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Steady-state Solution for Magnetohydrodynamic Rotating Flow of Generalized Burgers' Fluid in a Porous Medium

Faisal Salah, Zainal Abdul Aziz and Dennis Ling Chuan Ching
Department of Mathematics, Faculty of Science, University Technology Malaysia,
81310 UTM Skudai, Malaysia

Abstract: The aim of this study is to determine the exact steady state solution of magnetohydrodynamic (MHD) and rotating flow of generalized Burgers fluid induced by a constant accelerated plate. This is accomplished by using the Fourier sine transform. This result is then presented in equivalent forms in terms of exponential, sine and cosine functions. Similar solutions for Burgers', Oldroyd-B, Maxwell, Second grade and Navier- Stokes fluids can be shown to appear as the limiting cases of the present exact solution. The graphical results illustrate the velocity profiles which have been determined for the flow due to the constant accelerated of an infinite flat plate.

Key words: Exact solution, generalized Burgers' fluid, steady-state solution, fourier sine transform, rotating frame, porous space

INTRODUCTION

The studies of non-Newtonian fluids have received considerable attention because of numerous applications in industry, geophysics and engineering. Some studies are notably important in industries related to paper, food stuff, personal care product, textile coating and suspension solutions. A large class of real fluids do not exhibit the linear relationship between stress and rate of strain. Due to the non-linear dependence, the analysis of the behaviour of fluid motion of non Newtonian fluids tends to be much more complicated and subtle in comparison with that of Newtonian fluids. When the motion of a fluid is set up, the velocity field contains transients obtained by the initial conditions. These transients gradually disappear in time and the starting solution tends to the steady-state solution, which is independent of the initial conditions. Several researchers have discussed the flows of generalized Burgers' fluid in different configurations (Fetecau *et al*., 2009; Vieru *et al*., 2008; Shah, 2010; Hayat *et al*., 2006; Shah and Qi, 2010; Khan *et al*., 2010; Xue *et al*., 2008; Khan and Hayat, 2008). There are available few attempts in which the flows of non-Newtonian fluids have been investigated in different separate cases. Such attempts are made by Fetecau *et al*. (2006), Khan *et al*. (2008, 2009), Hayat (2006) and Hayat *et al*. (2008a,b).

The aim of the current study is to establish exact steady state solutions for the velocity field corresponding to flow induced by a constantly accelerating plate in generalized Burger fluid. The fluid is magnetohydrodynamic (MHD) in the presence of an applied magnetic field and occupying a half porous space, which is bounded by a rigid and non-conducting plate. Constitutive equations of a generalized Burgers fluid are used. Modified Darcy's law has been utilized. The steady-state solution to the resulting problem is attained by Fourier sine Transform, which contains as limiting cases the similar solutions for Burgers' fluid, Oldroyd-B, Maxwell, Second grade and Navier-Stokes fluids. The graphs are plotted in order to illustrate the variations of embedded flow parameters.

FORMULATION OF THE PROBLEM

We choose a Cartesian coordinate system by considering an infinite plate at z = 0. An incompressible fluid which occupies the porous space is conducting electrically by exerting an applied magnetic field $B_\circ$ parallel to the z-axis. The electric field is not taken into consideration, the magnetic Reynolds number is small and the induced magnetic field is not accounted. Both plate and fluid possess solid body rotation with a uniform angular Ω about the z-axis.

