VOT 71592

THE CONJUGATE GRADIENT METHOD FOR THE ADJUSTMENT OF CADASTRAL DATA IN GIS

(GUNAPAKAI KAEDAH CONJUGATE GRADIENT UNTUK PELARASAN DATA KADASTER DALAM GIS)

TAHER BIN BUYONG
ABDULLAH BIN DAUD

PUSAT PENGURUSAN PENYELIDIKAN
UNIVERSITI TEKNOLOGI MALAYSIA

VOT 71592

THE CONJUGATE GRADIENT METHOD FOR THE ADJUSTMENT OF CADASTRAL DATA IN GIS

(GUNAPAKAI KAEDAH CONJUGATE GRADIENT UNTUK PELARASAN DATA KADASTER DALAM GIS)

TAHER BIN BUYONG
ABDULLAH BIN DAUD

RESEARCH VOTE NO:
71592

Jabatan Geoinformatik
Fakulti Kejuruteraan dan Sains Geoinformasi
Universiti Teknologi Malaysia

2000

ACKNOWLEDGEMENTS

The authors ,wish to express their sincere appreciation to Prof. Madya Dr. Abdul Majid Abd. Kadir from Universiti Teknologi Malaysia and Associate Prof. Dr. Raymond Hintz from University of Maine, USA for their assistance and advice.

ABSTRACT

The use of measurements as the source of data for creating digital cadastral database supplemented by the adjustment of the measurements by the least squares method, undoubtedly results in accurate and up-to-date metric information. However the approach involves solving an extremely large system of linear equations, and subsequently increases the computing time and also requires large amount of computer processing storage. The problems are solved by applying an iterative conjugate gradient method (CGM). The proposed method reduces the storage by at least 70% compared to the prevailing usage of the banded storage scheme and significantly speed up processing time as compared to the usual Cholesky Decomposition method.

ABSTRAK

Penggunaan data ukur sebagai sumber asal untuk membina pengkalan data kadaster diukuti dengan pelarasan data ukur tersebut dengan kaedah ganda dua terdikit mampu mengeluarkan informasi paling tepat dan paling terkini. Namun demikian pendekatan ini melibatkan penyelesaian satu sistem persamaan linear yang amat besar, justeru itu melambatkan proses hitungan serta memerlukan terlalu banyak ruang komputer sewaktu menyelesaikan persamaan tersebut. Masalah ini diatasi dengan pengunaan kaedah lelaran kecerunan konjugat (CGM). Kaedah ini berjaya mengurangkan storan komputer sehingga 70% daripada kaedah skema berjalur dan juga berupaya mempercepatkan masa hitungan berbanding dengan kaedah yang kerap digunakan iaitu kaedah penghuraian Cholesky.

CONTENTS

V. DEVELOPING CGM COMPUTER PROGRAM

VI TESTS AND EVALUATION OF THE CGM PROGRAMS

VII CONCLUSIONS

LIST OF TABLES

LIST OF FIGURES

LIST OF SYMBOLS

A	-	Coefficient matrix of observation equations
a	-	Vector containing nonzero elements of matrix A
b	-	Right hand side vector of observation equations
d	-	Right hand side vector of normal equations
C	-	Partial differential matrix $\frac{dF}{dl}$
l	-	Vector of measurements
ICOL	-	Vector of column number for nonzero elements
IROW	-	Vector of row number for nonzero elements
IEND	-	Vector of diagonal adjacency numbers
N	-	Coefficient matrix of normal equations
n	-	Vector of nonzero elements of matrix **N**
N_T	-	Total nonzero elements in matrix **N**
T_A	-	Total nonzero elements in matrix **A**
v	-	Vector of residuals
W	-	Weight matrix of measurements
x	-	Vector of unknowns coordinates

CHAPTER I

INTRODUCTION

1.1 Problem statement and motivation

A geographic information system (GIS) can be described as an organized collection of computer hardwares, softwares, procedures, and personnel designed to efficiently capture, store, update, manipulate, analyze and display all forms of geographically referenced data/information. The system is designed to manage voluminous, varied and complex geographic data for solving problems that beset the earth and its inhabitant.

The power of GIS basically lies in its ability to integrate spatial and non-spatial components. This data integration opens the way for powerful and varied ways of looking and analyzing the data. The spatial component of a GIS data is often described as a series of layers, each of which contains map features that are related functionally (see Figure 1). Each layer is a set of homogeneous features that is

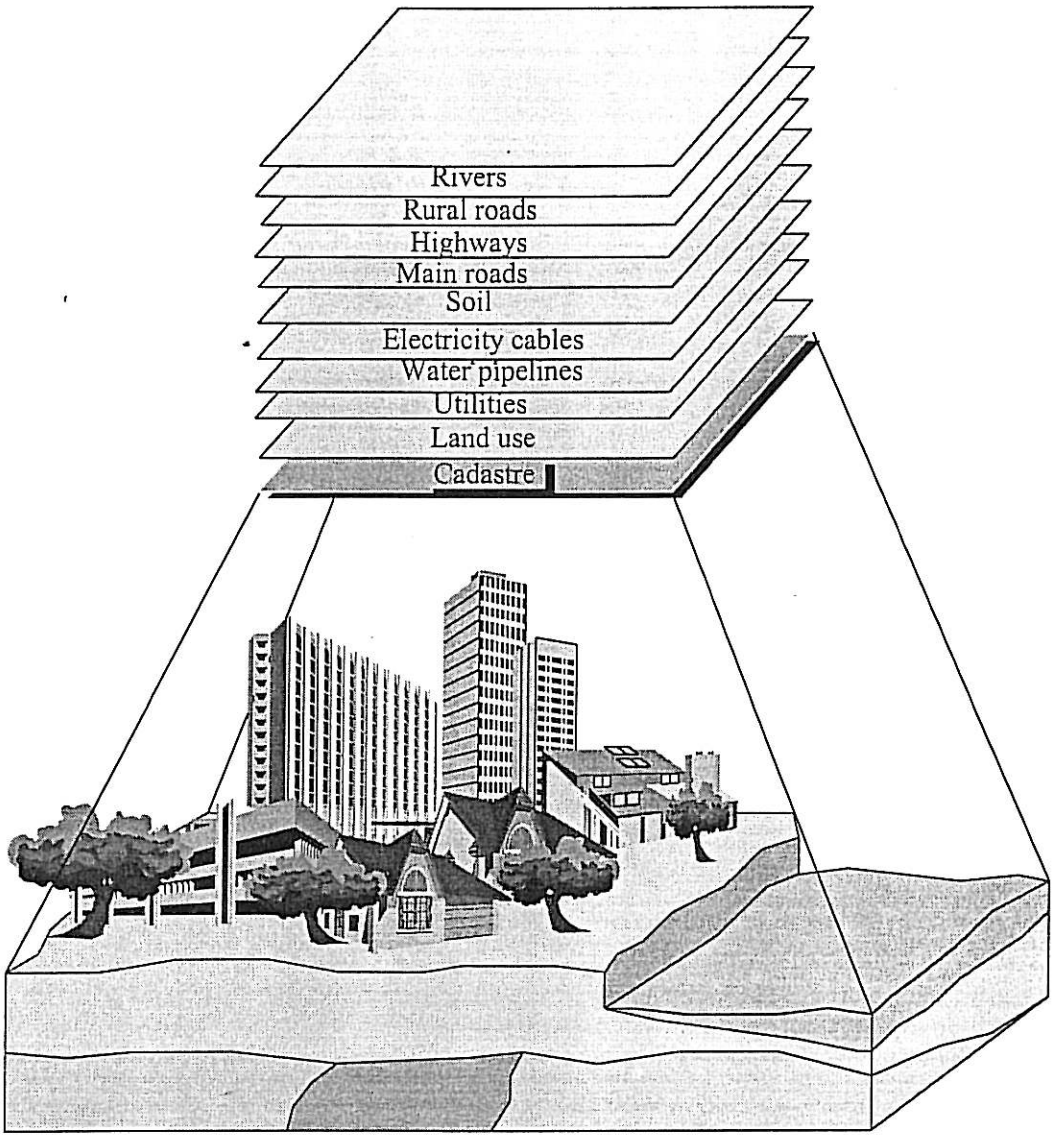


Figure 1: The layering concept in a GIS system

registered in term of position to the other data-base layers through a common coordinate system.

The base layer for most GIS applications throughout the word is the cadastral data layer (Dale & Mclaughlin, 1988; NRC, 1983). In the present super information infrastructure, which pools together various government agencies and private sectors, the cadastral data layer plays a wider role. For example, the study

carried out by Renong (1995) indicated that cadastral data is the most important data set for most users of the proposed National Land Information Infrastructure of Malaysia (NALIS).

1.1.1 Types of cadastral record

Cadastral information are the representation of land subdivided into units of ownership or parcels or sometimes known as lots. The cadastral record however can be described in three ways (Dale, 1976):

(1) verbal,

(2) numerical, and

(3) graphical.

Verbal description is a textual data that describes spatial information. It is significantly inferior to the numerical and graphical representation and produces little information of value for other applications. The numerical and graphical cadastral record systems are the basic resources for building up modern GIS.

The numerical record system is based on cadastral surveying and it involves measuring ground geometric values, i.e., the measurements. The measurements define the geometry of parcels. The most common measurements are distances and bearings or angles. They are measured accurately using instruments like theodolites

and steel tapes. The steel tapes are only effectives for short distances (say up to 50 meters). Nowadays, the use of modern surveying equipment incorporating electronic processors such as total stations are not only capable of delivering precise distance measurements, but also fast and reliable angular measurements. Typical accuracy for distance is several millimeters and the accuracy for angular measurement reaches seconds of arc. The process of acquiring such survey measurements can be much easier and faster when using satellite-based systems such as the Global Positioning System (GPS).

The third type of cadastral record is the graphical map or known as the cadastral map or cadastral plan. Maps are the end product of survey measurements. Maps are designed for displaying spatial information visually. During their production, measurements will undergo various processing stages, which includes computation reductions and adjustments, generalization, scale reduction, symbolization and other carthographic production processes. Each process degrades certain proportion of the contents and the quality of the measurements. Consequently, the derived graphical maps are far too inferior as compared to their original sources, i.e., the measurements.

1.1.2 Creation of digital cadastral database

An immediate action taken by most users in carrying out a GIS project is directed toward purchasing computer hardware and software. Lengthy discussions with vendors on the merits of various GIS components have been a standard practice. Yet, less attention has been given to the core of the system i.e. the data that goes into it. Although cadastral data can be found in three ways (i.e., verbal, numerical and graphical) the most popular source of data for creating spatial databases has been the graphical maps (Burrough, 1986 ; Aronof, 1989). A database created from graphical map is referred to as a map-based database.

The use of the graphical maps as a source data for building spatial databases has been adopted for two main reasons (Elfick, 1989). Firstly, GIS technology was developed in countries where cadastral is mostly based on maps. The technology has been mainly designed with the understanding that the spatial information comes from maps. Secondly, the users are subjected to the marketing forces of the computer industry that already investing heavily in developing CAD/CAM systems and the associated hardware.

Maps or plans are captured into computer systems either using keyboard entry, manual digitization or automatic scanning. The most reliable and popular method of data entry is by manual digitizing using digitizers. The digitized coordinate values, which may be of different scale, orientation and projection, are

later transformed to a common base with the help of control points coordinates. Latter, the captured data are stored in the system (Dangermond, 1989).

Although GIS is a sophisticated computer system and uses the latest technology in data capture, its graphical database which was derived from maps is essentially analogue in nature. Although the system permits users to bring together information from numerous spatial data sets into a composite form without the laborious manual processing, the derived information is still based on analogue maps. Therefore, such a system which uses maps as its source data is called map-based GIS system.

1.1.3 Weaknesses of the map-based database

Currently, many map-based databases have been successfully set up and applied in variety of GIS applications. However, they suffer several limitations, and some even fail to achieve the objectives for which they were designed, especially in parcel level implementations. The general concern of the problems is to maintain the integrity and accuracy of the spatial dataset (Shane, 1996 ; Taher 1992; Doytser & Eytan, 1995).

Metric information obtained from any map-based system is usually not up-to-date. Hard copy maps simply are unable to provide up-to-date information. Even

the latest map edition is in fact many years behind time. Inevitably, cadastral maps which usually serves as the base layer are subjected to upgrading and maintenance, as cadastral itself is a dynamic activity (Dale & McLaughlin, 1988 ; Hunter, 1991).

Maintaining and upgrading of the graphical information within the database are critical because the information which were initially obtained from maps is inherently of low quality. As new cadastral measurements become available, new measurements which are of higher quality must be used to update the database.

Integrating new measurements to the existing database is a challenge. Old measurements which is in the form of coordinates cannot be adjusted simultaneously with the new measurements. Coordinates are point value information which refer to the origin of a coordinate system. They are only functionally related to the origin and therefore have no ability to propagate information and cannot assimilate higher quality measurements available around them. To resolve these problems, a procedure that begins to gain acceptance, loosely known as *rubber-sheeting*, alters the existing coordinates to fit the new accurate measurements. Different authors came up with different methods to implement the concept of rubber-sheeting. For example, Crook (1990) described an implementation of the concept to update the New Zealand cadastral database.

The constant updating of the cadastral layers due to subdivision of parcels and other land developments leads to an apparent shift of a parcel layer with respect to other layers within a GIS (Figure 2). The shift introduces systematic errors.

Updating digitized coordinates of the parcel corners causes confusion among users. Such problems already occurred in utility data, and have been reported by several researchers (Hebblethwaite, 1989 ; Hadjiraftis and Jones, 1990).

Maps convey geometric information graphically. In practice, even the best drawn maps with infinitely sharp boundaries are far from perfect. Consider a 1mm-thick line on a 1:1000 map will represent a width of 1 meter on the ground. The error grows rapidly as map scale becomes smaller. Extra errors are introduced during digitizing process, as pointed out by Burrough (1986), Doytser & Eytan(1995). The errors that accumulate during the process include the digitizing errors, media stability (warping, stretching, folding and wrinkling of maps) and digitizer resolution errors. The magnitude of the digitizing error alone is in the order of meters (Masry and Lee, 1988; Henderson, 1989). It should be noted that besides these errors, another major source of error which makes the problems even worse is the inherent inaccuracy of the coordinate system itself. The resulting error may be up to tenths of meters depending on the quality of measurements and the adopted processing method (Vanicek & Krakiwsky, 1986 ; Schwarz, 1989).

The map-based approach which revolves around capturing graphical maps and storing them in databases for answering metric query is in fact an outdated methodology. The methodology was originally adopted due to the limitations of computing power and lack of efficient processing algorithms.

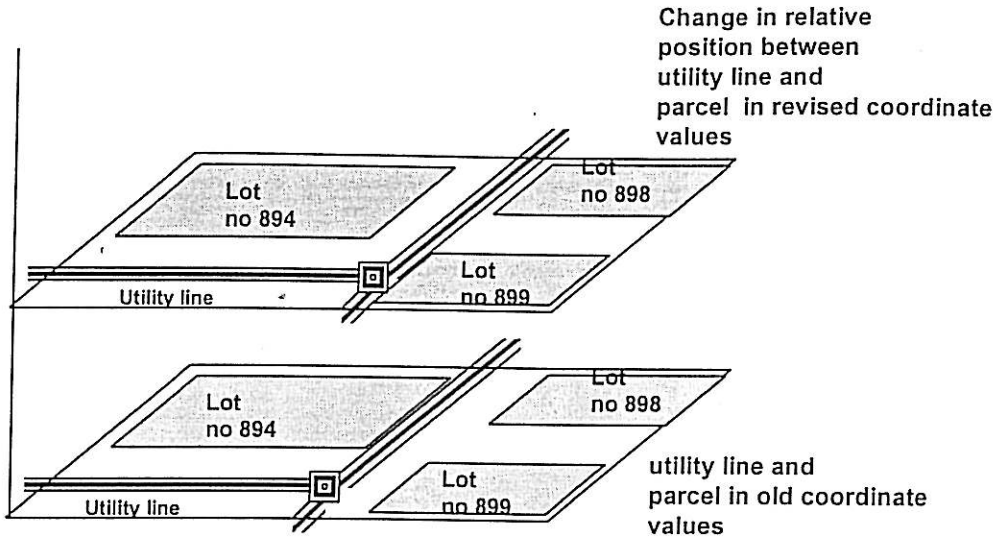


Figure 2: The effect of shifting a cadastral layer with respect to a utility layer.

1.1.4 A new methodology for creating cadastral database

The root of the database problems is attributed to the analogue nature of the source data within the map-based cadastral system. Thus, in order to overcome these problems, the following approaches are introduced.

1. Using measurements or derived coordinates for building graphical database
2. Processing the measurements using least squares method.

Many researchers during the last several years favours these approaches (Durgin, 1993; Weitzman, 1989). The new approach considers measurements instead of the hardcopy maps as source data for creating database. This idea has been proposed earlier by several researchers (Hintz et al, 1988; Clark, 1987; Kjerne, 1987 ; Taher, 1992). The rationale behind the concept is that measurement is the

most refined data that describes spatial information and it is the first data that records any changes to the spatial information. All other spatial information are derived products of measurements and therefore, measurements should be the basic entity of a graphical database. There are two main approaches that can be adopted to implement the database:

(a) the first approach stores measurements within the system. The system computes spatial information directly from the measurements. The user interacts with the measurements within the database and they are processed whenever an information is required. Since this approach produces information directly from the measurements, it is called a measurement-based cadastral database.

(b) The second approach stores the coordinates (instead of measurements) within the system. The system computes coordinates from the measurements but the measurements are kept outside the system. The measurements are brought to the system only for maintaining and updating the coordinates. Users interact with the coordinates and information is derived from the stored coordinates. The system is referred to as coordinated-cadastral database.

The measurement-based of the coordinated-based database possesses several advantages and benefits over the familiar map-based system. Updating the system is achieved by simply adding new measurements into the database. The system is

not dictated by the limitations and errors inherent in maps. Some of the main advantages of the approach are listed as follows:

1. Metric information derived directly from the measurements is obviously accurate (depending on the accuracy of the measurements) since field measurements are the most accurate spatial information. Accurate information has always been the ultimate aim of any digital database implementations. In fact, the validity and the integrity of any information from the database depends entirely on the validity of its source data. GIS is a powerful tool. However, if the data input is inaccurate, the end product of the system will be useless. Furthermore, metric information derived from measurements is up-to-date information since measurements are truly the most up-to-date data. Meaningful analysis depends entirely on the up-to-date information.

2. Other advantages of the approach are attributed to the inherent potentials in the measurements and the flexibility of handling measurements within a database. The database is almost free from errors being accumulated in the hard copy maps and it is also free from the introduction of new errors that are built up during the process of converting analogue maps to digital maps.

3. The introduction of this new approach has several other advantages unchallenged by the map-based system: the employment of measurements within the system allows ease of updating, incremental implementations, systems as by-product of standard activities, improvement of accuracy over time, correct integration of different layers, preservation of background

information, as well as several economic benefits. These advantages had been described in detail by Taher (1992).

This promising approach has attracted many researchers to further develop the concept. Many aspects of the methodology have been studied and proposed. Jacobi (1988) for instance, dealt with the updating of maps using photogrammetric methods. The work was extended to include ground measurements as the basis for land and geographic information system (Kjellberg, 1990). A system that retains the source elements within the system was proposed by Clark (1987). Kjerne(1987) presented object oriented paradigm to secure the measurements.

A system for effective management of large cadastral measurement has been studied by many researchers (Hintz et al, 1988; Hintz and Onsrud, 1990; Elfick, 1989). A comprehensive study of the measurements concept, including a clear and precise methods of introduction of the system has been studied by Taher (1992). The central idea of the above research revolves around the new role measurements within the GIS. Conceptually, the new system has the necessary potentials to overcome the problems faced in the old map-based system.

