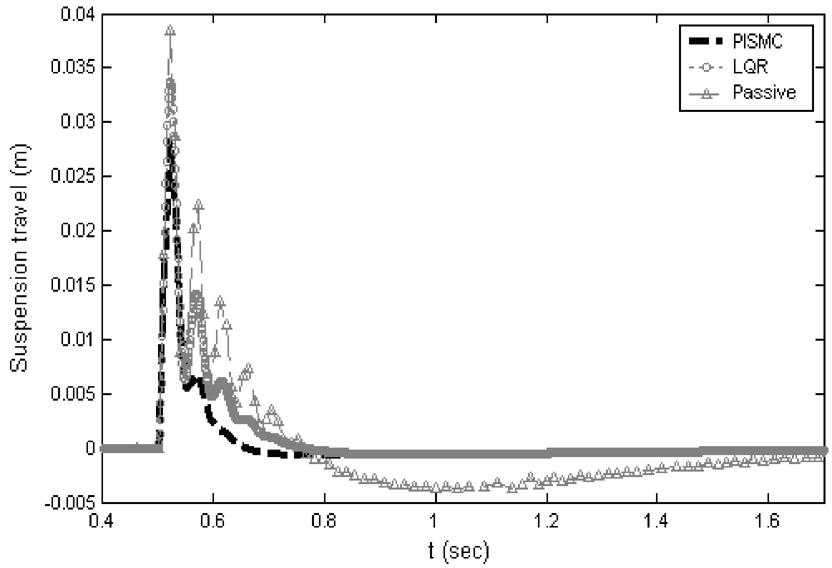

Figure 4.9: Suspension deflection

It can be seen that the magnitude of the wheel displacements for the active system of both PISMC and the LQR controllers are better than the passive system as shown in Figure 4.10. Roughly, the overshoot of wheel displacement of the passive system is about 5 % larger than the active system. But, it is clear that both the rise time and the settling time of wheel-hop for the active system with PISMC is better than the counterparts. The root mean square values of wheel displacement for PISMC, LQR and the passive system are 9.029e-2 m, 9.140e-2 m and 9.201e-2 m, respectively.

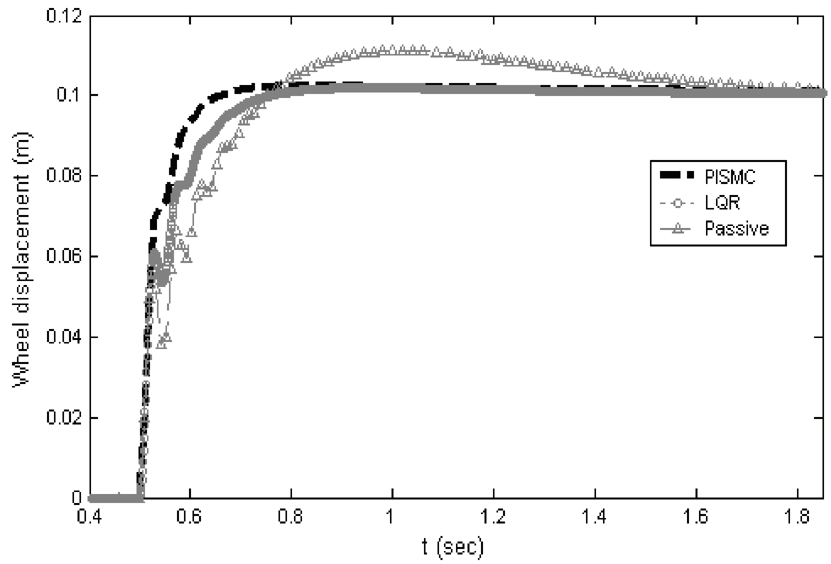

Figure 4.10: Wheel displacement

In this study, the angular position of control arm is included as one of the performance criteria. This is due to the fact that rotational effect of control arm is included in the quarter car model considered in this study. The similar trend was found on the performance of angular position of control arm as shown in Figure 4.11. From the figure, the active system with PISMC shows significant performance in reducing both amplitude and the settling time of control arm angular position compared with the LQR controller and the passive system. It is also noted that the active system with PISMC is able to improve the dynamics of angular motions of the control arm. The root mean square values of angular position of control arm for PISMC, LQR and the passive system are 7.370e-3 deg, 9.113e-3 deg and 1.188e-2 deg, respectively.