The governing flow equation is given by Hayat *et al*. (2008a,b).

$$\rho\left(1+\lambda_1\frac{\partial}{\partial t}+\lambda_2\frac{\partial^2}{\partial t^2}\right)\left(\frac{\partial F}{\partial t}+2i\Omega F\right)=\mu\left(1+\lambda_3\frac{\partial}{\partial t}+\lambda_4\frac{\partial^2}{\partial t^2}\right)\frac{\partial^2 F}{\partial z^2}$$
$$-\sigma B_o^2\left(1+\lambda_1\frac{\partial}{\partial t}+\lambda_2\frac{\partial^2}{\partial t^2}\right)F-\frac{\mu\phi}{k}\left(1+\lambda_3\frac{\partial}{\partial t}+\lambda_4\frac{\partial^2}{\partial t^2}\right)F \quad (1)$$

Corresponding Author: Zainal, A.A., Department of Mathematics, Faculty of Science, University Technology Malaysia, 81310 UTM Skudai, Malaysia

In which $F = u + i\upsilon$, u and υ are the velocity components in x and y directions, respectively, ρ is the fluid density, μ is the dynamic viscosity, σ is the finite electrical conductivity of the fluid and ϕ, k are the porosity and permeability of the porous medium, respectively, λ_1 and λ_3 are correspondingly the relaxation and retardation times and λ_2, λ_4 are the material constants having the dimensions as the square of time.

The initial and boundary conditions for a constant accelerated plate are:

$$u = \upsilon = 0 \quad \text{at} \quad t = 0, z > 0, \tag{2}$$

$$u(0,t) = At, \ \upsilon(0,t) = 0 \quad \text{for} \quad t > 0, \tag{3}$$

$$u, \frac{\partial u}{\partial z}, \upsilon, \frac{\partial \upsilon}{\partial z} \to 0 \quad \text{as} \quad z \to \infty, t > 0 \tag{4}$$

where A has dimension of $\frac{L}{T^2}$.

SOLUTION OF THE PROBLEM

Introducing the following dimensionless quantities:

$$\xi = z\left(\frac{A}{\nu^2}\right)^{\frac{1}{3}}, \ \tau = t\left(\frac{A^2}{\nu}\right)^{\frac{1}{3}}, \ G = \frac{F}{(\nu A)^{\frac{1}{3}}}, \ \beta = \lambda_1\left(\frac{A^2}{\nu}\right)^{\frac{1}{3}}$$
$$\alpha = \lambda_2\left(\frac{A^2}{\nu}\right)^{\frac{2}{3}}, \ R = \lambda_4\left(\frac{A^2}{\nu}\right)^{\frac{2}{3}}, \ Q = \lambda_3\left(\frac{A^2}{\nu}\right)^{\frac{1}{3}}, \tag{5}$$
$$M = \frac{\sigma B_o^2}{\rho}\left(\frac{\nu}{A^2}\right)^{\frac{1}{3}}, \ \omega = \left(\Omega\frac{\nu^{\frac{1}{3}}}{A^{\frac{2}{3}}}\right), \ \frac{1}{B} = \frac{\phi}{k}\left(\frac{\nu^2}{A}\right)^{\frac{2}{3}}, \ c = 2i\omega + M$$

where v is kinematic viscocity.

The problem statement (1) reduces to:

$$\left[\beta + \alpha\left(c + \frac{\partial}{\partial\tau}\right)\right]\frac{\partial^2 G}{\partial\tau^2} + \left[1 + \beta c + \frac{Q}{B}\right]\frac{\partial G}{\partial\tau} + \left[c + \frac{1}{B}\right]G = \left[1 + Q\frac{\partial}{\partial\tau} + R\frac{\partial^2}{\partial\tau^2}\right]\frac{\partial^2 G}{\partial\xi^2} \tag{6}$$

$$G(0,\tau) = \tau, \quad \tau > 0 \tag{7}$$

$$G(\xi,\tau), \frac{\partial G(\xi,\tau)}{\partial\xi} \to 0 \ \text{as} \ \xi \to \infty; \tau > 0 \tag{8}$$

Upon using Fourier sine transform, Eq. 6-8 yield:

$$\alpha\partial_\tau^3 G_s(\eta,\tau) + \left[\beta + \alpha c + R\eta^2\right]\partial_\tau^2 G_s(\eta,\tau) + \left[1 + \beta c + Q\left(\frac{1}{B} + \eta^2\right)\right]\partial_\tau G_s(\eta,\tau) + \left[c + \frac{1}{B} + \eta^2\right]G_s(\eta,\tau) = \sqrt{\frac{2}{\pi}}\eta[Q + \tau], \ \eta,\tau > 0 \tag{9}$$