1.1.5 Challenges in implementing numerical database.

The main constraint for implementing the cadastral database based on measurements (either the measurement-based system or the coordinated system) is the problem of processing huge amount of measurements usually found in a cadastral system. A typical cadastral system may have several millions measurements and they are to be processed within a reasonably short span of time.

The universally accepted method for adjusting survey measurements is the least squares method. The method produces homogeneous coordinates needed for GIS implementations. The method generates a system of linear equations from the cadastral measurements which must be solved to produce the coordinates. Since a cadastral system involves a large volume of measurements, the number of derived equations is also very large. However, these cadastral equations possess certain properties which can be exploited to optimize the computation process. The coefficient matrix of the equations are sparse and they are positive definite and symmetric. Therefore, the main challenge lies in the approach that can optimize the cadastral computation and it can be divided into four interrelated parts:

1. selecting a suitable algorithm for solving the large system of equations derived from the measurements,
2. minimizing computer storage for large volume of measurements,
3. optimizing processing speed, and
4. designing computer program specifically for the unique nature of cadastral data processing.

This research employs an iterative conjugate gradient method (CGM) to solve the problems. The method is implemented in a computer program which is specially designed to take full advantage of the greatly sparse nature of the coefficient matrix created from the cadastal measurements.

1.2 Potentials of the CGM

The least squares method involves the creation and solution of systems of linear equations. There are many methods for finding solutions of a linear equation and there is no single method that can be considered best for all situations (Landesman & Hestenes,1991). The most widely used method for surveying and geodetic computations is the Cholesky Decomposition method (Cross, 1983). However, there are situations when it is useful to use the CGM (Hestenes, 1980, Landerman & Hestenes 1991). For example, the method has the advantage that in certain circumstances, as in the case of a sparse, symmetric and positive definite matrix, a good estimate of the solution is obtained early in the computation. The method is not only capable of solving the equations, but it can reduce the storage and the time of processing the equations generated by the least squares process. It is the most efficient method among the gradient methods (Press et al, 1987). Furthermore, the method is expected to have several practical advantages when dealing with cadastral networks namely:

(1) optimizing a sparse system of equations,

(2) minimizing the storage requirement,

(3) solving the parameters directly from the observation equation, and

(4) solving the standard normal equation.

The CGM is known to be very efficient in handling a sparse system of equations. This method can produce parameters from the equations without using the zero elements at all. It only deals with the small proportion of nonzero terms found in the coefficient matrix. The patterns of cadastral networks generate the most sparse system among surveying networks and this unique feature is exploited in this research using the CGM.

The amount of computer storage for solving such a system of equations depends on the storage scheme used for storing and manipulating the elements of the equations. A sparse system organized by an efficient storage scheme can minimize computer storage and therefore more measurements can be processed simultaneously in any given time.

The least squares process begins with the creation of a system of linear equations called observation equations. The standard parameter estimation process requires the equations to be transformed into another system of equations known as normal equations, before they are solved for the unknowns. However, the CGM is able to solve the unknowns directly from the observation equations. The solution of the observation equations is expected to save tremendous amount of arithmetic operations.

Besides solving the observation equation, the CGM is also capable of obtaining the unknowns in the usual way, i.e. solving the normal equations. The coefficient matrix of the normal equations is symmetric and positive definite. These features can be used to simplify the CGM arithmetic process and therefore has potential to speed up the estimation time.

1.3 Research aim and objectives

The main aim of this research is to optimize least squares computations in cadastral systems using CGM. The investigation includes the implementation of a computer program designed to take the advantage of the sparse nature of linear equations derived from cadastral measurements. To achieve the goal the following objectives have been set:

1. to evaluate the validity (correctness of the results) of the CGM in solving cadastral survey network,
2. to estimate the magnitude of rounding off error in the iterative CGM,
3. to develop storage schemes that minimizes of computer storage,
4. to further develop the CGM for minimizing the processing time,
5. to implement a non-hierarchical adjustment concept needed in cadastral systems.

Since the research concentrates on the applications of the CGM to solve cadastral processing problems, detailed theoretical and mathematical derivations of the method is beyond the scope of this research. The subject is thoroughly described in many mathematical text books on numerical methods (Hageman & Young, 1981; Landesman & Hestenes, 1991; Hestenes, 1980) and research journals (Shariff, 1995; Psimarni, 1994). A brief mathematical derivation of the method is only presented in this report where necessary.

This research concentrates on the computation of the basic network parameters, i.e. station coordinates, measurement residuals and the network unit variance. These parameters provide sufficient information for evaluating validity and performance of the CGM which is also the main aim of this research. Rigorous tests were carried to ensure the results of the method are valid. These issues are fundamental since the CGM is new to cadastral applications. As such, the computation of other derived parameters such as error ellipses and statistical information, are beyond the scope of this thesis. These additional information can be obtained by several approaches for example by a method called "kerplunking" developed by Hintz (1994). The approach uses only small subsets of the large data set in a sequential fashion in the effort to resolve the problem of handling massive computation of the variance-covariance parameters.

1.4 Research significance

The significance of this research is the realization of a precise and up-to-date cadastral database system. Many countries have moved towards implementing the measurement-based database concept, either in the form of coordinated-cadastral system or a fully measurement-based cadastral system. The success of both systems depends fundamentally on the efficiency of their measurement processing component. The main contribution of this research is the development of such processing component, which is badly needed for implementing the measurement-based cadastral database for GIS applications.

The Malaysian Surveying and Mapping department is considering, in principle, to adopt the coordinated cadastral system (JUPM, 1995). Under the system, a parcel corners will be defined rigidly by a pair of easting and northing coordinates. These values are processed by the least squares method using measurements obtained from the existing bearings and distances of the boundaries together with newly observed GPS coordinates as control points. The coordinated cadastral system does not only benefits cadastral activities, but also the database which will be incorporated in GIS system that permits diverse applications. Although there are several issues that need to be resolved, the fundamental and the one immediate issue that must be solved is the processing of huge measurements from the existing system.

A coordinated cadastral system has been introduced in New South Wales Australia (Carr, 1990). This system offers the potential to significantly improves the State's cadastral survey system. Furthermore, it forms part of the New South Wales Land Information System. A similar coordinate system has been implemented in South Australia (Smith, 1988).

The purely measurement-based concept is also gaining acceptance and some jurisdictions in United States are beginning to adopt the system (Kopak, 1993; Durgin, 1993). The approach reduces the size of the simultaneous equations within the system by adjusting a block at a time and then combining together the individual adjusted blocks.

CGM is a new tool in cadastral network adjustment. Nearly all computations dealing with least squares problems utilised Cholesky Decomposition method through the solution of the derived normal equations. The present research is the first known attempt at solving the observation equations generated by cadastral measurements using CGM.

The developed computer program based on the CGM is expected to serve as a foundation for creating digital database. The database has several advantages over the map-based implementations. The data within the system is up-to-date and accurate. Furthermore, the system also allows ease of updating and maintaining the database. The task becomes simple and a routine process of entering new measurements into the system.

1.5 Expected findings

The CGM is expected to be an efficient method for solving least squares problem in cadastral surveying due to the inherent potentials of the method described in Section 1.2. The optimization strategies to be developed in this research will resolve the processing problems related to large cadastral networks. The storage of the equations derived from the measurements will be minimized by only storing the nonzero elements. Consequently, the arithmetic operations will only referred to the stored nonzero elements.

This implementation of the CGM will serve as a basis for setting up cadastral database based on the measurements as the source data. The updating of a database is accomplished simply by adding new measurements into the system. The continuous addition of measurements that will be used to update the coordinates will result in an enhancement of the overall quality of the system.

In the present work, the CGM will be rigorously tested using several real and simulated networks. These tests should reveal the correctness of the method. CGM solves least squares equation in two ways. One way is to solve the observation equations and secondly is to solve normal equations. The results obtained from the two approaches are expected to be similar and of correct values. The CGM results will be compared to the Cholesky Decomposition method and also to the standard trigonometric formula. The doubt about the effects of rounding off error inherent in most iterative methods is expected to be invalid for CGM.

The method for setting the observation equations is expected to save substantial computer storage by storing only the nonzero elements found in the coefficient matrix in a vector. The locations in the vector is determined from the geometry of the survey network. This information allows the necessary storage schemes to be allocated in advance prior to the actual least squares computation.

1.6 Organization of report

Chapter II highlights the needs for a specialized least squares adjustment for the unique cadastral networks. This chapter presents the overview of current strategies for optimizing least squares computation and discusses the deficiencies of the approaches in relation to cadastral applications in GIS. This is followed by the introduction of the CGM to solve the cadastral adjustment in GIS. This chapter shows that CGM has the potentials needed by cadastral system for its current adjustment work.

Chapter III presents a method that can optimize the creation and storage of the basic observation equations generated by the measurements. The development of the method is described in details beginning with the computation of the contributions of a single measurement into the coefficient matrix of the equations. This is followed by the arrangement of the combined contributions of all the

measurement within a network. This chapter then describes the optimized structure to store the coefficients in a vector. Finally, this chapter presents the CGM algorithm to solve the equations for the parameters.

Chapter IV presents the method for optimizing the construction of normal equations. The development of this method begins with the computation of the contribution of a single measurement into the symmetric coefficient matrix of the normal equations. This is followed by the arrangement of the combined contributions of all the measurements within a network, using graph theory. The resulting coefficients within the upper triangle of the matrix are stored in a vector. Finally this chapter presents the CGM algorithm for solving the equations.

Chapter V focuses on the development of a computer program to implement the methods presented in chapters III and IV. This chapter begins with the description of the designed strategy and continues with the translation of the design into a source code. This chapter finally describes the compiled program.

Chapter VI describes three methods for testing the validity of the CGM in solving cadastral data. This chapter continues with the evaluation of the performance of CGM in terms of the utilization of computer storage and also the speed of execution. Finally, Chapter VII provides summary of the research and highlights major findings. This chapter also includes recommendations for future research directions.

CHAPTER II

REVIEW OF ADJUSTMENTS IN CADASTRAL SURVEYING

2.1 Introduction

Survey points are classified according to their purposes and the information they represent, namely: geodetic points, engineering points and cadastral points (Hamilton, 1982). The use of geodetic points is to provide control coordinates for mapping programme, whilst the use of engineering point is to provide control for monitoring physical development. The primary information contained in these two classes of points is position information which forms the basic data for various users. Research activities on these types of control points have always been directed towards improving their respective position information to the highest level of accuracy.

The uses of the cadastral points are not only confined to the provision of position information as the geodetic and engineering control points, but also they provide a variety of other information which include boundary information, bearings, distances, utility services, legal rights and other cadastral information. The

information have always been part of a cadastral system since the beginning of cadastre (Dale, 1976; Dale and Maclauglin, 1988).

The basic data collected from any survey network is the measurements. These measurements undergo several processing steps before they can be represented as a useful information. The measurements must be calibrated, validated, and adjusted. Calibrations and validations involve procedures for removing of systematic errors and blunders which contaminate the measurements. Finally, these measurements are combined in an adjustment process to produce the desired parameters.

The methods for adjusting survey measurements can be divided into two groups namely rigorous and non-rigorous. The most popular rigorous method is the least squares method. It is a powerful method and it allows various types of measurements with different qualities combined in a simultaneous adjustment computation. Although the method involves massive computations, it produces homogeneous and unique results (Krakwisky, 1985; Cross, 1983). Furthermore, the results can be used to compute statistical information. Non-rigorous methods on the other hand, involve simple adjustment procedures and the results are not unique and statistically incorrect. Two most popular methods of non rigorous adjustment are Bowditch and Transit rule.

Traditionally, the rigorous least squares method has been used mainly in geodetic and precise engineering survey computation and analysis. Many computer programs have been developed specifically for these networks. Simple non-rigorous

methods can be considered as sufficient for adjusting cadastral networks, since the computation are concerned with relatively small area as compared to the coverage of the geodetic networks. Consequently, adjustment methods such as Bowditch and Transit rule have been used as the standard methods in cadastral applications.

However, current developments in cadastral system and its role in GIS requires a new way in dealing with the processing of the cadastral measurements. The simple non-rigorous methods are no longer considered satisfactory for processing cadastral measurements. It is necessary to use the rigorous least squares method to obtain homogenous coordinates within a cadastral region. The roles and advantages of the least squares method in cadastral computation are many, and they are described in Section (2.3). Although the least squares method has been extensively applied in geodetic surveying and precise engineering surveying, their implementations are not directly applicable to current cadastral systems.

This chapter reviews the related works and the features of cadastral network adjustments. The chapter begins with a description of the current least squares implementations and points out various reasons why the approach is considered inefficient for a cadastral system. Next, the current implementations of the conjugate gradient method (CGM) are presented. This is followed by the description of the unique cadastral network and its adjustment requirements.

2.2 Review of related works.

Many aspects of the least squares computations have been studied, but they were mainly concerned either with geodetic networks or engineering surveying networks. The system of linear equations derived in least squares process are invariably solved by Cholesky decomposition method. Thus, most researches in the least squares method were directed mainly towards optimizing the process related to Cholesky decomposition method. CGM has never been applied in cadastral computations although the method has been successfully used in many engineering applications.

In this section an overview of the related works is presented. The discussion includes significant findings, status, and relationships of the work to this thesis. In addition, the deficiencies of the methods related to cadastral computation will be highlighted. The review is presented according to the following topics:

(1) coordinates upgrading,

(2) least squares implementation,

(3) least squares optimization strategies,

(4) hierarchical adjustment strategy, and

(5) CGM and its applications.

2.2.1 Coordinate upgrading

Network upgrading is an improvement process with respect to new technology which permits massive data acquisition. There has been an increasing interest in upgrading national control networks to keep pace with new developments in technology. Many countries throughout the world have carried out the upgrading programmes to improve the overall consistency of geodetic networks.

The National Geodetic Survey of United States for instance has completed the readjustment of the National Geodetic Reference System (NGRS) to the North American Datum of 1983 (Schwarz, 1989). The overall relative accuracy was more than 1:200,000. However some areas experienced relatively poor accuracy of less than 1:100,000. The NGRS must be consistent for the whole nation because the primary purpose of the network is to provide a geodetic reference frame for various uses covering large parts of the nation. This requires readjustments of the entire NGRS network in order to improve the overall coverage.

The Malaysian geodetic network has evolved through several stages and readjustments. The geodetic network at the national level was started in 1948 with the combination of three old triangulation schemes known as the Perak system, Asa system and Repsold triangulation. The network consisted of 77 geodetic, 240 primary, 837 secondary and 51 tertiary stations. To cater for a new map projection for the mapping program, the geodetic network was revised and was renamed as Malaysia Revised Triangulation (MRT).

The networks were later strengthened by remeasuring three baselines using the Satellite Doppler observations. The process of readjustment was repeated several times in various scientific studies. The computation reduced the distortion and improved the overall accuracy, with distortions on average of 4 meters on northern region and 2 meters in southern region.

As the GPS technology develops and is applied to surveying and mapping, there has been an increasing interest in upgrading the geodetic networks. Similar scientific computations were carried out using the latest GPS technology (Abd Majid et al, 1993). The final accuracy of baselines is in the order of 1 to 2 ppm for two-dimensional, while 3-4 ppm for three-dimensional information. Although the geodetic system has improved tremendously, the cadastral system is still based on the 1948 MRT framework.

The concept of geodetic upgrading for improving the accuracy of positional information is in line with the present research. Whilst geodetic computations focus on the overall improvement and consistency over a nation, this research seeks to improve the quality of cadastral networks which in turn concerns primarily to providing consistency of a particular area of cadastral interest. The detailed nature of this localised adjustment is described in Section (2.5.2).

Besides the upgrading programmes, geodetic networks are also revised periodically. The system requires a revision after a suitable interval of time has elapsed (Bomford, 1980; Ethridge, 1989). On the face of it, this appears methodical.

Its main drawback, however, is that the final coordinate is not current. Furthermore, in practice the cyclic interval is always too long since the updating cost is prohibitively high, especially for poor countries. In fact, it has been very hard to persuade policy makers to provide the necessary fund, since the immediate economic benefit of the upgrading and maintenance programs of the geodetic network is not very clear to them (Ethridge,1989). Thus, if the cycle is a 10-year one, it would be anything up to 10 years before important changes are incorporated.

The geodetic concept of cyclic updating is simply not applicable to cadastral applications. In practice, the geodetic updating cycle is too long for cadastral activities. Present cadastral information in GIS applications and analysis requires up-to-date positional information. The present research aims at a continuous updating of cadastral coordinates. Cadastre is truly a continuous system as it involves day-to-day activities (see Section 2.5).

2.2.2 Least squares implementation

Many sophisticated computer programs based on the least squares method have been developed and implemented for various surveying organizations (Abdullah, 1990; Cross, 1981; Mepham & Krakiwsky, 1984). These implementations adopted specialised strategies in their effort to optimize the specific nature of geodetic or engineering networks.

The main objective of the engineering monitoring computer program is to detect movement of points or a group of points of interest. The technique is used for monitoring large structures such as dams and tall buildings. A few examples of such dedicated programs are the OPTUN, a program for extended pre-analysis and adjustment (Grundig & Bahndorf, 1985), and CANDSN, the Computer Aided network analysis and adjustment system (Mepham & Krakiwsky, 1984). DETECT is another example of such applications (Halim, 1995). Although these programs utilize the least squares method, their emphasis is mainly for engineering applications, thus may not be suitable to cadastral applications.

Several governments and private commercial organizations possess highly sophisticated geodetic network adjustment programs. Such programs are highly suitable for adjusting different types of national, continental triangulation control networks and spatial networks obtained from observations to artificial satellites. Few examples of the computer programs are the TRAV10 Horizontal Network Adjustment Program (Schwarz, 1978), the HAVAGO Three-dimensional Adjustment Program (Vincenty, 1979), and the 3DSUITE Combined terrestrial and satellite adjustment program (Abdullah, 1993).

The above geodetic least squares programs have been developed in line with the functions of geodetic networks. A geodetic network is in a form of a framework of points known as geodetic control points. These control points are designed to serve many users and uses, ranging from reconnaissance surveying, through small and large scale mappings, engineering surveying to scientific

research as for example in geophysics, and the study of plate tectonics. To fulfill these diverse applications, geodetic computational modeling involves not only geometrical parameters, but also includes parameters for monitoring physical phenomena such as earth gravity fields and geoidal surfaces.

A cadastral network on the other hand, is a specialized network which is concerned with parcels and their relative locations. As such, cadastral systems use simple coordinate systems and computation models. For instance, Malaysia is using Cassini Soldner projection for its cadastral system. The main parameters in this model are the coordinates defined in two-dimensional plane. Therefore, the concept of geodetic model and its adjustment program is considered not suitable to cadastral applications.

Currently, the least squares adjustment programs are designed mainly to provide solutions to GPS survey networks combined with existing conventional measurements. STAR*NET computer program developed by Starplus Software Inc. and GeoLab developed by BitWise Ideas Inc. are two examples of dedicated programs for such applications. Besides coordinate computations and analyses, the programs also produce other information including coordinate transformations parameters, geoidal undulation contours and parameters for linking satellite-based information to local networks. Although these parameters provide useful inputs into cadastral network computations, these programs themselves are not sufficient in fulfilling the roles and functions of cadastral systems.

2.2.3 Least squares optimization strategies

The routine steps in implementing the least squares adjustment are:

1. forming observation equations from measurements,
2. transforming observation equations into normal equations,
3. solving the normal equations, and
4. updating the parameters and repeating step 1 to 3 until the corrections converged to insignificant values.