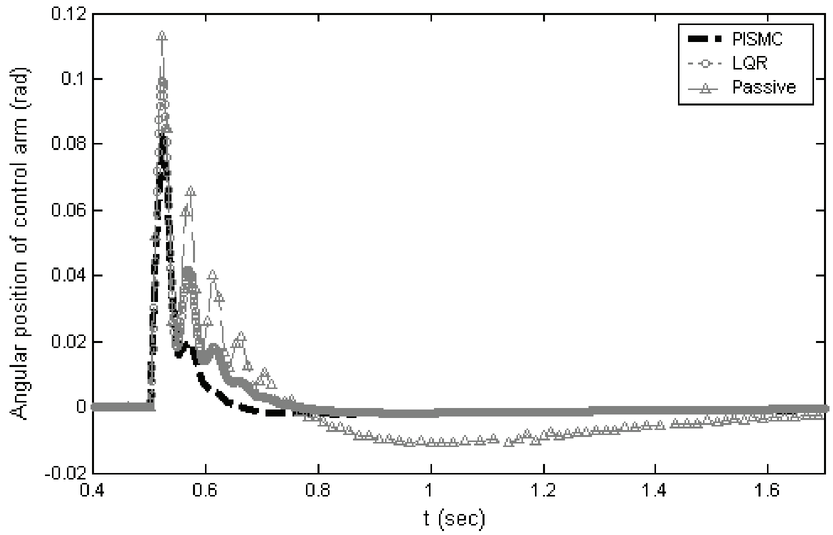

Figure 4.11: The angular dynamics of control arm

The force tracking performances of the inner loop controller for the specified bump input are shown in Figures 4.12 and 4.13. From the figures, it is demonstrated that the hydraulic actuator is able to provide the actual force close to the optimum target force for both PISMC and the LQR controllers. From the force tracking performance of PISMC and LQR controllers, it is noted that the peak-to-peak values of actuator force for PISMC is slightly higher than LQR. On the contrary, the RMS value of actuator force for LQR is also higher than PISMC. This is due to the fact that the actuator force produced by LQR controller has slower settling time than PISMC.

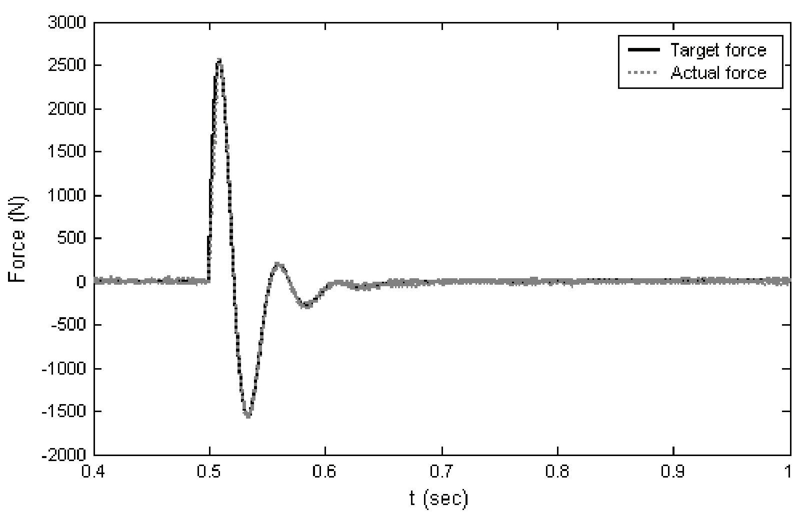

Figure 4.12: Force tracking performance of PISMC

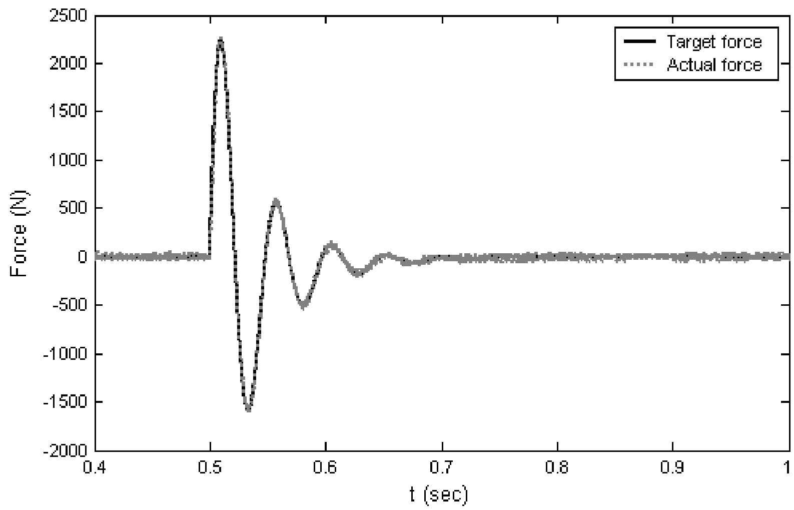

Figure 4.13: Force tracking performance of LQR controller

The robustness of the proposed controller to sprung mass variation is presented in Figure 4.14. From the figure, it is clear that additional sprung mass from passenger's weight will improve vehicle dynamics performance, particularly in the frequency transmissibility above body natural frequency. It can be seen that the RMS values of body acceleration, body displacement and suspension working space tend to decrease with additional sprung mass from passenger's weight in the frequency transmissibility above body natural frequency. Conversely, the RMS values of body acceleration, body displacement and suspension working space slightly increase with additional sprung mass in the frequency transmissibility below body natural frequency. Sprung mass variation does not influence the wheel acceleration performance at all frequency range.