Solving the ordinary differential Eq. 9 and inverting the result by means of the Fourier sine transform, we can write the velocity field G (ξ, τ) as a sum of the steady- state and transient solutions, i.e.

$$G(\xi,\tau) = G_s(\xi,\tau) + G_\tau(\xi,\tau)$$

The steady-state solution, which is valid for large values of time, has the form:

$$G(\xi,\tau) = e^{-\xi Y}\left[\theta\sin(\zeta E) + M\cos(\zeta E)\right] \tag{10}$$

where

$$\theta = \frac{\left[(Q+\tau)\left[(1-\alpha)c + b^2 R\right] + \tau\left(\frac{1}{B} - \beta - b^2\right) - \left[1 + \beta(c+Q) - \alpha\right]\right]}{U\left[Q^2 + (1-R)^2\right]} \tag{11}$$

$$M = \frac{\tau - (Q+\tau)R}{Q^2 + (1-R)^2} \tag{12}$$

$$U^2 = \left[\frac{Q\left((1-\alpha)c + \frac{1}{B} - \beta\right) - (1-R)\left(1 + \beta c + \frac{Q}{B} - \alpha\right)}{Q^2 + (1-R)^2}\right]^2 \tag{13}$$

$$b^2 = \left[\frac{Q\left(1 + \beta c + \frac{Q}{B} - \alpha\right) + (1-R)\left((1-\alpha)c + \frac{1}{B} - \beta\right)}{Q^2 + (1-R)^2}\right] \tag{14}$$

$$2Y^2 = \sqrt{b^4 + U^2} + b^2$$
$$2E^2 = \sqrt{b^4 + U^2} - b^2 \tag{15}$$

The above expressions for a MHD Burgers' fluid (λ_4) in a porous space take the form:

$$G(\xi,\tau) = e^{-\xi Y_1}\left[\theta_1\sin(\zeta E_1) + M_1\cos(\zeta E_1)\right] \tag{16}$$

where

$$\theta_1 = \frac{\left[Q[(1-\alpha)c - \beta] + \tau\left((1-\alpha)c + \frac{1}{B} - \beta - b^2\right) - (1 + \beta c - \alpha)\right]}{U_1\left(Q^2 + 1\right)} \tag{17}$$

$$M_1 = \frac{\tau}{\left(Q^2 + 1\right)} \tag{18}$$

$$U_1^2 = \left[\frac{Q\left((1-\alpha)c + \frac{1}{B} - \beta\right) - \left(1 + \beta c + \frac{Q}{B} - \alpha\right)}{Q^2 + 1}\right]^2 \tag{19}$$

$$b_1^2 = \left[\frac{Q\left(1+\beta c+\frac{Q}{B}-\alpha\right)+\left((1-\alpha)c+\frac{1}{B}-\beta\right)}{Q^2+1}\right] \tag{20}$$

$$\begin{aligned}2Y_1^2 &= \sqrt{b_1^4+U_1^2}+b_1^2\\ 2E_1^2 &= \sqrt{b_1^4+U_1^2}-b_1^2\end{aligned} \tag{21}$$

The result (Eq. 10) for a MHD Oldroyd-B fluid ($\lambda_2 = \lambda_4 = 0$) in a porous space takes the form:

$$G(\xi,\tau) = \left[\tau+\frac{Q\xi}{2}\left(\sqrt{c+\frac{1}{B}}\right)-\frac{\left(1+\beta c+\frac{Q}{B}\right)\xi}{2\left(\sqrt{c+\frac{1}{B}}\right)}\right]e^{-\xi\sqrt{c+\frac{1}{B}}} \tag{22}$$