The bulk of the time in least squares process lies in the solution of the normal equations. Research in this area has been concentrated mainly on the optimization of the solution of the equation using the well-known Cholesky Decomposition method. The optimization strategies are:

(1) Cholesky decomposition method,

(2) Restructuring normal equation, and

(3) Minimizing fill-in effect.

2.2.3.1 Cholesky decomposition method

There are two general classes of numerical methods for solving systems of equations: direct or iterative methods. A typical iterative method involves the initial selection of an approximation $\mathbf{x}^{(1)}$ to $\mathbf{x}$, and the determination of a sequence $\mathbf{x}^{(2)}$, $\mathbf{x}^{(3)}$

,..... such that limit $x^{(j)} = x$. On the other hand, direct methods provide the solution directly after a finite number of arithmetic operations have been performed.

Cholesky decomposition method for instance, is a direct method. The method aims at factoring or decomposing the given symmetric, no-singular coefficient matrix **N** of the normal equations. The decomposition process express **N** as a product of two factors, a lower triangular matrix **L** multiplied by the upper triangular matrix $\mathbf{L}^T$. Each factor is the transposed of the other, so that $\mathbf{N}=\mathbf{L}\,\mathbf{L}^T$ (see Figure 3).

A brief outline for solving the system of normal linear equations is as follows:

Consider the system of linear normal equations $\mathbf{N}\,\mathbf{x} = \mathbf{d}$.

Substituting $\mathbf{N} = \mathbf{L}\,\mathbf{L}^T$

one gets $\mathbf{L}\,\mathbf{L}^T\mathbf{x} = \mathbf{d}$

Letting $\mathbf{y} = \mathbf{L}^T\mathbf{x}$

then one gets $\mathbf{L}\,\mathbf{y} = \mathbf{d}$

The forward solution determines the value of **y** which is then used in backward substitution to obtain the parameter **x**. Detailed explanation of the method can be found in most textbooks on numerical methods and matrix analysis; for examples Pissanetzky (1984) and Landesman & Hestenes(1991).

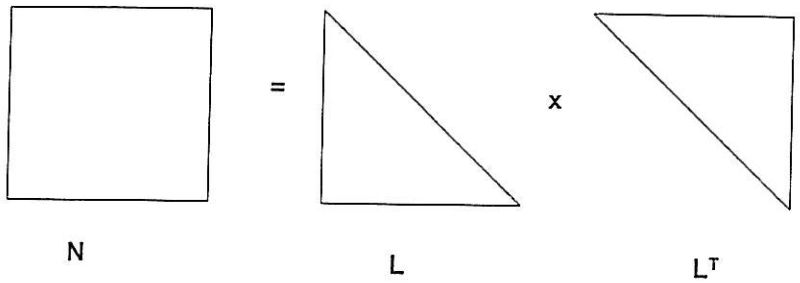


Figure 3: Cholesky decomposition of matrix **N**.

The method is considered as the most efficient method for solving positive definite symmetric coefficient matrix which occurs in least squares surveying problems (Krakiwsky, 1982). However, in practical implementations involving huge and sparse matrices, the method fails to completely eliminate the zeros terms. This is because the process of elimination within the method can make the zero elements nonzero. Hence, the arithmetic and storage operations are not strictly limited to nonzero terms only. This phenomenon is prohibitive for large matrices and in those cases, it may be worthwhile to consider " iterative" methods (Hageman and Young, 1981).

By adopting the iterative CGM, the zero elements of the coefficient matrix are totally eliminated from the system. In addition the method does not change the sparsity pattern during the process of obtaining the parameters. The ability to operate only on the nonzero elements is obviously a major advantage of the CGM.

2.2.3.2 Restructuring the normal equations

The coefficient matrix of an equation is classified as dense or sparse, depending on the percentage of the nonzero elements in the matrix. If most elements of the matrix are nonzero then the matrix is said to be dense, while if most of the elements are zero, then the matrix is said to be sparse. It is difficult to give a precise value of the fraction of the nonzeros below which, the matrix can be

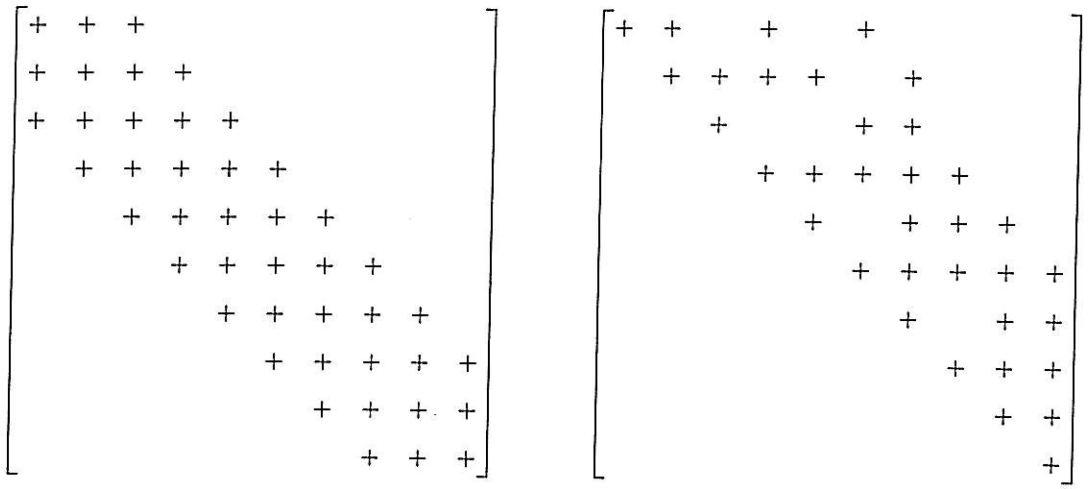
considered to be sparse. Generally, a matrix is said to be sparse if it is worthwhile to take explicit advantage of the existence of many zeros which depend on several interrelated factors such as the nature of the coefficient matrix, the algorithm and the computer being used (Pissanetsky, 1984). Accordingly geodetic networks can be categorized as sparse systems. Interestingly, cadastral networks are even more sparse than the geodetic networks (see Section 2.5.2).

As the performance of a solution depends heavily on the way nonzero terms are stored and processed, a number of storage schemes have been developed. These efforts are basically aimed at trying to structure the random nonzero terms into a well-defined pattern. The pattern is then linked to their original matrix locations by some indexing method. The nonzero terms are normally stored in a vector.

The most common and simplest storage scheme for exploiting the zeros of a matrix is the banded matrices scheme, see Figure 4. The scheme arranges all its nonzeros elements into a band formed by diagonals. When the matrix is symmetrical as in the case of survey networks, a semi-band can be used to further reduce the storage. The upper semi-band consists of all the elements in the upper portion of the band and the lower semi-band consists of all the elements in the lower portion of the band.

The total elements in the banded scheme depend on the width of the band. A large bandwidth stores greater number of elements than a small bandwidth. The dimension of the width depends on the order in which the rows and the columns are

arranged. One may thus seek a permutation of the rows and a permutation of the columns to make the resulting bandwidth small. The goal of the strategy is towards reducing the size of the band, but generally, the effort achieves little progress as the so called narrow band turns out to be not so narrow, after all.



Banded storage scheme

Enveloped storage scheme

Figure 4: The banded and enveloped storage schemes

A band of high order may still have a large quantity of zeros. The banded storage of such a matrix is wasteful. Jennings (1966) proposed an alternative scheme known as the envelope or variable band scheme (see Figure 4). All the elements which belong to an envelope are stored in an orderly fashion (row by row) in a one- dimensional array.

Although the bandwidth scheme and the envelope scheme manage to reduce the storage allocations and hence reducing the number of arithmetic operations, zero elements are still present within both schemes. The number of zero elements present

within the system depends on the size of the band for banded scheme or the length of the elements in each row for the envelope scheme.

2.2.3.3 The fill-in effect

The zero elements existed within the band or within the envelope may turn into nonzero values during the elimination process. The changing of zeros into nonzero values is called "fill-in". To keep the fill-in to a minimum level the technique of "nested dissection" was developed by George & Joseph (1981). Although the method manages to reduce the effect of the fill-in, it then relies on a peculiar ordering scheme of 'blocking' the unknowns, which in turn requires a complicated indexing system, and thus creates large overhead.

Another method called optimal ordering has been developed to reduce the fill-in effect (George & Joseph, 1981). The method is capable of reducing the effect of the fill-in. However, the strategy still requires elaborate storage and addressing techniques, which also requires large programming overhead on its own. What is gained by the strategy, eventually, is used up by the overhead. Such a complicated scheme may reduce the efficiency in handling large volumes of cadastral measurements.

On the other hand, the arithmetic operations in the iterative CGM allows a simple and efficient storage scheme. The scheme can be easily designed so that the effects of the fill-in phenomena are eliminated. This is because the CGM does not involve forward and backward substitutions as in the case of the direct Cholesky method. Furthermore, the CGM retains the pattern of nonzero elements throughout the computation process.

2.2.4 Hierarchical adjustment strategy

In practice, geodetic framework divides the survey network in several ranks or hierarchy to reduce the computational load (Vanicek & Krakiwsky, 1986). Each rank has its own groups of stations and measurements. In view of the ranking, a hierarchical adjustment strategy has been adopted in most geodetic computations.

Hierarchical adjustment is a kind of step by step adjustment (Figure 5). Points are grouped into several ranks such as primary, secondary, tertiary, or sometimes known as first order, second order and so on, and they are adjusted separately within respective groups. Points included in the first step adjustment are considered as the most accurate ones although they are seldom true in practice (Hintz et al, 1988). The proceeding adjustment with a different set of points must adopt the previously adjusted points as controls with fixed coordinates. The adjustment steps are repeated until all points within the network are adjusted.

The ranking system seems to be based on sound principle of the 'whole to part'. However, new development in data acquisition equipments challenges this principle. In fact, there are valid reasons to propose a reverse principle of 'part to whole', Measurements obtained within the 'part' may be as precise as or better than the measurements obtained around the 'whole'. Nowadays, devices like total stations for measuring distances and angles, GPS for measuring relative positions are not only employed for primary control survey, but they are used within the 'part' zone as a day-to-day devices. In the past the classical hierarchical adjustment strategy is partly due to lacking of computational tools which make impossible for handling large amount of equations. Current advances in computer technology is questioning the continued use of this concept, particularly those concerning cadastral system (Hintz et al, 1988). Today, network adjustment need not labeled their points with hierarchical order.

The notion of networks of control points and hierarchical adjustments contributes a fair share to the problems inherent in the present implementation of cadastral system. The criteria for classifying points are not based on mathematical principle. They are normally classified according to the spacing between points and their locations within the network. Points situated around the perimeter are considered strategic and are considered as high order points compared to points inside the network. Points situated within the networks are not considered strategic points. This principle was valid during the pre-GPS and EDM era, where the entire

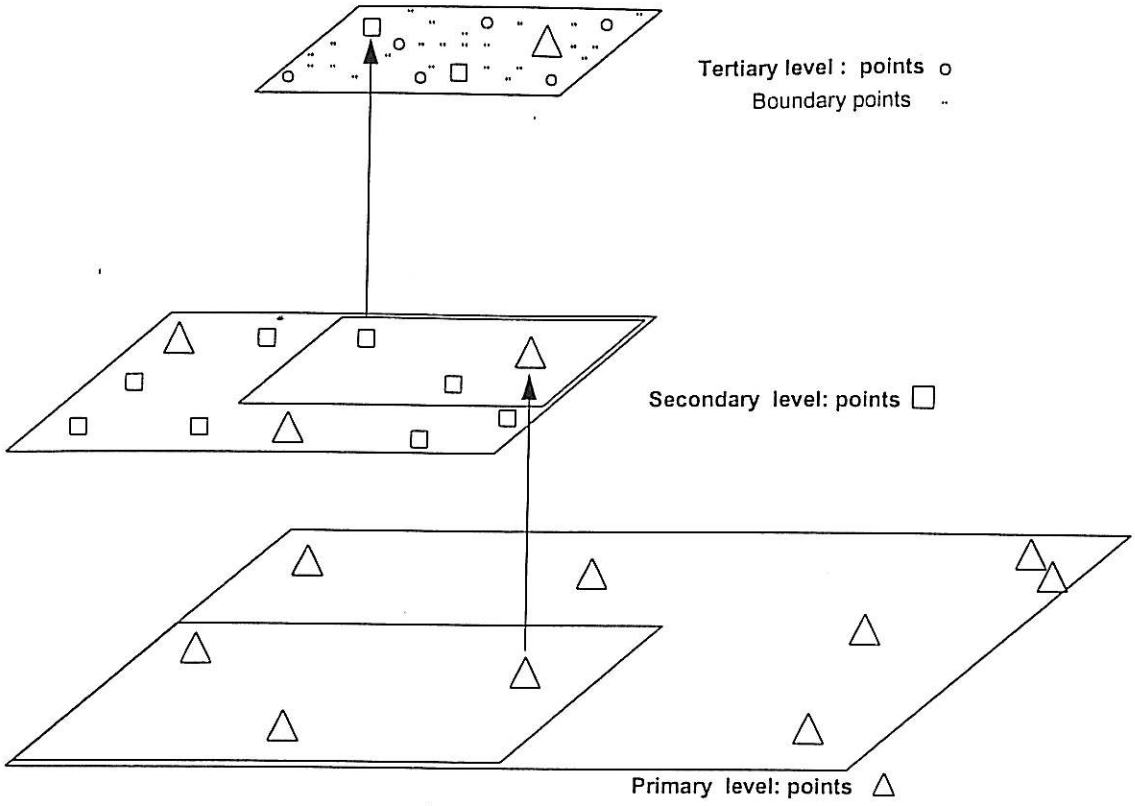


Figure 5: Hierarchical adjustment.

network is filled up with angular observations. Angular observations seemed to give better results for longer lines, though no evidence was actually found that this would hold in general.

Under the criteria, cadastral networks will always fall under lower order points. It is ironic that cadastral lines are all short lines and are mostly situated within the interior of a geodetic network. Forcing cadastral measurements to fit within a set of control points, which may not be of higher accuracy, is a direct conflict with the principle of adjustment. Good quality measurements should not be downgraded to fit into existing coordinate values (White & Griffin, 1985). If points need to be

classified, they must be made according to their accuracy. Although cadastral lines are short, they are accurate as they are measured according to strict observation procedures and using modern devices like EDM or by precisely-calibrated steel tapes.

2.2.5 Development of the CGM

The CGM was first proposed by Hestenes and Steifel (1952) as an algorithm for solving large systems of linear equations. The method becomes quite popular because empirically it was found to have a much faster rate of convergence than the gradient methods (Polak, 1973).

In 1969, a slightly different adaptation of the Hestenes & Steifel method was introduced (Polak, 1973). The latter contained proofs of convergence for convex functions. Klessig & Polak (1972) showed that the Polak's version of the method converges in a finite steps. It converges when the number of iterations reaches the number of the unknowns. For this reason, the method is considered as semi-iterative method.

The CGM algorithm is short and simple (Press et al, 1987) . It demands only two subsidiary calculations: (a) calculating the gradient of the function $\mathbf{dF(x)}$ at an

arbitrary point, and (b) minimizing the **F** along a specified ray, that is, find the value **L** that minimizes the expression **F(x+Lu)** for a specified **x** and **u** (Press et al,1987).

Now lets consider the function

$$\mathbf{f(x)} = \frac{1}{2}|\mathbf{A.x} - \mathbf{b}|^2 \qquad \text{eq (2.1)}$$

The above function has only a single minimum, that is at a value **x** that satisfies the linear set of equation **A x = b**. A CGM gradient minimization will therefore solves that set of equations.

The two subsidiary calculations involved are:

$$\nabla\mathbf{f(x)} = \mathbf{A}^{\mathbf{T}}\mathbf{.(A.x - b)} \qquad \text{eq.(2.2)}$$

$$\mathbf{e} = \frac{-\mathbf{U}.\nabla\mathbf{f}}{|\mathbf{A.u}|^2} \qquad \text{eq.(2.3)}$$

What has this to do with sparse matrices? Equations (2.2) and (2.3) make only two kinds of references to matrix **A**, namely multiplying the matrix by a vector and multiplying its transpose by a vector. For sparse matrix, this multiplication can be reduced substantially. Since the coefficient matrix **A** for a cadastral application is extremely sparse, the arithmetic operations can be minimized by excluding all the operations involving zero elements.

The CGM is applied in many applications particularly for systems which require high speed computations. The method is found to be very efficient in term of storage and speed. The method has been used in finite elements computations in frequency domains for three-dimensional problems (Gijzen, 1995). The method is

also used for testing carried out in varying performance of supercomputer (Dongarra et al, 1992).

The CGM has been found to be very efficient in solving linear equations related to structural engineering analysis (Suarjana & Lau, 1994). The method improves the computing time and the solution is more stable compared to direct methods. Similar works have been carried for solving system of linear equations with multiple right-hand side using block CGM (Schemit & Lai, 1994).

The CGM has also been used for solving system of linear equations found in the studies related to fluid flows and thermal convection (Psimarni, 1994; Tang, 1995). The elements of the coefficient matrix of the equation are symmetric and positive definite. This is similar to normal equations found in a cadastral network problem. Other applications of CGM include solving problems in electronics signal processing (Bose & Chen, 1995; Fu & Dowling, 1994), aerodynamic studies (Burgreen & Baysal, 1994) and other scientific studies such as reactor kinetic (Yang et al, 1993) involving nonsymmetric linear equations. It is reported that the performance of the CGM is remarkable compared to direct methods. Although the CGM has been found to be efficient in many engineering and scientific researches, applications in cadastral computations have yet to be developed, and forms the main aim of the present work.

2.3 Roles of least squares adjustment

Adjustment is usually applied for computing the 'best' position of points within a survey network involving redundant measurements, according to a specified criterion. The most widely used method is the least squares method. The method is based on the criterion that the best parameters are obtained when the sum of squares of the resulting residuals is minimum. The residual is the difference between the adjusted measurements and the original observed measurements.The parameters of the adjustment are mainly the position information, i.e., the coordinates. The mathematical proof of the principle can be found in many surveying adjustment text books such as Mikhail & Gracie (1981) and Vanicek & Krakiwsky (1986).

The theoretical justifications of the least squares method which yield optimum results has been described by several authors (Cross, 1983; Mikhail, 1976). There are also a number of practical reasons why the method has been universally accepted:

(1) The method is extremely easy to apply because it yields a linear set of equations.

(2) It is unique, i.e. there is only one solution to a given problem. The so-called non-rigorous methods yield a number of solutions depending on the subjective choice of computation.

(3) The method leads to an easy quantitative assessment of quality, i.e. via the residuals and the covariance matrix of parameters.

The adjustment is necessary because the number of measurements within a network is purposely acquired more than the minimum required for the unique determination of the parameters. The extra measurements which are called redundancy can be used to detect the existence of blunders and also provide statistical information about the adjusted parameters. The least squares adjustment method is able to deal with such redundant set of measurements efficiently.

The least squares computation produces homogenous and unique coordinates. The usage of homogeneous coordinates is a pre-requisite for GIS data analysis, since the system combines various data layers obtained from a myriad of sources. The overlay operations are only valid if the layers are of the same coordinate system and the coordinates of objects are homogeneous.