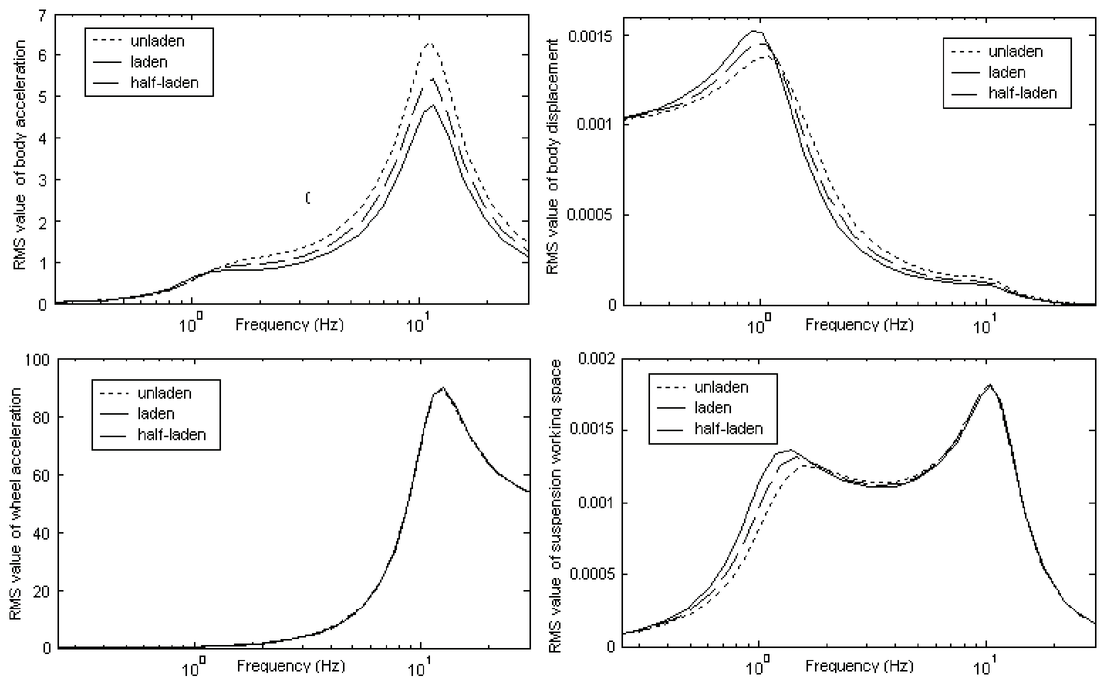

Figure 4.14**:** Robustness evaluation of proposed controller to sprung mass variation.

4.2 Virtual Reality Animation

Previously, the modeling and simulation of the car suspension system with different type of controller have showed satisfactory result. However, the outputs are in term of 2D graphs only and the user can only view the result after the simulation is complete. By including a virtual world of a car going on a maneuver, the user now can view the simulation in three-dimensional animation during the simulation run time. This will make it easier to understand the simulation output data.

The Virtual Reality Toolbox viewer is automatically loaded when the related Simulink file is opened.