The Eq. 22 for a MHD Maxwell fluid ($\lambda_2 = \lambda_3 = \lambda_4 = 0$) in a porous space is now of the form:

$$G(\xi,\tau) = \left[\tau-\frac{(1+\beta c)\xi}{2\left(\sqrt{c+\frac{1}{B}}\right)}\right]e^{-\xi\sqrt{c+\frac{1}{B}}} \tag{23}$$

The result (Eq. 10) for a MHD second grade fluid ($\lambda_1 = \lambda_2 = \lambda_4 = 0$) in a porous space takes the form:

$$G(\xi,\tau) = e^{-\xi f}\left[A_1\sin(\zeta L)+a\tau\cos(\xi L)\right] \tag{24}$$

with

$$a = \frac{1}{Q^2+1},\quad A_1 = \frac{\left[s(\tau+Q)-x-\tau\left(\frac{Qx+s}{Q^2+1}\right)\right]}{x-Qs}, \tag{25}$$

$$x = 1+\frac{Q}{B},\ s = c+\frac{1}{B} \tag{26}$$

$$\begin{aligned}2f^2 &= \sqrt{\left(\frac{Qx+s}{Q^2+1}\right)^2+\left(\frac{x-Qs}{Q^2+1}\right)^2}+\left(\frac{Qx+s}{Q^2+1}\right)\\ 2L^2 &= \sqrt{\left(\frac{Qx+s}{Q^2+1}\right)^2+\left(\frac{x-Qs}{Q^2+1}\right)^2}-\left(\frac{Qx+s}{Q^2+1}\right)\end{aligned} \tag{27}$$

The above expressions Eq. 24-25 for MHD viscous fluid ($\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = 0$) in a porous space now become:

$$G(\xi,\tau) = e^{-\xi f_1}\left[\tau\cos(\xi L_1)-\sin(\xi L_1)\right] \tag{28}$$

where

$$2f_1^2 = \sqrt{s^2+1}+s \tag{29}$$

$$2L_1^2 = \sqrt{s^2+1}-s \tag{30}$$

RESULTS AND DISCUSSION

Here, we present the graphical illustrations of the velocity profiles which have been determined for the flow due to the constant accelerated of an infinite flat plate. The emerging parameters here are the rotating parameter w, magnetic field parameter M and parameter of the porous medium B, the material constants parameters are E and R. In order to illustrate the role of these parameters on the real and imaginary parts of the velocity G, the Fig. 1- 6 ha ve been displayed. In these Fig. 1-6 panels (a) depict the variations of [-Re[G(ξ, τ)]] for generalized Burgers' fluid and panels (b) indicate the variations of [Im [G(ξ, τ)]].

Figure 1a shows that the real part of the velocity profile decreases for various values of rotation w, with respect to the increase in ξ. As w increases, the velocity profile decreases. Figure 1b indicates that the magnitude of imaginary part of the velocity profile increases initially and later decreases for various values of rotation w, with respect to the increase in ξ. As w increases, the velocity profile also increases. Similar result is obtained (Hayat *et al*., 2008a,b).

Figure 2a is prepared to see the effects of magnetic on the real part of velocity profile. Keeping R, E, B, Q, P, w, τ fixed and varying M, it is noted that the real part of velocity profile decreases by increasing the magnetic parameter M. Figure 2b also is prepared to see the effects of magnetic on the imaginary part of the velocity profile. Keeping R, E, B, Q, P, w, τ fixed and varying M, it is noted that the imaginary part decrease initially and later increases. Similar result is obtained (Hayat *et al*., 2008a).

Figure 3a indicates that the variation of porosity parameter Keeping R, E, M, Q, P, w, τ fixed. It is found that by increase in the porosity parameter is lead to increase the real part of the velocity profile.

Figure 3b Keeping R, E, M, Q, P, w, τ fixed and varying M, it is noted that the imaginary part increases initially and later decrease.