Data quality assessment is a major concern in a cadastral system (Chrisman, 1989; Hunter & Goodchild, 1995; Hunter, 1991). Several studies on error modeling in spatial databases over the past few years have been performed. The communication of spatial data errors may take many forms, ranging from the use of epsilon bands (Chrisman, 1984) and other descriptors of errors such as map reliability diagrams and fuzzy logic (Leung et al, 1992), probability surfaces (Lowell, 1992), variability diagrams (MacLean, 1992) simulation techniques (Fisher, 1991; Goodchild,1991) and advanced computer graphics animation and audio effects (Fisher, 1993). Such approaches, however, often fail to offer convenient means of keeping track of the effects of error propagation and also fail to establish estimates of the accuracy and reliability of their information products

(Gregory et al, 1995). A major obstacle is the absence of error estimates in cadastral data within the system. Only least squares estimation method together with the measurements and their accuracy can truly support statistical quality assessment. The propagation of errors in the measurements is based on the well known Gauss variance-covariance propagation (Cross,1983).

Above all, the main advantage of the least squares method is the ease of updating and maintaining a spatial database. The method allows the use of all measurements with different qualities to be processed simultaneously. New measurements can be easily entered into the system and their incorporation into the existing measurements improves the quality of the information. Thus, the difficult task of database upgrading and updating is only a matter of entering new measurements and re-computing the coordinates.

2.4 Nature of cadastral measurements.

Before going into the optimization aspects in cadastral least squares adjustment, first consider the nature and characteristics of these cadastral measurements. The characteristics of the measurements influence the strategy and the process of the adjustment. This factor is critical to the success of any computation, particularly when a large volume of data is involved.

A cadastral parcel is a unit of land ownership. Each parcel is made up of several sides which form the boundaries of a parcel. The number of sides in a parcel depends upon the shape and the size of the parcel. The surveys are mostly traversed along the parcel boundaries. The most common types of measurements are bearings, angles and distances. These angles and bearings are obtained by using theodolites, while distances are normally obtained by using EDM instruments. Steel tape is still widely used for measuring distances especially for short lines. The angular accuracy is better than ten seconds for modern theodolites. The accuracy of distance measurements is better than one centimeter for EDM and steel tapes. Nowadays, the Global positioning system (GPS) is becoming a standard tool for cadastral control networks. Relative accuracy of GPS is around one centimeter.

The role and nature of a cadastral network system is unique in comparison to geodetic and engineering networks. The main differences are:

(1) amount of measurements,

(2) geometry of networks, and

(3) redundant measurements.

2.4.1 Amount of cadastral measurements

Land is the source of all material wealth. Cadastral parcels are created through the legal process which is related to several other factors such as social,

cultural, and historical backgrounds. To cope with these human and physical developments a very large number of parcels are normally created.

, The number of parcels or lots in an average-sized jurisdiction may be up to several hundred thousands. Each lot must be surveyed and recorded. Since each lot is made up of several sides, the number of measurements are several times greater than the number of lots in a jurisdiction. A typical cadastral jurisdiction may contain up to two million measurements (Taher, 1992).

The field data acquisition in a cadastral system is a day-to-day activity. This is because cadastre deals with a dynamic system interrelation: government, people and law (Dale, 1976). Cadastral survey activities include subdivision, partitioning, amalgamation, public utilities and all kind of land reserves: road, forest, etc. These activities produce a continuous flow of new measurements that must be added into the system.

The cadastral survey activity does not only generate a continuous flow of new measurements but it also deals with deleting old measurements, as old parcels may be replaced by new ones (e.g. amalgamation). The very dynamic nature of dealing with cadastral measurements creates a special problem of updating and maintenance.

2.4.2 Geometry of cadastral survey networks

The geometry of a cadastral network is unique. Geometrically, a parcel is a polygon with several sides, and the most common shape is a four-sided figure. The lengths of the sides are usually short, that is, in the range of 10 meters to 100 meters. As the objective of the survey is to obtain measurements of the parcel boundaries, the survey is carried out mostly by traversing along these boundary lines. The measurements across the parcel are not observed except for isolated checking. This pattern of observation scheme creates a unique geometry where cadastral networks appear as rectangular blocks with measurements mainly along the sides.

This is in contrast to geodetic and engineering survey network, where the scheme is designed in such as way that they contain many interconnections between points. These connections are purposely created to attain the highest level of accuracy. Thus, it is why geodetic and engineering networks are mainly made up of overlapping triangles. A dense observation scheme is the unique feature of geodetic and engineering survey networks. Observation schemes for cadastral networks are not as dense as those of the geodetic and engineering networks, but they are very large.

2.4.3 Redundant measurements and sparse system

Although a cadastral network comprises of a very large volume of measurements, their observation scheme is not as compact as in the geodetic or engineering networks. Compact observation schemes represent the existence of large number of redundant measurements. On the other hand, loose observation schemes represent the existence of smaller number of redundant measurements. The redundant measurements in geodetic network can be as high as 60 percent of the total measurements (Bomford, 1980). The redundant measurements in precise engineering networks are usually greater than the geodetic networks.

Since a cadastral network is far less compact than the geodetic and engineering networks, it has a very small proportion of redundant measurements. A compact observation scheme with large redundant measurements leads to a very dense system of equations, and vice versa. In this respect, a cadastral network with a small proportion of redundant measurements generates a very sparse system of equations.

2.5 Cadastral survey network adjustment for GIS

As pointed out earlier, the method of adjustment in cadastral survey networks should be designed in view of the role of a cadastral system. A cadastral

network system not only required positional information as in the case of geodetic networks, but also concerns with legal matters. The validity of cadastral information is fundamental in a spatial analysis and the decision making process.

The advent of powerful and sophisticated GIS has changed the traditional role of cadastre. Nowadays, it becomes part of the sophisticated GIS system. To fulfill the new role, the cadastral processing requires new approaches as follows:

- a non-hierarchical adjustments,
- local adjustments, and
- global adjustments.

2.5.1 Non-hierarchical adjustment

Cadastral measurements should be treated using a non-hierarchical adjustment. In contrast to a hierarchical adjustment, a non-hierarchical adjustment is a simultaneous adjustment that treats all measurements according to their respective accuracy. A non-hierarchical adjustment categorizes points according to their respective levels of accuracy. High-quality points are points with more accurate position values, i.e. with small variances. Alternatively low-quality points are those with less accurate position values, i.e. with large variances. The measurements are not distinguished by their lengths or their historical system of adjustments. This

approach benefits cadastral networks greatly because cadastral measurements are all short lines measured between parcel corners.

The cadastral survey which includes subdivision, partitioning, party wall, road reserves and other land development projects are essentially a kind of fill-in survey, where new boundaries are created within the existing parcels . The newly subdivided parcels have to fit within the old survey although their accuracy may be lower than the new ones. Only a non- hierarchical adjustment allows a simple and practical introduction of new measurements into the existing system.

2.5.2 Local adjustment

The model for metric information in a measurement-based cadastral system is based on the notion that the processing of surveying measurements is no longer expensive and difficult. Therefore the task should be easy and can be repeated frequently. In other words, the adjustment should be carried out with high efficiency when an information is required.

The total number of measurements in a typical cadastral database may be in the order of millions (see section 2.4.1). It is impractical to adjust all the measurements in the database each time new measurements are incorporated into

the database. One of the strategies to handle the problem is by introducing two-phase adjustments: local adjustment and global adjustment.

The local adjustment is the adjustment using measurements around the neighbourhood of the interested area that significantly influence the desired results (Taher & Kuhn, 1990). Users can decide on the level of accuracy they need for their applications. Although this strategy seems to break the network into small manageable blocks, the blocks are not fixed and depend on the location of the area of interest. Therefore local adjustment is a dynamic strategy for selecting the relevant measurements for a localized computation. This sort of computation is particularly useful in cadastral applications where relative accuracy is important.

2.5.3 Global adjustment

The global adjustment is a periodic adjustment of all measurements in a database. This is necessary to create consistency between the coordinate copy and the stored measurements. The coordinate copy constitutes another view of a metric information. This coordinate provides an instant information for query which does not require the most up-to-date information. As new measurements are added to the database, inconsistency may be significant. When this happens, a re-computation is necessary to bring back the view coordinates to their up-to-date base coordinates.

One of the methods for constant coordinate updating seeks adjustment using the method of sequential adjustments (Krakiwsky, 1982). However, the adjustment requires storage of the covariance of the coordinates, which will then be very large. The method has another problem in that the resulting coordinates may deviate from the true least squares coordinates over time, due to rounding off errors (Mikhail and Helmering, 1973). Although researches to improve the situation in a specific case have been promising (Veres and Youcai, 1987), a general formulation that would work for all practical cases has not been created. Even if the method of sequential adjustment is adopted, a periodic adjustment of the entire measurements in a database will be necessary to bring the coordinate back to their ''correct'' values.

The strategy that allows the consistency between coordinate copy and the base measurement to relax is known as quasi-copy. In a quasi-copy, derived values are allowed to deviate from their true values in a controlled manner (Alonso et al, 1988). Thus, coordinate copies are allowed to differ from their true values within a specific condition or period, for example, until one hundred measurements have been added or until a period of one week is reached. So, it is unnecessary to update the coordinate every single time a measurement is added or deleted. When the limit of inconsistency is intolerable for a particular purpose, a single global readjustment of the entire measurements in a database will bring the coordinate's copy up-to-date.

CHAPTER III

THE SOLUTION OF THE OBSERVATION EQUATIONS BY CGM

3.1 Introduction

In cadastral surveys, the parameters to be estimated are coordinates of boundary points, usually defined in a plane rectangular coordinates system. The estimated coordinates serve as basic data for displaying graphical information, for computing other parameters such as adjusted measurements and areas of parcels as well as for other cadastral geometric information and analysis.

The normal practice in the cadastral survey is measuring and recording the measurements and then correcting them for physical effects such as atmospheric refraction and instrumental error. The resulting numerical values represent the elements in a selected plane projection system. These corrected measurements are passed on to an adjustment process to produce the required parameters.

The least squares adjustment process begins with the creatic linear equations called observation equations. Each measurement w equation and it relates measurement to its respective parameters. T over- determined because the number of measurements is always g parameters. The coefficient matrix of the system is then transform positive-symmetric coefficient matrix of the normal equation.

As mentioned in Section 1.2 the CGM is a method that solves least squares equations in two ways. The first approach is the solution of observation equations and the second approach is the solution of normal equations. The present work investigates both approaches of the CGM for the following reasons:

- To verify the validity of results obtained from both approaches. This is crucial because to-date the CGM has not yet been applied especially in cadastral applications.
- To determine the most efficient approach for CGM in cadastral applications.
- To determine which approach requires the least computer storage.

This chapter concentrates on the optimization and the solution of the observation equations while the optimization of the normal equations and its solution will be described in the next chapter. This chapter begins with the development of observation equations in the least squares method. Next, the chapter describes the locations in the coefficient A matrix affected by the contribution of a measurement. The subsequent section develops a method to organize the nonzero elements in the coefficient matrix into a vector. The indexing

information specifies for each element in the vector its original location in A matrix. Finally, the chapter presents the CGM for solving the observation equations.

3.2 Linear equations in least squares method

Since measurements are the basic data, hence we begin by defining a vector **L** containing several types of measured quantities. The most common cadastral measurements are grid bearings, horizontal angles and horizontal distances. GPS measurements and other satellite-based coordinates may be used as part of measurements. However the three-dimensional coordinates of the satellite-based system should be reduced to horizontal angles and distances, or in terms of plane projection coordinate, like Cassini projection system, before the coordinates are introduced to least squares adjustment.

Let the true value of measurements **L** be contained in a vector $\mathbf{L}_T$:

$$\mathbf{L}_T = \mathbf{L}+\mathbf{v} \qquad \text{eq (3.1)}$$

where **v** is a vector containing residuals.

It is required to estimate a set of parameters, **x**, from the measurements. There must always be a known mathematical relationship between the measurements and the parameters. This relationship constitutes the basic mathematical model and is expressed as a general vector equation:

$$\underline{F(x,l)} = \underline{0} \qquad \text{eq(3.2)}$$

which is,

$$\begin{aligned} F1(x1,l1) &= 0 \\ F2(x2,l2) &= 0 \\ F3(x3,l3) &= 0 \\ &\cdots \\ Fr(xr,lr) &= 0 \end{aligned}$$

Some examples of the common function $Fi(xi,li) = 0$ for a cadastral measurement are as follows.

(a) For an observed bearing, a, from station j to station k in a plane coordinate model,

$$Tan(a) - \frac{(E_k - E_j)}{(N_k - N_j)} = 0$$

where E_k, N_k, E_j and N_j would be parameters

(b) For an observed distance, d, from station j to station k in a plane coordinates model,

$$d - \sqrt{(E_k - E_j)^2 + (N_k - N_j)^2} = 0$$

where E_k,N_k,E_j, and N_j would be parameters.

3.2.1 Linearization of the mathematical model

The basic equations (3.1) are non-linear. The practical approach to solve these non-linear equations is by linearizing them into a system of linear equations. The linearization process always requires estimation of the provisional (or approximate) values of the quantities involved. In a cadastral computation, these quantities are obtained from simple coordinate computation methods, such as the Bowditch or transit rule. Let these provisional values be $\mathbf{x}_o$ and let $\mathbf{x}_T$ be the true coordinates and the relationship between these two quantities is

$$\mathbf{x}_T = \mathbf{x}_o + \mathbf{x}, \qquad \text{eq (3.3)}$$

where it is necessary to estimate the small quantities $\mathbf{x}$. Hence, upon substituting (3.3) and (3.1) in (3.2) , one can write

$$\underline{F}(\underline{x}_T, \underline{l}_T) = F(x_{o,} + x, l + v) = 0$$

and, applying the Taylor series expansion to first differentials, we obtain

$$\underline{F}(\underline{x}_T, \underline{l}_T) = F(x_0, l) + \frac{\partial F}{\partial x} x + \frac{\partial F}{dl} v = 0$$

and can be written in matrix notation

$$\mathbf{-b + Ax + Cv = 0} \qquad \text{eq (3.4)}$$

Equation (3.4) is a system of linear equations, where $\mathbf{b}$ is coefficient vector, and $\mathbf{A}$ and $\mathbf{C}$ are coefficient matrix and they are described in the following sections:

(1) The vector -**b** is denoted by $F(\mathbf{x}_0,\mathbf{L})$. It contains the values of $F(\mathbf{x},\mathbf{L})$ computed at the known points $\mathbf{x}_0,\mathbf{L}$. The negative sign being introduced merely for convenience.

Hence, the vector -**b** is :

$$-\mathbf{b} = \begin{bmatrix} f_1(x_0,l) \\ f_2(x_0,l) \\ f_3(x_0,l) \\ . \\ . \\ . \\ f_r(x_0,l) \end{bmatrix}$$

(2) $\frac{\partial F}{\partial l}$ is a matrix of order (r x m) and is denoted by the letter **A**. Each row will simply be the partial differentials of $F(\mathbf{x},\mathbf{l})$ with respect to $x_1,x_2.....x_m$.

$$A = \begin{bmatrix} \frac{\partial F_1}{\partial x_1} & \frac{\partial F_1}{\partial x_2} & . & . & . & \frac{\partial F_1}{\partial x_m} \\ \frac{\partial F_2}{\partial x_1} & \frac{\partial F_2}{\partial x_2} & . & . & . & \frac{\partial F_2}{\partial x_m} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ \frac{\partial F_r}{\partial x_1} & \frac{\partial F_r}{\partial x_2} & . & . & . & \frac{\partial F_r}{\partial x_m} \end{bmatrix}$$

(3) $\frac{\partial F}{\partial l}$ is a matrix or order (r x n) and is denoted by the latter **C**. Each row will simply be the partial differentiation of $F(\mathbf{x}, \mathbf{l})$ with respect to $L_1, L_2....L_r$.

and hence,

$$C = \begin{bmatrix} \frac{\partial F_1}{\partial l_1} & \frac{\partial F_1}{\partial l_2} & \cdots & \frac{\partial F_1}{\partial l_n} \\ \frac{\partial F_2}{\partial l_1} & \frac{\partial F_2}{\partial l_2} & \cdots & \frac{\partial F_2}{\partial l_n} \\ \vdots & \vdots & \cdots & \vdots \\ \frac{\partial F_r}{\partial l_1} & \frac{\partial F_r}{\partial l_2} & \cdots & \frac{\partial F_r}{\partial l_n} \end{bmatrix}$$

Matrices **A** and **C** are often referred to as design matrices, while **b** the right-hand side vector.

3.2.2 The observation equations

The system of equation $\mathbf{Ax + Cv - b = 0}$ is a general linearized version of the basic model (3.2). A special case of the system occurs when the model contains one observed quantity for each equation. This type of system of equations is called observation equation and is commonly used for cadastral networks. In this case, the equation (3.2) can be written as $F(x) - l = 0$. Clearly, if we differentiate the matrix **C** with respect to **l**, we will get **C=-I**.

Thus the final linearised set of observation equations is written as

$$\mathbf{A}_{(m,n)}\, \mathbf{x}_{(n)} = \mathbf{b}_{(m)} + \mathbf{v}_{(m)} \qquad \text{eq (3.5)}$$

where;

A is the rectangular matrix of coefficient, also known as design matrix,

x is the vector of correction to unknowns,

b is the vector of (observed-computed) values, and

v is the vector of residuals.

This equation is the initial linear equation of the least squares method. Standard least squares process requires this equation to be transformed into a new system of equations called normal equations before it can be solved for unknowns.

3.2.3 The normal equations

The least squares method is a mathematical optimization to find the expressions for **x** and **v** which make

$$\mathbf{v}^T\mathbf{W}\mathbf{v} = \text{minimum} \qquad \text{eq (3.6)}$$

subject to constraints of the eq (3.5)

$$\mathbf{A}\mathbf{x}+\mathbf{v}-\mathbf{b}=\mathbf{0}$$

where **W** is the inverse of the covariance matrix of the observations, and **A**, **b** are also known values.

The optimization process produces a new system of linear equations (Cross, 1983):

$$\mathbf{A}^T\mathbf{W}\mathbf{A}\mathbf{x} = \mathbf{A}^T\mathbf{W}\mathbf{b}$$

$$\mathbf{Nx} = \mathbf{d} \qquad \text{eq (3.7)}$$

where

$$\mathbf{N} = \mathbf{A}^T \mathbf{W}\, \mathbf{A},$$

$$\mathbf{d} = \mathbf{A}^T \mathbf{W} \mathbf{b}.$$

Equation (3.7) is called normal equations The mathematical proof of equation (3.7) is a standard optimization process and can be referred in many survey text books such as Mikhail and Gracie (1981), Vanicek and Krakiwky (1986) and Mikhail (1976). The coefficient matrix **N** is always symmetrical and positive- definite. These special properties of matrix **N** allows further reduction of storage requirement. The setting up of the normal equations will be described in Chapter IV.

3.3 Creating the coefficient matrix of observation equations

It is clear from the previous section that matrix **A** represents the partial derivatives of the functional model of each observation with respect to the parameters. The functional model relates a measurement to its respective parameters (i.e. the coordinates of the stations from which the measurements were measured). Therefore, coefficient matrix **A** is a measurement-parameter incident matrix, see Table 1.

Table 1: Coefficient matrix **A**

	PARAMETERS			
	x_1	x_2	x_3	x_4
Measurement ,1	$\frac{\partial F_1}{\partial x_1}$	$\frac{\partial F_1}{\partial x_2}$	$\frac{\partial F_1}{\partial x_3}$	$\frac{\partial F_1}{\partial x_4}$
Measurement, 2	$\frac{\partial F_2}{\partial x_1}$	$\frac{\partial F_2}{\partial x_2}$	$\frac{\partial F_2}{\partial x_3}$	$\frac{\partial F_2}{\partial x_4}$
Measurement .	.	.	.	.
Measurement, m.	$\frac{\partial F_m}{\partial x_1}$	$\frac{\partial F_m}{\partial x_2}$	$\frac{\partial F_m}{\partial x_3}$	$\frac{\partial F_m}{\partial x_4}$

3.3.1 Locations contributed by a single measurement

Measurements are relative geometrical values between stations. The stations are points which are either occupied or targeted by a survey instrument during the acquisition of the measurements. A row in matrix **A** represents a measurement and the columns along the row represents the unknowns associated with that measurement. The number of coefficients contributed by a measurement is twice the number of stations from which the measurement was observed. For example, a

bearing which involves two stations will have 4 contributions. The number of contributions generated by common cadastral measurements are shown in Table 2.