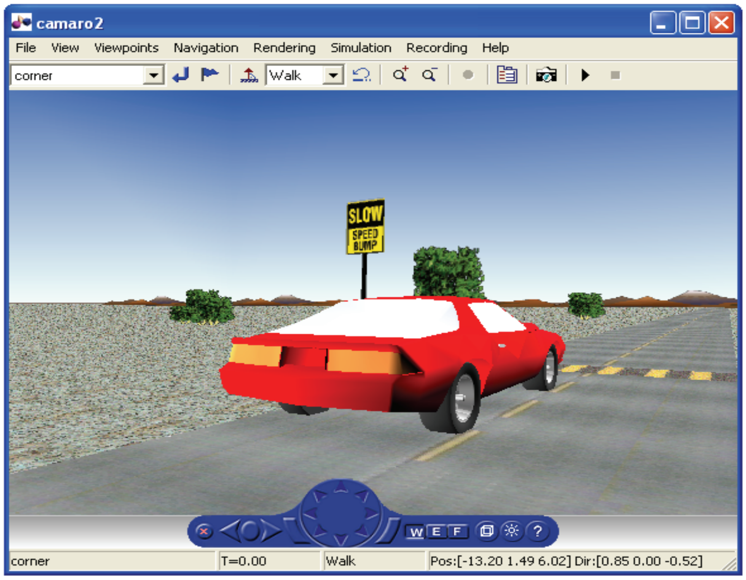

Figure 4.15: The virtual world during simulation

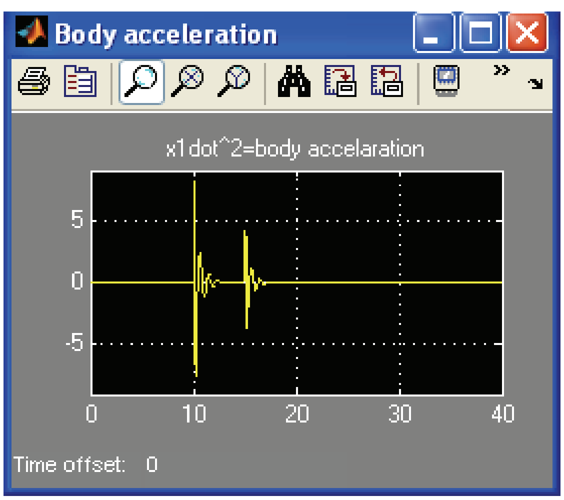

Figure 4.16: The output plotted from graph

User can view the virtual reality animation simultaneously with the graph output during the simulation run time. The animation shows that the active suspension system model with PISMC controller has better performance compared to the passive suspension system. The car experienced lesser translational movement when hitting the road bump. This animation result is also tally with the plot shows on the graph.

Besides that, the virtual world animation can be recorded as movie file during the simulation run time using the Virtual Reality Toolbox viewer. This file later can be played on a platform without MATLAB installed. This will assist in studying and sharing of information related to the suspension system.

CHAPTER 5

CONCLUSION AND SUGGESTION

5.1 Conclusion

The study presents a robust strategy in designing a controller for an active suspension system based on variable structure control theory, which is capable of satisfying all the design requirements within the actuators limitation. The mathematical model of a non-linear quarter car model is presented in a state space form. A detailed study of the proportional integral sliding mode control algorithm is presented and used to reject the effects of road induced disturbances on the non-linear quarter car model. The performance characteristics and the robustness of the active suspension system are evaluated and then compared with the LQR controller and the passive suspension system.

The result shows that the use of the proposed proportional integral sliding mode control technique is effective in controlling a vehicle and more robust compared to the counterparts. From the simulation results, it can be seen that the proposed controller shows significant improvement in reducing both magnitude and settling time of the body acceleration, body displacement, suspension displacement, wheel displacement and the angular motion of control arm. The proposed controller is capable of satisfying all the requirements for active suspension design.

Force tracking performance of the non-linear hydraulic actuator model was also investigated. Proportional Integral control was implemented for force tracking control of the hydraulic actuator. The results of the study show that the hydraulic actuator is able to provide the actual force close to the target force with acceptable force tracking error for sinusoidal, square, saw-tooth and random functions of target forces. It can also be noted that the force tracking control is also able to closely follow the target force produced by PISMC and LQR controller.

The development of the car in the virtual environment on the other hand can provide an insight into the actual condition when a car hits the road bump, thus provides more intuitive way to verify the system response. This will allow the user to better imagine and relate to the real plant that being modeled.

5.2 Suggestion for Future Research

For future work, it is suggested that the PISMC controller design technique can be improve. In this study, the constants in the matrix C are determined using trial and error approach. Thus, it is suggested that specific and more reliable method can be used such as using the optimization technique. In addition, the proposed control strategy could also be validated with the experimental study.

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APPENDIX

The following papers have been submitted and published:

i.) Sam Y.M., Huda K. and Osman J.H.S., (2006). " Proportional-Integral Sliding Mode Control of A Hydraulically Actuated Active Suspension System: Force Tracking and Disturbance Rejection Control on Nonlinear Quarter Car Model". *International Journal of Vehicle System Modeling and Testing*. (Conditional accepted).

ii.) Sam Y.M., Huda K. and Osman J.H.S., (2006). " Proportional-Integral Sliding Mode Control of An Active Suspension System with Force Tracking of Hydraulic Actuator". *International Journal of Control and Cyberntics.* (under review).

iii.) Sam, Y.M, and Huda K. (2006). "PI/PISMC Control of Hydraulically Actuated Active Suspension System", Proceedings of 1st Regional Conference on Vehicle Engineering and Technology, Kuala Lumpur, July 03-05, 2006.

iv.) Sam, Y.M, and Huda K. (2006). " Modeling and Force Tracking Control of Hydraulic Actuator for An Active Suspension", Proceedings of 1st IEEE Conference on Industrial Electronics and Applications, Singapore, Mei 24 -26, 2006, pp 316-321.