Figure 4a show the effects of material parameter E of G. Burgers' fluid on the real part of velocity profile when R, B, M, Q, P, w, τ are fixed. It interesting to note that by increase in the material constant parameter E is lead to increase the real part of velocity profile.

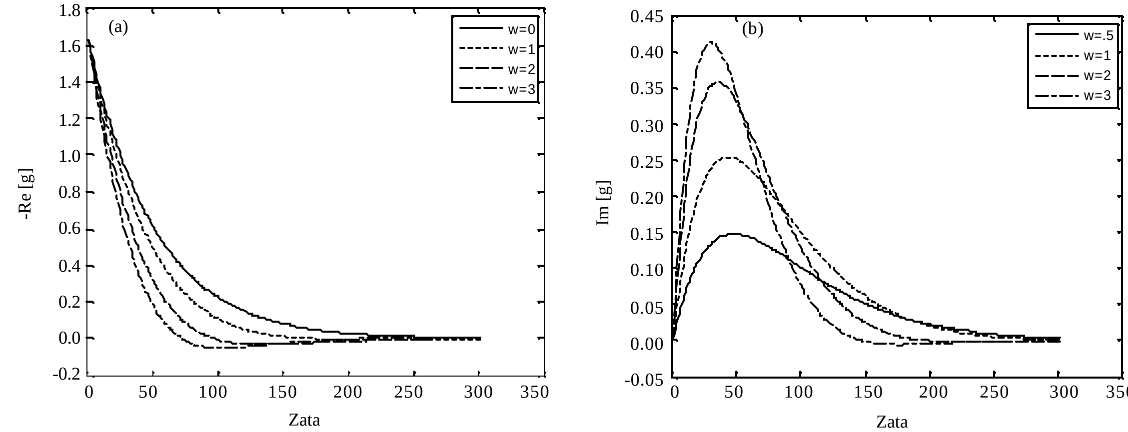

Fig. 1: (a, b) The variation of velocity profile G (ξ, τ) for various values of rotation w when (R = 1.3, E = 1.5, B = 1, Q = 1, P = 2, M = 2, τ = pi/2)

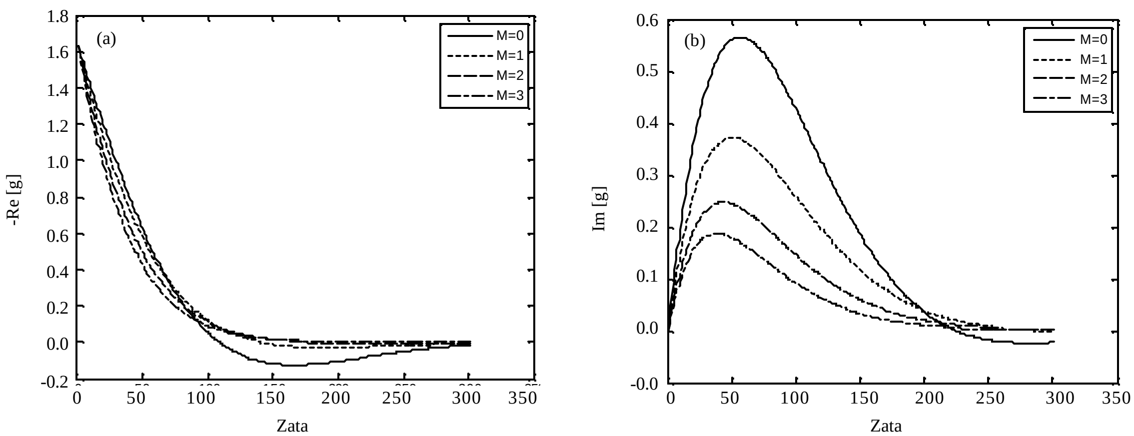

Fig. 2: (a, b) The variation of velocity profile G (ξ, τ) for various values of (MHD) M when (R = 1.3, E = 1.5, B = 1, Q = 1, P = 2, w = 1, τ = pi/2)