After knowing the number of contributions from a measurement, the next important question is where these contributions fit into matrix **A**. The locations effected by a measurement are the locations (along the row assigned for the measurement) which are incidented with the parameters associated with that measurement.

Table 2: Number of contributions from measurements into matrix **A**

Type of measurements	number of stations occupied	number of coefficients
distance	2	4
bearing	2	4
angle	3	6
northing	1	1
easting	1	1

This can be further explained by the following example. An observation (say the qth observation) is related to station j and station k. For each station there are two unknowns (i.e. easting and northing). Let 'a' and 'b' be the unknown

locations for easting and northing, respectively, of station j. Similarly let 'c' and 'd' be the respective locations for easting and northing of station k (see Figure 6).

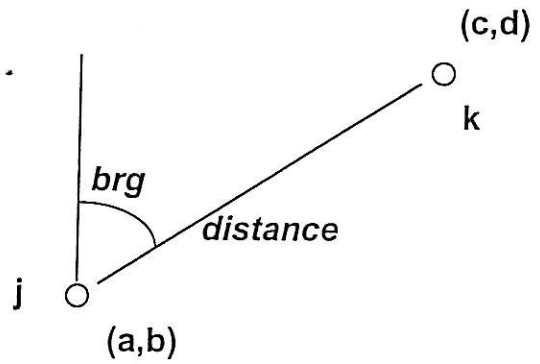


Figure 6: A measurement between station j to station k.

The locations of the unknown (i.e. the column number) is derived from station number for the measurement, and they are as follows:

$$a = j \times 2 - 1$$

$$b = j \times 2 \quad \text{for station j.}$$

and $$c = k \times 2 - 1 \qquad \text{eq (3.8)}$$

$$d = k \times 2 \quad \text{for station k}$$

For the bearing measurement, it occupies locations at columns a,b,c and d in the q^{th} row. The rest of the column along the row are not effected and therefore remain zero. The position of coefficients for a measurement, say the 'q^{th}.' measurement are shown in Table 3.

Table 3: Locations in matrix **A** contributed by the q^{th} measurement.

	station 1 . .		station .j a^{th}	b^{th}	station th ..th .th		station k c^{th}	d^{th}	station . . .	
measurement 1										
measurement 2										
measurement 3										
measurement q th.			A_{qa}	A_{qb}			A_{qc}	A_{qd}		
measurement ..th .										

3.3.2 Structure of elements in matrix A

The design matrix is formed when the coefficients for all measurements are assembled in their respective positions in the matrix. Each measurement contributes their coefficients to specific locations in the design matrix as described in the previous section. A location receives a contribution only once. Once the location is filled, it will not receive any more contribution from any measurement. There is no overlapping of contributions from the measurements. Therefore, the total elements in matrix **A** is simply the sum of contributions from all measurements of a network. For common cadastral measurements listed in Table 4, the total elements in a network, T_A are given by,

$$T_A = 4 \times (N_b + N_d) + 6 (N_{ang}) + N_E + N_N \qquad \text{eq (3.9)}$$

where T_A is the total elements in matrix **A**,

N_b is the total number of bearing,

N_d is the total number distances,

N_{ang} is the total number of angles

N_E is the total number of eastings

N_N is the total number of northings

A typical cadastral network survey consists of mainly bearings and distances. Other types of measurements, such as coordinates and angles are rarely used. Therefore the total number of elements T_A is about four times the total number of bearings and distances. The detailed discussion on amount of computer storage saving based on (Eq 3.9) are found in Section 6.4.1.

The construction process of matrix **A** is completed when all the measurements in a network update their respective contributions into the matrix. The process is illustrated by an example using a simple cadastral network. The network consists of four stations with ten measurements (see Figure 7). The measurements are listed in Table 4.

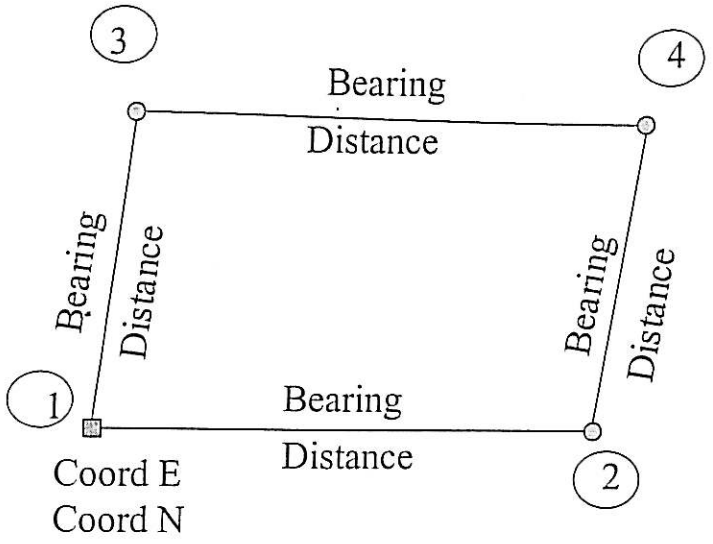


Figure 7: A cadastral network.

Table 4: Measurements of the network in Figure 7

No	measurement type	Occupied station	Forward station
1	Coordinate Easting	1	
2	Coordinate Northing	1	
3	distance	1	2
4	bearing	1	2
5	distance	2	4
6	bearing	2	4
7	distance	4	3
8	bearing	4	3
9	distance	3	1
10	bearing	3	1

Following the method described earlier in this section, the measurements for the above network fill up their respective locations in matrix **A** as shown in Table 5. It is clear that each measurement will occupy a row, and the locations of the parameters associated with the measurement will fill up their respective locations along its designated row. Note that the number of rows of matrix A is equal to the number of measurements in the network, i.e., 10 rows.

The total number of elements in matrix **A** is obtained by applying equation (3.9). Using the equation (3.9) the total elements in matrix **A** of the network which made up of one easting, one northing, four distances and four bearings will contain 34 nonzero elements (T_A) i.e.

$$T_A = 4 \times (4+4) + 6 \times (0) + (1) + (1)$$

$$= 34$$

Table 5: The locations of nonzero elements in matrix **A**.

	Station 1		station 2		station 3		station 4	
	1	2	3	4	5	6	7	8
1. Easting stn 1	1,1	.	.	.	.	.	.	.
2. Northing stn 1	.	2,2	.	.	.	.	.	.
3. Distance 1-2	3,1	3,2	3,3	3,4	.	.	.	.
4. Bearing 1-2	4,1	4,2	4,3	4,4	.	.	.	.
5. Distance 2-4	.	.	5,3	5,4	.	.	5,7	5,8
6. Bearing 2-4	.	.	6,3	6,4	.	.	6,7	6,8
7. Distance 4-3	.	.	.	.	7,5	7,6	7,7	7,8
8. Bearing 4-3	.	.	.	.	8,5	8,6	8,7	8,8
9. Distance 3-1	9,1	9,2	.	.	9,5	9,6	.	.
10. Bearing 3-1	10,1	10,2	.	.	10,5	10,6	.	.

3.4 Storing nonzero elements in vector a

Having established the locations of the elements in matrix A effected by the contributions of the whole set of measurements, the next step is to re-organize the elements into a structure so that only non-zero elements are stored and referenced. This is done by ordering the elements row by row in an ascending order. The scheme reduces the matrix size into a vector containing only non-zero elements. The resulting ordering of the 34 non-zero elements of matrix A is shown in Table 6.

Table 6: Nonzero elements arranged in ascending order row by row

	Station 1		station 2		station 3		station 4	
	1	2	3	4	5	6	7	8
1. Easting stn 1	1	.	.	.	.	.	.	.
2. Northing stn 1	.	2	.	.	.	.	.	.
3. Distance 1-2	3	4	5	6	.	.	.	.
4. Bearing 1-2	7	8	9	10	.	.	.	.
5. Distance 2-4	.	.	11	12	.	.	13	14
6. Bearing 2-4	.	.	15	16	.	.	17	18
7. Distance 4-3	.	.	.	.	19	20	21	22
8. Bearing 4-3	.	.	.	.	23	24	25	26
9. Distance 3-1	27	28	.	.	29	30	.	.
10. Bearing 3-1	31	32	.	.	33	34	.	.

3.5 Linking nonzero elements to original matrix A

Storing exclusively the nonzero terms in a vector undoubtedly leads to a scheme with minimum storage locations. However, these elements must be linked to their respective locations in the original matrix. The linking is established by an indexing system. The indexing system for the matrix **A** consists of two pointers.

(a) a pointer telling the column number for each non-zero elements, and

(b) a pointer telling the total number of the nonzero terms for each row.

By adopting the above indexing system, the matrix **A** is truly represented by a vector which holds all the nonzero element of matrix **A**. To preserve the standard notation, the array is denoted by vector **a.** All arithmetic operations involving matrix **A** can be replaced by its representative vector **a**. It should be noted that without the use of the indexing information, the arithmetic operation is meaningless. The elements and indexing values of vector **A** is shown in Table 7. The pointer telling the column number for each element is ICOL, and the pointer telling the sum of nonzero elements in each row is IROW.

3.6 Solving observation equations by CGM

The residual (**v**) of the observation equations is

$$\mathbf{v} = \mathbf{A}\mathbf{x} - \mathbf{b} \qquad \text{eq (3.10)}$$

The residual ($\mathbf{v}'$) from the normal equations is

$$\mathbf{v}'=\mathbf{Nx}-\mathbf{Ab} \qquad \text{eq (3.11)}$$

From equation (3.10 and 3.11) the relation between vector $\mathbf{v}'$ and vector $\mathbf{v}$ is

$$\mathbf{v}' =\mathbf{A}^{T}\mathbf{v} \qquad \text{eq (3.12)}$$

Table 7: Storing and indexing vector **a**

Pointer number	Coefficient of matrix A	Column Number ICOL	Total elements in each row IROW
1	A(1)	1	1
2	A(2)	2	1
3	A(3)	1	4
4	A(4)	2	4
5	A(5)	3	4
6	A(6)	4	4
7	A(7)	1	4
8	A(8)	2	4
9	A(9)	3	4
10	A(10)	4	4
11			
.			
30	A(30)	6	
31	A(31)	1	
32	A(32)	2	
33	A(33)	5	
34	A(34)	6	

The CGM is able to solve least squares equation without having to compute coefficient matrix N (Dragomir et al, 1982) by means of a recurrence relation

$$r^{(j)} = r^{(j-1)} + f(Nh^{(j)}) \qquad \text{eq (3.13)}$$

where f is

$$f^{(j)} = \frac{r^{(j-1)T} r^{(j-1)}}{(Ah^{(j)T})(Ah^{(j)})}$$

and h is

$$h^{(j)} = -r$$
$$h^{(j)} = -r^{(j-1)} c^{(j-1)} h^{(j-1)}$$
$$c^{(j-1)} = \frac{r^{(j-1)} r^{(J-1)}}{r^{(j-2)T} r^{(j-2)}}$$

One eliminates $\mathbf{N=A^TA}$ for unweighted measurement by equation 3.12 in equation (3.13.)

$$\mathbf{ATv^{(j)}=A^Tv^{(j-1)}} + f(\mathbf{A^TAh^{(j)}}) \quad \text{or}$$

$$\mathbf{v^{(j)} = v^{(j-1)}} + f(\mathbf{Ah^T})$$

The algorithms for solving the residuals by CGM begin with the starting values of parameters **x** and the residuals being computed. The general algorithm for implementing the CGM is as follows.

For the first iteration, since $\mathbf{x} = 0$ then equation becomes $\mathbf{v}=\mathbf{b}$.

$$
\begin{aligned}
r(j) &= A^T v(j) \\
c(j) &= \frac{r(j)^T r(j)}{r(j-1)^T r(j-1)} \\
h(1) &= -r(1) \\
h(j+1) &= -r(j) + c(j)h(j) \\
f(j) &= \frac{r(j)^T r(j)}{(Ah(j))^T (Ah(j))} \\
x(j) &= x(j-1) + fh(j) \\
v(j) &= v(j-1) + f(Ah(j))
\end{aligned}
\qquad \text{Eq (3.14)}
$$

Based on the above CGM equations, the step-by-step algorithm for computer programming has been developed in this research as outlined in Table 8.

Table 8: CGM for solving observation equations

No	ALGORITHM	REMARK	NOTE
1	Let $\mathbf{x}=0$ and $\mathbf{v}=\mathbf{l}$	Initialize vectors	
2	$\mathbf{r} = \mathbf{A}^T \mathbf{v}$	start iteration	Matrix by vector
3	$a= \mathbf{r}^T \mathbf{r}$		Vector by vector
4	$b = \mathbf{r}^T \mathbf{r}$		Vector by vector
5	c= a/b		Scalar
6	$\mathbf{h} = -\mathbf{r}$ $\mathbf{h}= -\mathbf{r} + c\,\mathbf{h}$	first iteration only	 scalar by vector
7	$\mathbf{d}= \mathbf{A}\,\mathbf{h}$		Matrix by vector
8	$e = \mathbf{d}^T \mathbf{d}$		Vector by vector
9	f= a/e		Scalar
10	$\mathbf{x} = \mathbf{x} + f\,\mathbf{h}$	Update vector x	scalar by vector
11	$\mathbf{v} = \mathbf{v} + f\,\mathbf{d}$	Update vector v	scalar by vector

The CGM algorithm for solving observations is found to be simple and short. Among the eleven steps, there are only two major operations (i.e. step 2 and 7). The two steps involve multiplication to matrix **A** by a vector. However, since matrix **A** is reduced to vector **a** whose length is about four times the number of measurements, the matrix operations are thus reduced to only the multiplication of four vectors by a vector.

The above operations consider each measurement as having equal weights of value one. For weighted measurements, the values of **A** matrix and vector **L** must be premultipled by weight (**W**), i.e.

$$\mathbf{A} = \mathbf{A} \times \mathbf{W}$$

$$\mathbf{L} = \mathbf{L} \times \mathbf{W}$$

and the rest of the computation follows exactly the above algorithm.

3.7 Summary

The method of setting up the observation equations developed in this chapter has achieved the goal of identifying and storing only the nonzero elements. These elements are stored in a vector. The method also manages to relate the elements in the vector with their true locations in the regular matrix **A** using a simple indexing scheme. The indexing scheme replaces matrix **A** with vector **a** and it

permits all arithmetic manipulation and operation involving matrix **A** replaced by the vector **a** without any lost of information.

The CGM solves the observation equations for the least squares parameters and residuals. The algorithm is short and it involves simple vector operations except for the two matrix operations involving the multiplication of matrix **A** by a vector. However, the method minimizes these operations by using vector **a** instead of matrix **A**.

The CGM uses vector **a** without altering the structure of the elements in vector **a** throughout the whole least squares estimation process. This means that the operations do not require new elements to be allocated to the vector. The vector is free from the complicated fill-in effect. Therefore, the method of setting up the observation equations developed in this chapter achieves three leading advantages, i.e. storing only the minimum nonzero terms, operate only on the stored nonzero terms, and always preserve the sparsity pattern.

CHAPTER IV

THE SOLUTION OF THE NORMAL EQUATIONS BY CGM

4.1 Introduction

To recapitulate, the least squares estimation method begins with the creation of observation equations in the form of $\mathbf{Ax} = \mathbf{b}$. The equation is transformed into a new system of linear equations called the normal equations. The system is in the form of $\mathbf{Nx} = \mathbf{d}$, where $\mathbf{N}$ is the coefficient matrix, $\mathbf{x}$ = unknown vector and $\mathbf{d}$ is the right-hand vector. Matrix $\mathbf{N}$ is derived from design matrix $\mathbf{A}$ and weight matrix $\mathbf{W}$, i.e. $\mathbf{N} = \mathbf{A}^T\mathbf{WA}$ while vector $\mathbf{d}$ is obtained by $\mathbf{d} = \mathbf{A}^T\mathbf{Wb}$.

Chapter 3 focused on the optimization of observation equations, while the present chapter focuses on the normal equations. Both system of equations created from a cadastral network are very sparse. However the two system of equations have several distinct features, particularly, in the sparsity pattern and process of constructing the coefficient matrix. The pattern of coefficients in matrix $\mathbf{A}$ is

random, while the pattern of coefficients in matrix **N** is always symmetric. The features of coefficient matrix **N** are explored in this chapter with the effort to optimize the solution of the normal equations.

This chapter begins with the description of the relationship between a measurement and their locations in regular matrix **N**. Next the problem of building up the nonzero terms in matrix **N** is discussed. Graph theory is introduced to resolve the problem of arranging the locations within matrix **N**. Next, the chapter describes the development of a mathematical formula to directly relate the measurements to the locations in a vector which is designed to store only the nonzero elements found in the upper triangle of matrix **N**. The subsequent section presents the CGM algorithm for solving the normal equations. The last section summarizes a few important observations.

4. 2 Characteristics of matrix N

Before presenting the optimization method for solving normal equations, consider the characteristics of matrix **N** and the relationship between a measurement and the effected locations in matrix **N**. The difference between matrix **N** and matrix **A** is also described to clarify the nature of matrix **N**. Understanding the structure and the nature of elements in a matrix is necessary in order to develop the most effective approach for solving the equations.

Matrix **N** is a parameter-parameter incident matrix. This is in contrast to matrix **A**, which is a measurement -parameter matrix. On the other hand matrix **N** is square matrix whose dimension is the total number of unknowns, i.e. **N**(n,n), where 'n' is the total number of unknowns.

The process of building matrix **N** is different to that of matrix **A**. In matrix **A**, the elements of a measurement occupy at the locations in a row designated to that measurement, whereas in matrix **N**, the elements of a measurement occupy at the locations scattered at several rows and columns depending on the stations associated with that measurement.

In matrix **A**, a location receives a coefficient only once. This means that once a location has received a contribution, the location will not receive any more contributions from any measurement. Thus a location can be reserved for one contribution. This is in contrast to matrix **N** where a location may receive several overlapping contributions from several measurements.

Since a measurement is referred to a row in matrix **A**, the pattern of the elements entering the matrix depends upon the sequence of measurements entered into the system. The elements in matrix **N** is symmetrical and sparse (Cross, 1983). The distribution of elements in the matrix **N** depends upon the structure of observation scheme. The following sub-sections further describe the process for constructing matrix **N**.

4.2.1 Number of contributions to matrix N

A station carries two coordinate parameters, i.e. easting and northing values. Stations are connected by measurements. There always exist a mathematical relationship between measurements and the parameters. Since matrix **N** is a parameter-parameter matrix, the number of coefficients contributed by a measurement is equal to the squares of parameters associated with the measurement. For example, a bearing which is a two-station measurement will have four parameters and the total number of locations that will contribute to matrix **N** will be 16 (4x4) elements. Similarly, a distance which is also a two-station measurement, will contribute 16 elements.

Based on the above relationship, the number of contributions to matrix **N** generated by common cadastral measurements are summarized in the Table 9.

Table 9: Number of contributions to **N** for standard cadastral measurements

Type of measurements	Number of stations	Number of parameters	Number of contributions
Northing	1	1	1
Easting	1	1	1
bearing	2	4	16
distance	2	4	16
angle	3	6	36

4.2.2 Locations in N effected by a measurement.

After knowing the number of contributions of the cadastral measurements to matrix **N**, the next question is where those contributions go into the matrix. The affected locations are the intersection of the unknowns generated by the individual measurements. The following discussions will further illustrate the point.

The relationship between a measurement and its parameters was described in Section 3.3.1. Following the same notation as in Figure 6 and using eq (3.8), the parameter numbers (a, b, c and d) for a measurement measured between two stations j and k are:

a = j x 2-1 (easting station i)

b = j x 2 (northing station i)

c = k x 2-1 (easting station k)

d = k x 2 (northing station k)

The four parameter numbers will generate 16 elements (i.e. 4 x4 parameter) in matrix **N**. The positions of those contributions in matrix **N** are depicted in the Table 10. The intersections of these four parameters provide the information about the locations of the contributions.

for row a N_{aa}, N_{ab}, N_{ac}, N_{ad}

for row b N_{ba}, N_{bb}, N_{bc}, N_{bd}

for row c N_{ca} N_{cb}, N_{cc}, N_{cd}

for row d N_{da}, N_{db}, N_{dc}, N_{dd}

Table 10: Locations in **N** effected by measurements between station j and station k

	a	b	.	.	c	d	.	.
.								
a	N_{aa}	N_{ab}			N_{ac}	N_{ad}		
b	N_{ba}	N_{bb}			N_{bc}	N_{bd}		
.								
.								
c	N_{ca}	N_{cb}			N_{cc}	N_{cd}		
d	N_{da}	N_{db}			N_{dc}	N_{dd}		

The above example shows the effected locations in matrix **N** from a distance measurement. Note that the locations are directly related to the stations from which the measurement was captured. The elements in matrix are built up as more measurements update their respective locations in matrix **N**. The resulting pattern of matrix **N** is obtained when the contributions from the whole set of measurements is incorporated.

The pattern of elements in matrix **N** describes distribution of measurements within a network, that is the observation scheme. The scheme shows where the stations are to be occupied and where the station to be targeted. In addition, the

scheme also shows the type of measurement and what level of accuracy to be achieved. Therefore, the observation scheme can be used to determine the structure of matrix **N**. By knowing the structure of matrix **N**, one can prepare an optimized storage scheme that will only receive the nonzero coefficients contributed by a measurement.

4.2.3 Overlapping of contributions

Note that the locations in matrix **N** are directly related to the stations from which the measurements were captured. Therefore, if more than one measurements occupy a particular station, which normally occurs in a cadastral network, the contributions of these measurements will be overlapping. For example, any new measurements observed between station 'j' to 'k' will occupy the same locations as shown in Table 10.

for row a N_{aa}, N_{ab}, N_{ac}, N_{ad}

for row b N_{ba}, N_{bb}, N_{bc}, N_{bd}

for row c N_{ca} N_{cb}, N_{cc}, N_{cd}

for row d N_{da}, N_{db}, N_{dc}, N_{dd}

The number of overlappings in a location depends upon the frequency of the point being occupied or targeted during the data acquisition process. Therefore, if some points were visited or targeted k times during the data acquisition process,

the related location in matrix **N** will receive k overlapping contributions from those measurements.

4.2.4 Symmetric coefficient matrix N

The coefficient matrix **N** is symmetric and therefore their values of coefficients are symmetrical about the diagonal. Thus N(a,b)=N(b,a), N(a,c)=N(c,a) for all locations in **N**. The symmetrical pattern of the coefficients provides an opportunity to reduce computer storage by nearly half and consequently minimize computer arithmetic. This is because the matrix can be sufficiently represented either by elements in the upper triangle or in the lower triangle. In this thesis, the upper triangle of the matrix is used to represent the matrix.

Table 11: Locations in upper triangle effected by a measurement between stations j and k.

	a	b	.	.	c	d	.	.
.								
a	N_{aa}	N_{ab}			N_{ac}	N_{ad}		
b		N_{bb}			N_{bc}	N_{bd}		
.								
.								
c					N_{cc}	N_{cd}		
d						N_{dd}		

The upper triangle is made up of elements above the diagonal and also the diagonal elements themselves. The corresponding elements in upper triangular of matrix **N** of the above example are shown in Table 11.

4.2.5 Computation of coefficient values

The coefficient values of **N** are obtained by applying the equation $\mathbf{N}=\mathbf{A}^T\mathbf{W}\mathbf{A}$. Using this formula a single location **N**(i,j) as presented by (Cross, 1983) is given as

$$\mathbf{N}_{(i,j)} = \mathbf{N}_{(i,j)} + \mathbf{A}_{(q,i)} \times \mathbf{A}_{(q,j)} \times \mathbf{W}_{(q,q)} \qquad \text{eq (4.1)}$$

where i and j are the parameters numbers and q is the measurement number,

$\mathbf{A}_{(q,i)}$ the partial differential of the measurement with respect to i,

$\mathbf{A}_{(q,j)}$ the partial differential of the measurement with respect to j, and

$\mathbf{W}_{(q,q)}$ the weight of the measurement.

As an example, the value of all the coefficients in the upper triangle of matrix **N** affected by the qth measurement measured between stations j and k are computed using the following relationship:

The diagonal elements are

$$N_{(aa)} = N_{(aa)} + A_{(q,a)}\, A_{(q,a)}\, W_{(q,q)},$$

$$N_{(bb)} = N_{(bb)} + A_{(q,b)}\, A_{(q,b)}\, W_{(q,q)},$$

$$N_{(dd)} = N_{(dd)} + A_{(q,d)}\, A_{(q,d)}\, W_{(q,q)},$$

$$N_{(cc)} = N_{(cc)} + A_{(q,c)}\ A_{(q,c)}\ W_{(q,q)}.$$

and the off-diagonal elements are

$$N_{(ab)} = N_{(ab)} + A_{(q,a)}\ A_{(q,b)}\ W_{(q,q)},$$

$$N_{(cd)} = N_{(cd)} + A_{(q,c)}\ A_{(q,d)}\ W_{(q,q)},$$

$$N_{(ac)} = N_{(ac)} + A_{(q,a)}\ A_{(q,c)}\ W_{(q,q)},$$

$$N_{(ad)} = N_{(ad)} + A_{(q,a)}\ A_{(q,d)}\ W_{(q,q)},$$

$$N_{(bc)} = N_{(bc)} + A_{(q,b)}\ A_{q(q,c)}\ W_{(q,q)},$$

$$N_{(bd)} = N_{(bd)} + A_{(q,b)}\ A_{(q,d)}\ W_{(q,q)}.$$

4.2.6 Locations of coefficient in vector d

The right-hand side of the normal equation is a vector, denoted by **d**. It is a filled vector and its dimension is equal to the total number of unknowns in the network. The locations of element in vector **d** are directly related to the locations of unknowns. Thus the effected locations due to contributions of a measurement are the locations of unknowns associated with that measurement. A coordinate measurement at station i will have two elements at locations $\mathbf{d}_{(2i)}$ and $\mathbf{d}_{(2i-1)}$. For a measurement between two stations i and j, the effected locations in **d** are $\mathbf{d}_{(2i)}$, $\mathbf{d}_{(2i-1)}$, $\mathbf{d}_{(2j)}$ and $\mathbf{d}_{(2j-1)}$. For an angle observation observed at i, reference target j and final target k, the effected locations in **d** are $\mathbf{d}_{(2i)}$, $\mathbf{d}_{(2i-1)}$, $\mathbf{d}_{(2j)}$, $\mathbf{d}_{(2j-1)}$, $\mathbf{d}_{(2k)}$ and $\mathbf{d}_{(2k-1)}$.

As an example, a bearing measurement between stations j and k is concerned with locations $\mathbf{d}_{(a)}$, $\mathbf{d}_{(b)}$, $\mathbf{d}_{(c)}$ and $\mathbf{d}_{(d)}$, such as are depicted in Table 12.

The values of elements in vector **d** are obtained by applying equation (3.7) $\mathbf{d}=\mathbf{A}^T\mathbf{W}\mathbf{b}$. Using this equation a single location $\mathbf{d}_{(i)}$ is given by

$$\mathbf{d}(i)= \mathbf{d}(i) + \mathbf{A}(q,i) \times \mathbf{W}(q,q) \times \mathbf{b}(q) \qquad \text{eq (4.2)}$$

where i the parameters number and q is the measurement number,

A(q,i) the partial differential of the measurement with respect to i,

W(q,q) the weight of the measurement,

b(q) the (measurement - computed) value.

Table 12: Locations in vector **d**

d(a)
d(b)
.
.
d(c)
d(d)
.

Based on the eq(4.2), the four coefficient values in vector **d** effected by the qth measurement whose parameter unknowns are a, b, c and d are as given by:

$$\mathbf{d}_{(a)} = \mathbf{d}_{(a)} + \mathbf{A}_{(q,a)} \times \mathbf{W}_{(q,q)} \times \mathbf{b}_{(q)}$$

$$\mathbf{d}_{(b)} = \mathbf{d}_{(b)} + \mathbf{A}_{(q,b)} \times \mathbf{W}_{(q,q)} \times \mathbf{b}_{(q)}$$

$$\mathbf{d}_{(c)} = \mathbf{d}_{(c)} + \mathbf{A}_{(q,c)} \times \mathbf{W}_{(q,q)} \times \mathbf{b}_{(q)}$$

$$\mathbf{d}_{(d)} = \mathbf{d}_{(d)} + \mathbf{A}_{(q,d)} \times \mathbf{W}_{(q,q)} \times \mathbf{b}_{(q)}$$

4.3 Structure of elements in matrix N

From the previous sections, it was clear that the location of contributions of a measurement to matrix **N** follows a different pattern from that of matrix **A**. Thus, the storage scheme which was designed for the observation equation is not applicable for storing the normal equations. In matrix **A** , a measurement will occupy only a row, whereas in matrix **N**, a measurement will fill up several rows and several columns according to the procedure described in Section 4.2.2.

In matrix **A**, a contribution will occupy a unique location and this location will not receive any other contributions from any other measurement. This is in contrast to matrix **N** where a location cannot be reserved for a single contribution. A location may receive several overlapping contributions from several measurements. Since stations are points occupied or targeted during the acquisition of measurements, the frequency of visiting a point depends on the observations scheme. Thus, certain points may be occupied several times with several types of measurements. Similarly, some points may be targeted several times during the data acquisition. This causes

overlapping of contributions over the locations in matrix **N** from several types of measurements.

To solve the above problems, a technique is therefore developed in this thesis to identify, organize and arrange the nonzero locations row-by-row in ascending order which existed in the upper triangular of matrix **N**, and store them in a vector called coefficient vector **N**. The method introduces a mathematical function relating the locations in the vector **n** to any measurements before the contributions of the measurements are passed into the predefined locations in the vector.

The basic tool behind the above steps is graph theory. The following sections introduce some terminology of the graph theory and its relationship to sparse matrix problems. A sparse matrix can be thought of as a matrix at all, but rather as a graph (Pissanetsky, 1984). This is why graph theory plays such an important role in problems which involve sparse matrix such as the cadastral least squares computation.

4.4 Introduction to graph theory

There has been a wide divergence in graph terminology (Harary, 1969). The terminology presented in this thesis is a selection of what is considered to be relevant to cadastral applications. Thus, the introduction of the graph theory is

directed towards handling sparse systems of equations found in cadastral application. For this purpose, the relevant principles of the graph theory and the cadastral surveying are established.

A graph G is a collection of points or vertices $x_1,x_2,.....x_n$ (denoted by the set X), and a collection of edges $e_1,e_2,.... e_m$ (denoted by the set E) joining all or some of these points. Synonyms for 'vertex' are point, node, junction, and station; while synonyms for edges are line, arc, branch, link, and measurement. The graph G is then fully described and denoted by the doublet G(X,E). We say edge $e_1(_{i,j})$ is connected between the vertices i and j, and that $(_{i,j})$ is incident with the vertex i and j or conversely that i and j are incident with $(_{i,j})$.

The graph is conveniently represented by a geometric diagram in which the vertices are indicated by small circles or dots, while any two of them, i and j, are joined by a continuous curve, or even a straight line, between i and j if and only if (i,j) is in E. As an illustration, consider the graph G(X,E) in which

$$X = \{ x_1, x_2 , x_3, x_4 \text{ and } x_5\}$$

$$E = \{ e_1,e_2,e_3,e_4, e_5,e_6,e_7,e_8,e_9 \text{ and } e_{10}\}$$

If the edges in E have a direction which is usually shown by an arrow, then the resulting graph is called a directed graph (see Figure 8). If the edges have no directions, then the graph is called nondirected or undirected graph (see Figure 9).

When an edge is denoted by the pair of its initial and final vertices (i.e. by its two terminal vertices), its direction will assume to be from the first vertex to the second. Thus, in Figure 8, (x_2,x_1) refers to arc e_1, and (x_2, x_4) to arc e_9.

Edges with a common terminal vertex are called adjacent edges. Also, two vertices xi and xj are called adjacent if either edge (xi,xj) or edge(xj,xi) or both exist in the graph. Thus in Figure 9 edges e1,e9,e10,e3 and e4 are adjacent and so are the vertices x1 and x2, x4 and x5 ; on the other hand edges e5 and e8 or vertices x1 and x3 are not adjacent. A loop is an arc whose initial and final vertices are the same, for example edge e2 form a loop (see Figure 8).

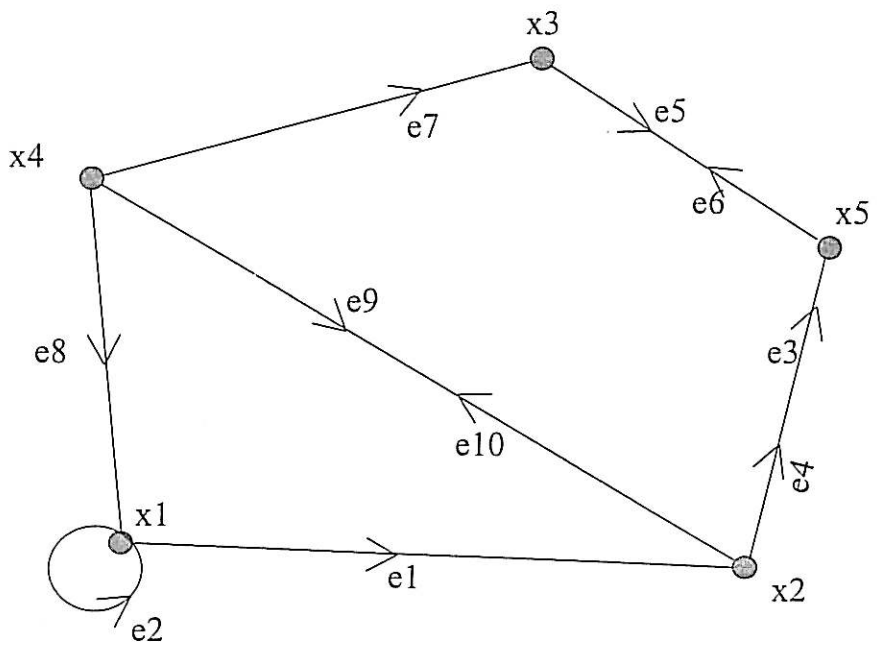


Figure 8: A directed graph.

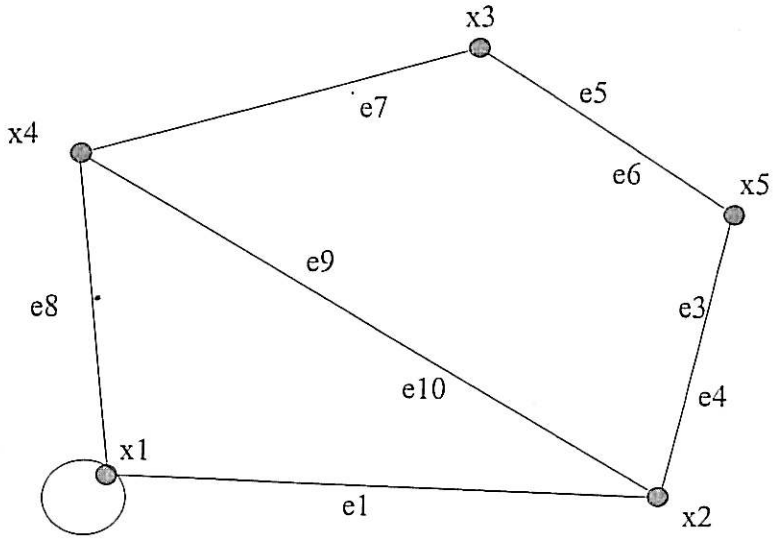


Figure 9: An undirected graph

The number of edges which have a vertex x_i as their initial vertex is called the outdegree of vertex x_i, and similarly the number of edges which have x_i as their final vertex is called the indegree of vertex x. Thus referring to Figure 8 the outdegree of x4, denoted by $do_{(x4)}=3$, and the indegree of x5 denoted by $dt_{(x5)}=3$. It is obvious that the sum of the outdegrees or indegrees of all the vertices in a directed graph is equal to the total number of edges of G. For an undirected graph, the degree of a vertex x_i is defined by the number of edges connected to vertex x_i either as their initial vertex or as their final vertex.

Graph theory also includes the existence of several distinct edges for a given pair of nodes. These edges are indicated by the symbols $(j,k)_1,(j,k)_2.....(j,k)_i$ and they are called parallel edges of G. In Figure 9 edges e5 and e6 are parallel edges.

A graph is said to be symmetric if, for every edge (j,k) of G, it is matched by an edge (k,j) of G. This would imply that, in G, if there are m edges directed from j to k then there are also m edges directed from k to j.

The above concepts and terminology of graph theory is considered as sufficient for use in the creation of coefficient matrix of the normal equations. The following sections concerns with the application of the graph theory in cadastral network computation.

4.4.1 Graph and matrices

Matrix is a convenient way of representing a graph algebraically (Harary, 1969). This is because computers are more efficient at manipulating numbers than at recognizing pictures. Thus, it is a standard practice to communicate the specification of a graph to a computer in matrix form. The use of matrices provides a far more efficient way of representing a large or complicated graph than a pictorial representation.

There are a number of matrices which can be associated with a graph. The most relevant matrix for this research is the adjacency matrix, denoted by matrix **G**. The matrix **G** is a vertex to vertex incident matrix, in which all pairs of adjacent stations are recorded. Thus, the elements of the matrix show the pairs of the vertices

that are adjacent. In order to distinguish the incidented locations from the non-incidented locations, the elements of incidented locations are denoted by a number, say 1, or else they are denoted by zero. Thus for example G(ij)=1, if vertex x_i is adjacent to vertex x_j in **G**, or else G(ij)=0, i.e. vertex x_i and x_j are not adjacent.

Table 13: Adjacency matrix **G** of the graph as shown in Figure 9

	x1	x2	x3	x4	x5
x1	0	1	0	1	0
x2	1	0	0	1	1
x3	0	0	0	1	1
x4	1	1	1	0	0
x5	0	1	1	0	0

4.4.2 Cadastral network and the graph

Since the objective of introducing graph theory is to facilitate the study of sparse matrices found in cadastral network, the relationship between graphs and cadastral networks is now established. Lets begin by defining cadastral network in terms of the graph terminology.

Vertices and edges of a graph are respectively equivalent to stations and measurements in a cadastral network. Thus, when an edge e_k is connected between the vertices i and j, it represents the k^{th} measurement between station i and station j. When an edge is shown with a direction, it describes a measurement and its direction of observation. Thus e_i with a direction reveals that station i was occupied by the instrument whereas the station j becomes the target during the observation. The structure of edges within a graph represents a measurement scheme in a cadastral network.