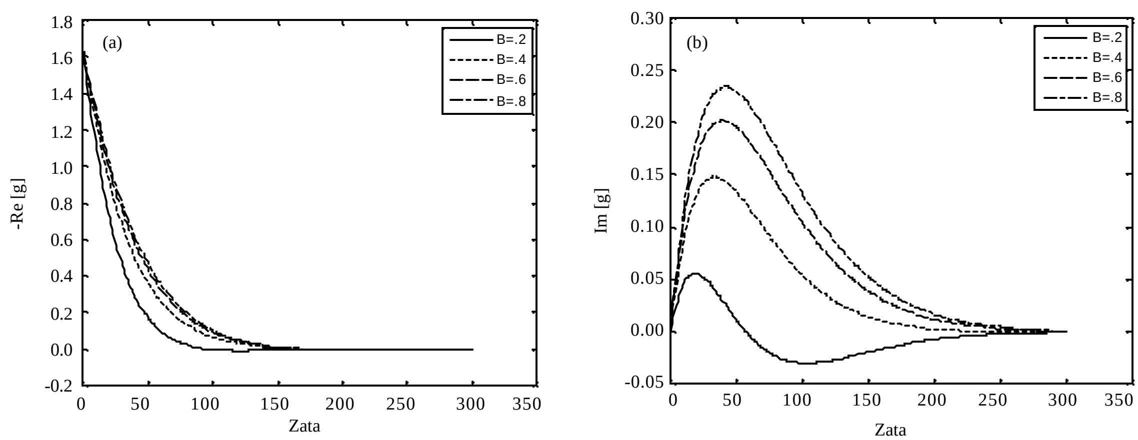

Fig. 3: (a, b) The variation of velocity profile G (ξ, τ) for various values of porosity parameter B when (R = 1.3, E = 1.5, M = 2, Q = 1, P = 2, w = 1, τ = pi/2)

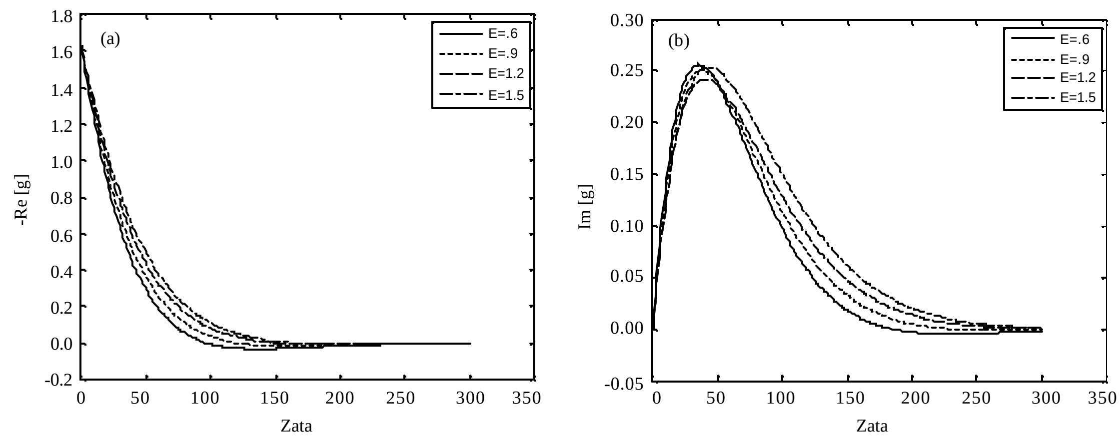

Fig. 4: (a, b) The variation of velocity profile G (ξ, τ) for various values parameter E when (R = 1.3, B = 1, M = 2, Q = 1, P = 2, w = 1, τ = pi/2)