The term outdegree of vertex x_i refers to the number of stations which had been used as targets from the instrument station x_i. On the other hand, indegree of vertex x_i represents the number of instrument stations which use station xi as one of their targets. The degree of a vertex xi represents the number of stations that are connected to station xi (i.e., either x_i used as an instrument station or as a target from other stations). This feature is particularly important in cadastral because the elements of a row of coefficient matrix **N** corresponds to the degree of vertices.

The concept of parallel edges in a graph corresponds to a line measured by more than one type of measurement. For example, two types of measurements such as distance and bearing are measured between a pair of stations. A loop at a vertex x_i in a graph indicates that station x_i has coordinate measurement.

It is clear that the geometric figure of a graph represents a cadastral network. Referring to the undirected graph in Figure 9, its equivalent measurement scheme are presented in Table 14.

Table 14: Measurement scheme of the graph in Figure 9

No.	FROM STN Initial Vertex	TO STATION final Vertex	MEASURE- MENTS	EDGES
1	1	2	bearing	e1
2	1		coordinate	e2
3	2	5	distance	e3
4	2	5	bearing	e4
5	3	5	distance	e5
6	5	3	bearing	e6
7	4	3	distance	e7
8	4	1	bearing	e8
9	4	2	distance	e9
10	2	4	bearing	e10

4.4.3 Adjacency matrix of a cadastral network

The adjacency matrix of a cadastral network is a station to station incident matrix. A station is said to be incidented with another station when a measurement exists between them. In other words, the connections between stations within a network are recorded in the adjacency matrix. Therefore, the elements of adjacency matrix represent the connectivity structure of the network. The incidented location is denoted by a number, say 1, or else they are denoted by zero. Thus for example, G(ij)=1 if station x_i is linked by a measurement to station x_j, or else **G**(ij) =0, (i.e., no measurement exists between station x_i and station x_j).

The adjacency matrix of a graph is equivalent to the adjacency matrix of a cadastral network, except for the status of the diagonal elements. Diagonal elements are denoted by zero values in adjacency matrix of a graph whereas these elements are denoted by number 1 in the cadastral adjacency matrix **G**. Therefore, all diagonal elements are assigned as number 1(i.e. G(ii)=1, for all i), instead of zero as in the case of the standard adjacency matrix of a graph. This is because the diagonal elements are nonzero terms in a cadastral application. The corresponding adjacency matrix for the measurements in Table 14 is shown in Table 15.

In a cadastral network, each row of the adjacency matrix represents a station and the total number of elements within the row is the degree of the station. The elements in a row represent the connections that exist to or from the station. Each element represents a connection between two stations. Therefore, as an example, in

Table 15 the first row shows all the connections that are linked to station 1. From this station there exists two connections, i.e., stations 2 and 4 as indicated by G(1,2) and G(1,4) that is equal to 1. Similarly, the station 2 is connected to stations 1,4 and 5 as indicated by G(2,1)=G(2,4)=G(2,5)=1. Thus, the matrix provides the indication whether or not certain edges (i.e. measurements) between stations are present or not. This information provides the basic data for computing the nonzero locations.

Table 15: The adjacency matrix **G** of the network as in Table 14

	Stn x1	Stn x2	Stn x3	Stn x4	Stn x5
Stn x1	1	1	0	1	0
Stn x2	1	1	0	1	1
Stn x3	0	0	1	1	1
Stn x4	1	1	1	1	0
Stn x5	0	1	1	0	1

4.5 Adjacency number

The adjacency matrix describes the structure of the measurement scheme however, the use of 0 or 1 for presenting its elements is inefficient in subsequent matrix manipulations. A new system of numbering the elements is hereby proposed.

The adjacency matrix is more meaningful if its nonzero elements are arranged in a systematic manner. The numbering scheme that is suitable for presenting matrix N is an ascending order of row-by-row. Such a scheme is referred to as adjacency number and for example, the adjacency numbers of the measurements as shown in Table 14 can be tabulated as in Table 16.

The adjacency number is a sequence of connections of stations. Since stations are tied by measurements, each measurement can be described in terms of adjacency number. The relationship between an adjacency number and a measurement can be described as follows:

For a measurement at a single station, i, such as a coordinate measurement, the adjacency number is at G(i,i). For a measurement between station i and station j, such as bearing and distance, there are three adjacency numbers (i.e. two adjacency numbers at the diagonal and 1 adjacency number at the off diagonal location). Their respective locations are G(i,i), G(i,j) and G(j,j).

Table 16: Adjacency number of an adjacency matrix G

	x1	x2	x3	x4	x5
x1	1	2		3	
x2		4		5	6
x3			7	8	9
x4				10	
x5					11

For a measurement which involves three stations (i,j,k) such as angles, there are six adjacency numbers (i.e. three adjacency-number at diagonal and three-adjacency number at the off-diagonal locations). These locations are G(i,i), G(j,j), G(k,k), G(i,j), G(i,k) and G(j,k).

Considering the measurements shown in Table 14, the adjacency number for the coordinate measurement at station 5 is 11 (i.e., G(5,5)=11). The three adjacency numbers for any measurement between stations 2 and 4 are G(2,2)=4, G(2,4)=5 and G(4,4)=10. The six adjacency numbers for an angle measured at 1, reference target 2 and final target 4 are at G(1,1)=1, G(2,2)=4, G(4,4)=10, G(1,2)=2, G(1,4)=3 and G(2,4)=5.

4.6 Computing locations of coefficients in vector n

As mentioned earlier in the introduction, the aim of the method developed in this chapter is to identify all locations of nonzero coefficients situated in the upper triangle of the symmetric matrix **N** and arranging these locations in a systematic linear list in the coefficient vector **n** . The adjacency numbers associated with every measurement provide the key information for computing the designated locations in vector **n**.

The relationship between vector **n** and the measurement is then described as follows. Every adjacency number generates four locations in the coefficients vector **n**. However, for diagonal adjacency numbers only three locations are used. They are:

G(i,i) for all i has 3 elements (diagonal adjacency number)

G(i,j) $i \neq j$ has 4 elements (off-diagonal adjacency number).

A diagonal adjacency number generates three locations because one of its elements falls below the diagonal. Note that only the elements in the upper triangle are required to be stored. For a coordinate measurement at station ' i ' whose adjacency number is at G(i,i), there are three nonzero terms. For a measurement between stations *i* and *j* whose adjacency numbers are at G(i,i), G(j,j) and G(i,j), there are 10 nonzero terms (i.e 3+3+4 elements). For an angle measured at station *i*, reference target station *j* and final target *k* whose adjacency numbers are at G(i,i), G(j,j), G(k,k), G(i,j), G(j,k) and G(i,k) will produce 21 nonzero terms(i.e., 3+3+3+4+4+4 elements).

The relationship between the locations in vector **n** and the adjacency numbers has therefore been established. A location in vector **n** is a function of the three adjacency numbers in a row. The derived mathematical function is described as follows. Let q_1, q_2, q_3, and q_4, be the four locations in coefficient vector **n** derived from an adjacency number (say the Q^{th} for row i and column j) and these locations are tabulated as in Table 17.

Table 17: Locations generated by an adjacency number.

	Station i		Station j	
	easting	northing	easting	northing
easting x I	.	.	q_1	q_2
northing x I		.	q_3	q_4

The mathematical functions relating Q^{th} and q_1, q_2, q_3 and q_4 were developed using empirical method:

$$q_4 = 2 \times (Q_{last} + Q^{th}) - i$$

$$q_2 = q_4 - \{ 2 (Q_{last} - Q_{diag}) + 1 \} \qquad \text{eq 4.3}$$

$$q_1 = q_2 - 1$$

$$q_3 = q_4 - 1$$

where Q_{last} is the last diagonal adjacency number for row i,

Q_{diag} is the diagonal adjacency number for row i.

As an example, one of the adjacency number for a measurement between stations 2 and 4 is G(2,4)=5. The adjacency number 5 is in row 2 and for this row $Q_{diag}=4$, $Q_{last}=6$, the locations in vector **n** are therefore:

$q_4 = 2 \times (6 + 5) - 2 = 20$

$q_2 = 20 - \{2(6-4) +1\} = 15$

$q_3 = 15-1 = 14$

$q_1 = 20-1 = 19$

Equation (4.3) is the mathematical function that relates an adjacency number to their corresponding locations in coefficient vector **n**. By using the equation (4.3) each measurement can update their respective locations in the coefficient vector **n**, without having to create matrix **N**. Equation (4.3) is therefore a key tool for the optimization of construction of vector **N**. When all the measurements within a network complete their updating process, the coefficient vector which represents the matrix **N** is completed.

Table 18: The arrangement of location of non-zero elements in N

	stn 1		stn2		stn3		stn4		stn5	
	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4			5	6		
2		7	8	9			10	11		
3			**12**	**13**			**14**	**15**	16	17
4				**18**			**19**	**20**	21	22
5					23	24	25	26	27	28
6						29	30	31	32	33
7							**34**	**35**		
8								**36**		
9									37	38
10										39

Note that the locations are arranged as a simple list without any zero terms within the list. Considering the observation scheme in Table 14 the resulting location of matrix **N** is as shown in Table 18. For example, a measurement between stations 2 and 4 whose adjacency numbers are 4, 5 and 10, their respective elements to be updated in vector **n** are printed in bold. These locations are 12, 13, 14, 15, 18, 19, 20, 34, 35, and 36.

4.7 Linking coefficients with their true locations in matrix N

Since matrix **N** is now represented by the coefficient vector **n**, the elements within the vector must be linked to their true locations in the original matrix **N**. This is done by introducing an indexing scheme. The indexing scheme developed in this research is made up of two integer vectors:

(a) **ICOL** containing column numbers for each element, and

(b) **IEND** containing the diagonal coefficient numbers for each row.

The length of **ICOL** equals to the total number of elements, whereas the length of **IEND** is equals to the number unknowns. Using the example in Table 18, the 19 th element in vector **n** is in the 7th column and the diagonal coefficient number in row 4 is 18. Therefore for **n**(19), its indexing information are **ICOL** (19)=7 and **IEND** (4)=18. The vector **n** and their corresponding indexing information are listed in Table 19.

4.8 Computing coefficients values

Based on the general equation (4.3), and the indexing information the coefficient values of the vector **n** can be directly computed by:

$$\mathbf{n}_i = \mathbf{n}_i + A_a \times A_b \times W_q \qquad \text{eq (4.4)}$$

where $\mathbf{n}_i$ is a computed location in vector **n**, i is obtained using eq (4.3)

A_a is the partial differential of the measurement with respect to easting *j*

A_b is the partial differential of the measurement with respect to northing *j*

W*q* is the weight of the measurement.

To illustrate the application of equation (4.4), consider a measurement, says the ninth measurement, which was measured between stations 2 and 4 and has three adjacency numbers 4, 5 and 10. Using these adjacency numbers, the locations in vector **n** can be computed and the locations are found to be 12,13,14,15,18,19,20,34,35,and 36 which are shown in bold in Table 18. Using the equation (4.4), the coefficient values are computed as:

$$\mathbf{n}_{12} = \mathbf{n}_{12} + A_3\ A_3\ W_9$$

$$\mathbf{n}_{13} = \mathbf{n}_{13} + A_3\ A_4\ W_9$$

$$\mathbf{n}_{14} = \mathbf{n}_{14} + A_3\ A_7\ W_9$$

$$\mathbf{n}_{15} = \mathbf{n}_{15} + A_3\ A_8\ W_9$$

$$\mathbf{n}_{18} = \mathbf{n}_{18} + A_4\ A_4\ W_9$$

$$\mathbf{n}_{19} = \mathbf{n}_{19} + A_4\ A_7\ W_9$$

$$\mathbf{n}_{20} = \mathbf{n}_{20} + A_4\ A_8\ W_9$$

$$\mathbf{n}_{34} = \mathbf{n}_{34} + A_7\ A_7\ W_9$$

$$\mathbf{n}_{35} = \mathbf{n}_{35} + A_7\ A_8\ W_9$$

$$\mathbf{n}_{36} = \mathbf{n}_{36} + A_8\ A_8\ W_9$$

The coefficient values are updated directly at the predetermined locations without the need to construct and store the three matrices **N, A** and **W**. The related elements within those matrices are directly identified and used to compute the coefficient in the predefined location in vector **n**.

4.9 Number of elements in vector N

It is clear that the number of elements in the coefficient vector **n** depends on the total connections in a network which is given by the adjacency number in matrix **G**. Since each diagonal adjacency number generates four locations except for the off-diagonal adjacency number which generates three locations, the total elements in the coefficient vector **n** is therefore:

$$N_t = 4 \times \mathbf{G}_{last} \text{ - total stations,} \qquad \text{eq(4.5)}$$

where

N_t the total number of terms in vector **n**,

$\mathbf{G}_{last}$ the last connectivity number, and

Table 19: Vector N and its corresponding indexing vectors

Number	Coefficient of matrix N	Column Number ICOL	Starting row number IEND
1	n(1)	1	1
2	n(2)	2	7
3	n(3)	3	12
4	n(4)	4	16
5	n(5)	7	19
6	n(6)	8	23
7	.	.	
.	.	.	
.	.	.	
25	n(25)	7	
26	n(26)	8	
27	n(27)	9	
.	.	.	
.	.	.	
34	n(34)	7	
35	n(35)	8	
36	n(36)	8	
37	n(37)	9	
38	n(38)	10	
39	n(39)	10	

4.10 The CGM algorithm for solving normal equations

The CGM algorithm for solving normal equations, $\mathbf{Nx} = \mathbf{d}$ is a straight forward solution of the system of linear equations (Bhirud, 1975; Press et. al 1987).

1. Let $\mathbf{p}=\mathbf{r}=\mathbf{d}$
2. Change in $\mathbf{x} = e.\mathbf{p}$ and change in $\mathbf{r} = e.\mathbf{q}$

   where $\mathbf{q}=\mathbf{N}.\mathbf{p}$

   $c=\mathbf{p}^T.\mathbf{r}$

   $d=\mathbf{p}^T.\mathbf{q}$

   $e=c/d$
3. Change in $\mathbf{p} = g.\mathbf{p}$

   where $f=\mathbf{r}^T.\mathbf{q}$

   $g=-f/d$

Repeat step 2 to 3 until the magnitude of the changes in **x** and **r** are insignificant.

Based on the above CGM method, the step-by-step algorithms suitable for computer implementation has been developed (Table 20). The algorithm is simpler and shorter compared to the CGM version for solving observation equations. There is only one major operation (i.e., step 2), among the eleven steps. This step involves multiplication of the matrix **N** to a vector. However, since matrix **N** is replaced by vector **n** the multiplication process is reduced drastically.

Table 20: CGM algorithm for solving normal equations

No	ALGORITHM	REMARK	MATRIX OPERATION.
1	Let p=r=b and x=0	Initialize vectors	
2	q = N. p	Begin iteration	Matrix by vector
3	$\mathbf{c}=\sum_{1}^{n}\mathbf{p}_j.\mathbf{r}_j$		Vector by vector
4	$\mathbf{d}=\sum_{1}^{n}\mathbf{p}_j.\mathbf{q}_j$		vector by vector
5	$e=\frac{\mathbf{c}}{\mathbf{d}}$		scalar
6	x = x + e.p	Update parameter x	scalar by vector
7	r = r - e.q	Update vector r	scalar by vector
8	$f=\sum_{1}^{n}\mathbf{r}_j.\mathbf{q}_j$		vector by vector
9	$g=-\frac{f}{d}$		scalar
10	p = r + g.p	Update vector p	scalar by vector
11	repeat step 2 to 10 until p= 0		

4.11 Summary

The method of setting up the normal equations developed in this chapter has achieved the goal to identify the nonzero terms within upper triangle of the

coefficient matrix **N** and arranging these terms in a vector **n**. A mathematical function is developed to link the locations in vector **n** to the measurements.

The summary of the method used for constructing the normal equations are:

(a) recovering the connectivity of stations in the observation scheme,

(b) arranging the connectivity of stations in ascending order of row-by-row section,

(c) converting the connectivity numbers into locations numbers in vector **n**,

(d) linking the elements in the vector **n** to their original locations in matrix **N**, and

(e) computing the predetermined locations in the vector **n** for each measurement.

The CGM will therefore solves normal equations for least squares parameters and residuals. The algorithm of the method is shorter and simpler as compared to its version for solving observation equations. The CGM uses vector **n** without altering the structure of the elements in vector **n** throughout the whole least squares estimation process.This means that the operations do not require new elements to be allocated to the vector. The vector is also free from complicated fill-in effect. Therefore, the method of setting up the normal equations developed in this chapter has achieved three leading advantages in sparse matrix technology, i.e., storing only the minimum nonzero terms, operating only on the stored nonzero terms and always preserving the sparsity pattern.

CHAPTER V

DEVELOPING CGM COMPUTER PROGRAM

5.1 Introduction

The importance of having up-to-date and accurate cadastral information in GIS has been clearly emphasized in Chapter I. The best way to obtain such a quality information is to process raw cadastral measurements using the simultaneous rigorous least squares method. The outputs of the solution are the coordinates of the cadastral points which are needed in variety of applications, such in GIS.

Solving a problem using a computer requires that certain procedures to be strictly followed. It should be noted at the outset that despite the apparent complexity and power of the computer, it is merely a tool which requires systematic instructions in order for it to solve a problem. Thus, solving a problem using a computer requires a great deal of planning especially in building up the instructions. The process of

developing these instructions can be painstaking at times, but there are steps that can be adopted in order to minimize the problems that may arise. These steps include:

1. defining the problem,
2. developing a methodology for solving the problem,
3. designing the computer program,
4. writing the computer program,
5. submitting the program for compilation, and locating and correcting any errors the compiler detects, and
6. testing the program with sample data, and locating and correcting any remaining errors.

The first two steps are independent of the computer system and have been described in detailed in Chapter 1 and 2. This chapter, however, deals with the remaining steps to translate the methodology described in Chapters 3 and 4 into computer programs.

The most suitable and widely used language for engineering computation and also applicable to cadastral computation problem is FORTRAN. Therefore, this language is selected for implementing the CGM program. This chapter describes the development of the program from design stage to writing up the program. This chapter describes the operations involving sparse matrices of the observation and normal equations. The test and evaluation of the programs are described in Chapter 6.

5.2 Program design methodology

Programming is a process of 'instructing ' a computer what to do. The product of programming is a source code. Before the source code is written, one must decide on the design of the program. The design of a computer program is a process of structuring and desgning the algorithm of the program to such a level that the coding or the writing of the program can proceed without further description.

Before the design of the cadastral program is presented, a discussion on the concept and approach in designing a computer program proceeds. The general plan of design is called *top-down development* or *successive refinement*. One tries to decompose a problem into a simple sequence of sub-problems in order of increasing the details. Breaking the problem into such manageable sub-units at this point greatly simplifies the program-coding task that must be performed later.

The basic concept for implementing a program design is the systematic transformation of a statement to a detailed specification of action in FORTRAN. The basic steps of a top-down design are as follows:

1. break a problem into a sequence of sub-problems,
2. refine a statement into several finer statements with increasing details,
3. expand a statement of what has to be done into a specification of how it is to be done,
4. expand the high level commands such as "solve","find" or "compute" into lower-level statements of a programming language, and

5. translate a problem description into FORTRAN statements.

The highest-level module is the main control modules. These modules outline the major structure and the basic algorithm and it is further broken down into lower-level modules. Depending on the complexity of the problem, several levels of modules may be required. The lowest-level modules contain the greatest level of detail or the most refined algorithm. A module performs a specific function, and the combinations of several independent modules create a program which performs the required task.