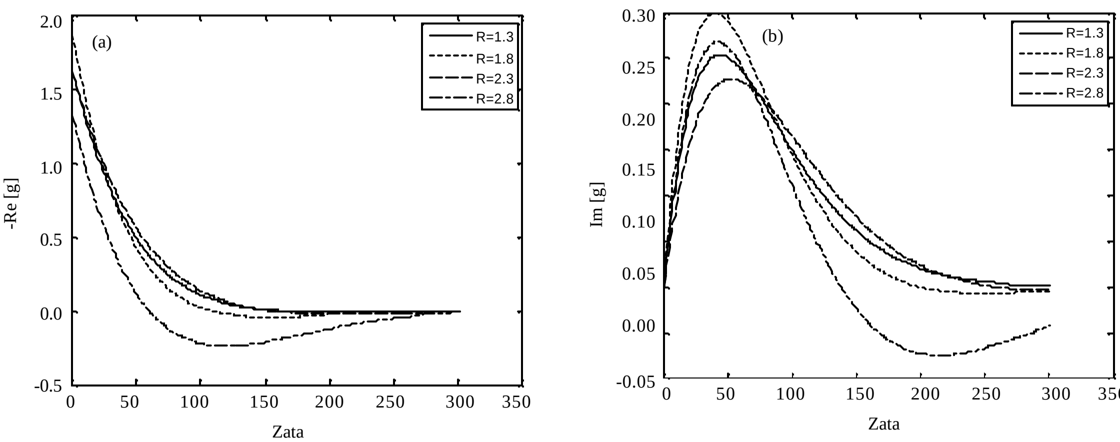

Fig. 5: (a, b) The variation of velocity profile G (ξ, τ) for various values parameter R when (E = 1.5, B = 1, M = 2, Q = 1, P = 2, w = 1, = pi/2)

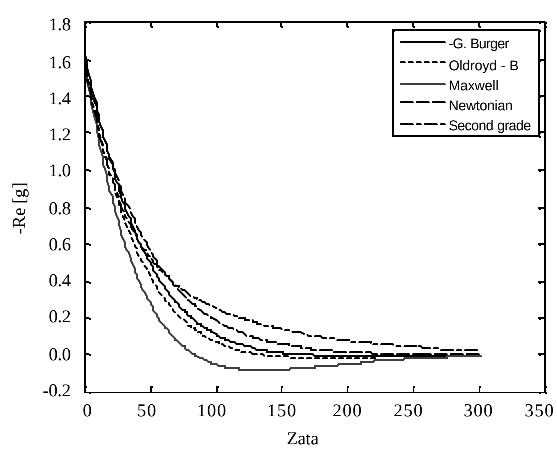

Fig. 6: The variations of velocity profile G(ξ, τ) for various fluids when (B = 1, M = 2, w = 1, τ = pi/2)

Figure 4b it is shown that when are fixed and by increasing the material constant parameter R, B, M, Q, P, w, τ is lead to imaginary part increases initially and later decrease.

Figure 5a show the effects of material parameter R of G. Burgers' fluid on the real part of velocity profile keeping R, E, M, Q, P, w, τ fixed. It is found that by increase in the parameter Ris lead to decrease the real part of the velocity profile.

Figure 5b is prepared to see the influence of material parameter R of G. Burgers' fluid on the imaginary part of velocity profile keeping R, E, M, Q, P, w, τ fixed. It found that by increase in the material constant parameter R is lead decrease the imaginary part of the velocity.

Figure 6 is prepared to show the variation of velocity profile for various fluids in comparison of G. Burgers' fluid. It is observe that real part of Oldroyd-B is quite same of G. Burgers' fluid.

CONCLUSIONS

The steady-state solution corresponding to the motion of generalized Burgers' fluid due to the constant acceleration of an infinite flat plate is established by means of the Fourier sine transforms. The solution for generalized Burgers' fluid and similar solutions (i.e., the limiting cases) for Burgers', Oldroyd - B, Maxwell, Second grade and Navier-Stokes fluids. Fetecau (2006) presented here in a simple form in terms of the elementary exponential and trigonometric functions. These satisfy all the above governing equations and all the above imposed boundary conditions.

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