The communication between the modules is of fundamental importance. For correct flow of communication the data in each module must be defined precisely. Thus in subdividing a problem into modules, the input and output of information within a module and passing of information between modules are the primary considerations. Depending on the degree of data and common coupling, the communication of data can be nontrivial operations and can cause a major problems in program designing.

During the process of designing a program errors may occur anywhere in any module. The errors of a particular module are apparent in the next higher modules. At this point, the structure and the algorithm of the particular module must be reexamined and corrected. If the error is still apparent, the next higher level modules or the previous module may have to be redesigned accordingly. This process is known as a stepwise refinement. This is a practical feed back mechanism

in the design which allows the design scheme to be refined gradually as it becomes more and more to completeness.

There are a number of visual aids commonly used to present the designed structure and algorithm. These visual aids include:

(1) structure chart,

(2) HIPO chart,

(3) flowchart, and

(4) pseudo code.

The structure chart is a hierarchical, or a tree-like diagram. It shows functions, their relationships and flow of control. HIPO is the acronym for Hierarchy plus Input-Process-Output, whereas structure chart emphasis only on the structure and functions. HIPO diagrams shows the inputs and outputs of program modules.

A flowchart is a means for describing, ordering, and expressing a solution in graphical form. The flowchart is most suitable for describing lower-level modules which contain the greatest level of details. At times, flowcharts may become lengthy and difficult to read, especially those for complex diagrams. In some cases, it is also difficult to express the logic of processing steps within the commonly used flowchart symbols. In these cases, a flowchart can be replaced by a pseudo code. A Pseudo code is an English-like description of the processing steps in a program and it is mainly used method for expressing program logic and algorithm.

5.3 Design of cadastral adjustment program.

The above design methodology is applied in the cadastral computation program. The design begins with the creation of five main control modules, i.e., INITIALIZE, READ, COMPUTE, WRITE and TERMINATE. The subdivision of the problem into these modules seems trivial, but their task and process have to be clearly defined. It should be clear that each module must perform three basic functions:

1. receive the input data,
2. process the input data, and
3. release the output.

The outline of the main control modules of the cadastral computation program can be summarized in Table 21. The design is depicted graphically using a combination of structure chart and HIPO chart. Although the program is divided into five main modules, the amount of processing task in each module varies according to nature and purpose of a program. For the cadastral computation program, which indeed specializes in data processing program, the largest and the most important module is the COMPUTE module.

Other minor modules (INITIALIZE, READ, WRITE, TERMINATE), are relatively small as compared to COMPUTE module. These modules can be easily designed by familiarizing with the FORTRAN language and writing down the necessary statements.

The INITIALIZE module initializes the necessary memory locations for the program. The READ module opens previously created measurement data file. The file contains the cadastral data input which includes measurements and their respective standard errors, station numbers and provisional coordinates of the stations.

Table 21: Main modules of a least squares computer program

MODULE	INPUT	PROCESSING	OUTPUT
INITIALIZE	load program	initialize vectors for receiving input data, processing data and output parameters.	vectors has been created (a) measurements,related stations, weights, (b) vectors for storing linear equations, (c) vectors for output parameters.
READ	open mea-surements file	(a) read stations, (b)read coordinates, (c)read measurements and related stations (d)standard deviations.	(a)station numbers and coordinates are in memory, (b)measurements and their stations are in memory.
COMPUTE	reorder stations and coordinates	(a) compute the coefficients of each measurement, (b) Indexing the coefficients, locations in a vector, (c) build linear equation, (d) Solve the equation.	(a) The solution is repeated until the corrections to parameters insignificant. (b) Obtain the parameters and the residuals of measurements.
WRITE	reorder the parameters	formatting the parameters according to users requirement.	present results.
TERMINATE	decide to stop or continue	terminate program.	work ended.

The program allows random network station numbering so the program has to re-order the stations according to the new number scheme which is used throughout the computation process. The WRITE module writes the output parameters and TERMINATE module simply stops the running of the program, when all the necessary criteria are satisfied.

5.4 Design of COMPUTE module for observation equations

The COMPUTE module for creating observation equations consists of two sub-modules:

1. create observation equations, and
2. solve observation equations.

The statement "creates observation equation" is itself a module which obviously requires further refinements. Each type of measurements is read and processed one by one to generate the observation equations. Figure 10 is a flowchart of the COMPUTE module and the refinement of the module for the distance measurement is given in Figure 11.

The steps outlined in Figure 11 are sufficiently detailed, in that it requires no further definition, except for the PROCESS 1,2 ,3 and 4. The four processes, one for each type of measurements (i.e., coordinate, distance, bearing and angle) need

further definition. These are the processes that actually build up the observation equations **Ax=b**. It computes the coefficient values of each element at precomputed locations (i.e., each jth element) in matrix A. Each jth element is linked to its true locations in matrix A by indexing information in IROW and ICOL, as described in section 3.4. It also computes the right-hand side of the equation.

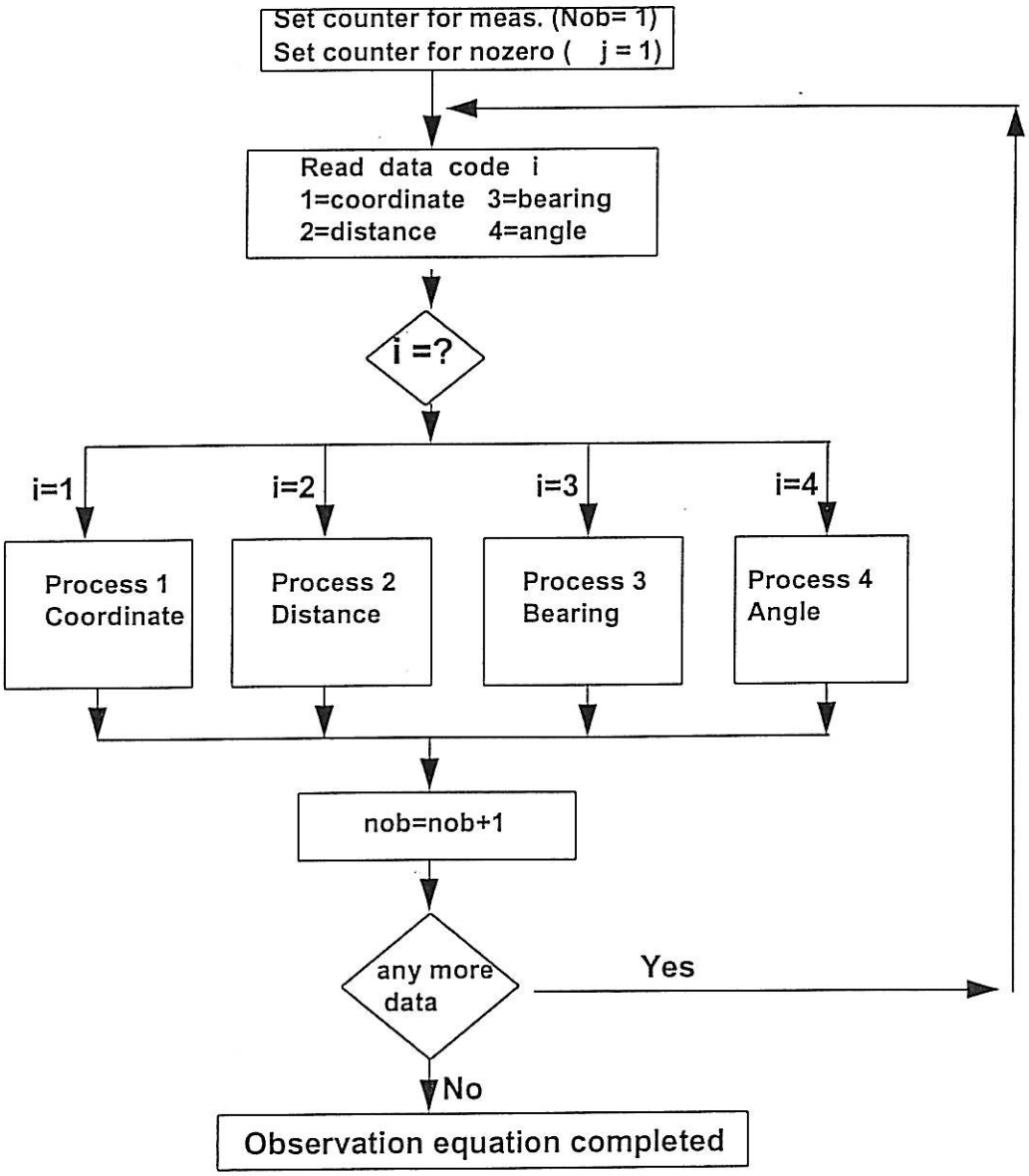
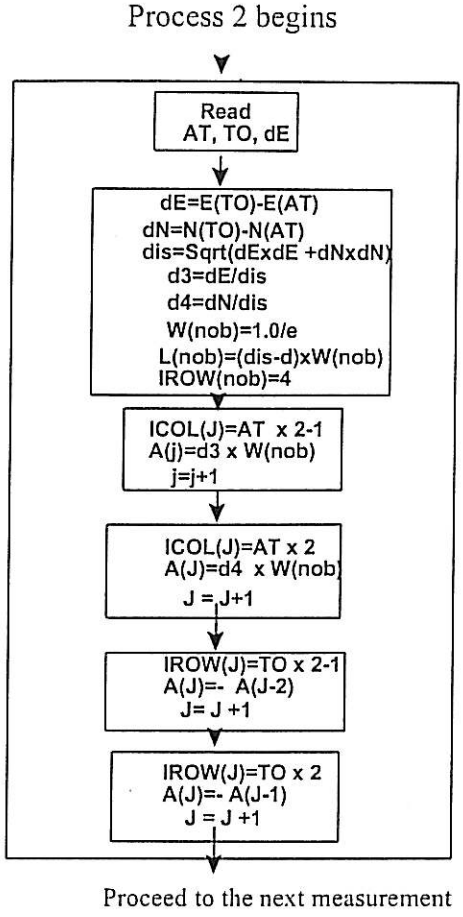


Figure 10: Flowchart showing the creation of the observation equations



INPUT ARGUMENTS		OUTPUT ARGUMENTS
D = Measurements		W= Weight
dE = Standard error		L = Computed - Observed
AT = Occupied station		IO = Number of elements for a row
TO = Target station		IB = Column number for each coefficient
N = Prov. Northing (from Figure 10)		A = Coefficient values
E = Prov. Easting (from Figure 10)		
NOB = Counter for meas. (From Figure 10)		

Figure 11: Contribution of a distance to observation equations

The flowchart for PROCESS 2 manipulates distance measurements, and is given in Figure 11. The flowchart for the other measurements is not shown as they are basically the same except for the functions for partial differentials.

5.4.1 Flowchart of the CGM for solving Ax=b

The solution of equation **Ax=b** using the CGM algorithm has been described in Section 3.6. The flowchart of the algorithm is depicted in Figure 12.

The algorithm consists of several standard vector multiplications, except for operations involving vector **A** (i.e., $R= \mathbf{A}^T v$ and BH= **A** h) which require further definition. This is because vector **A** is not a standard, filled vector, but it is the optimized vector that contains only the nonzero terms of the original matrix **A,** and has been described in details in Chapter 3. The corresponding flowcharts are modified versions of vector multiplication $\mathbf{R= A^T v}$ (Hintz, 1994) as shown in Figure 13, and the manipulation of **BH= A h** is shown in Figure 14.

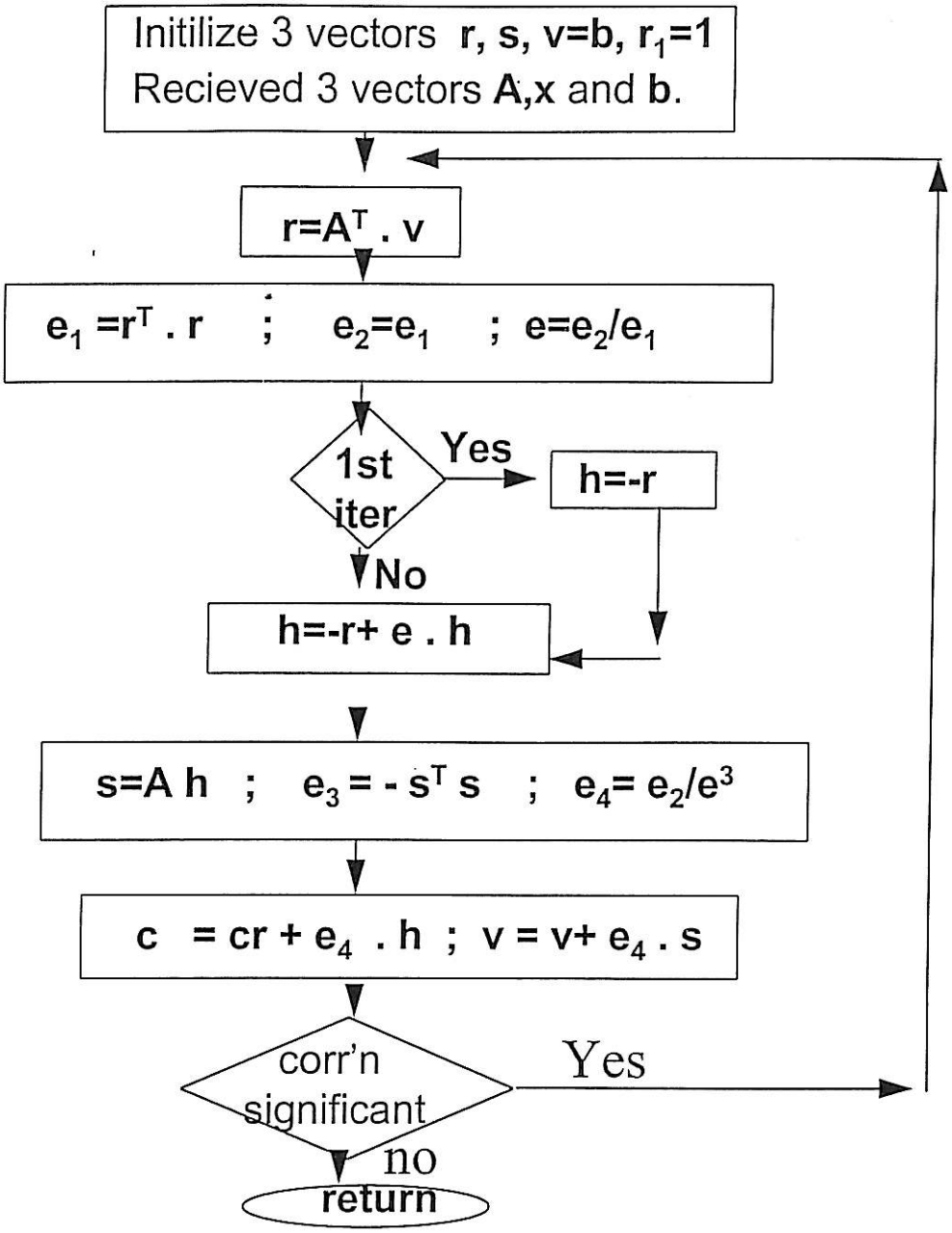


Figure 12: Flowchart of CGM steps for solving observation equations.

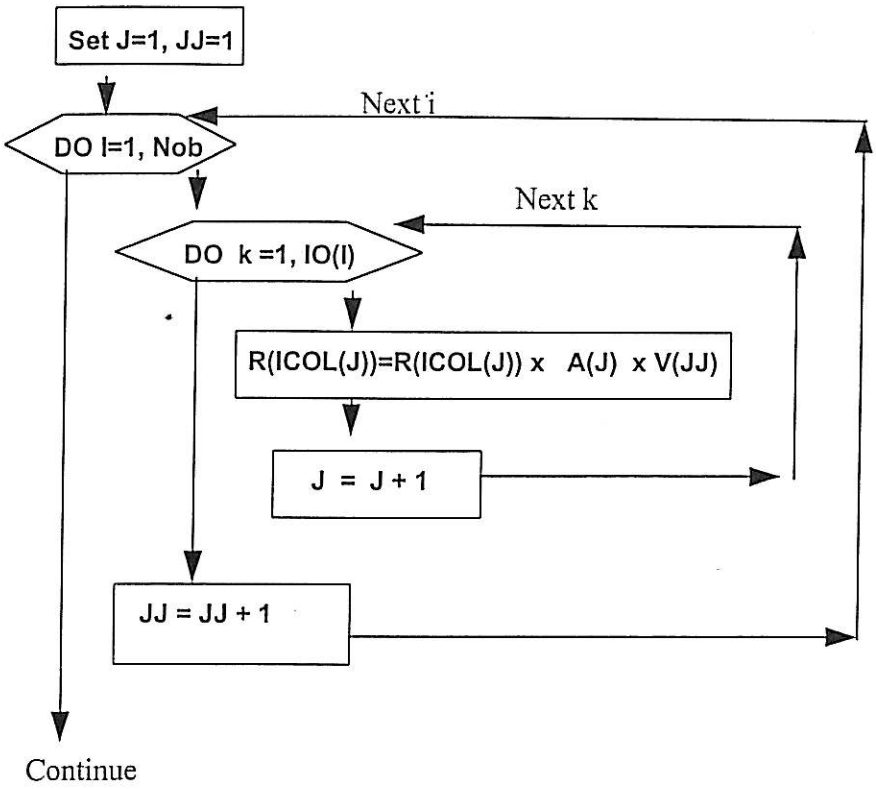


Figure 13: Flowchart for computing $R=A^T v$

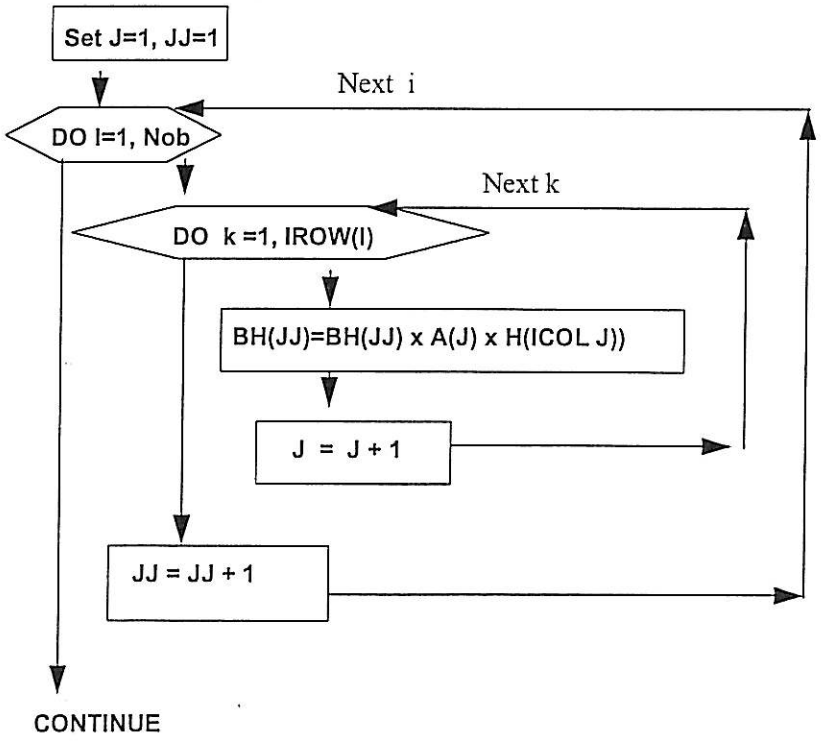


Figure 14: Flowchart for computing BH= **A h**

n. The elements in vector **n** are then linked to their true locations in matrix **N** through the indexing information in IROW and ICOL.

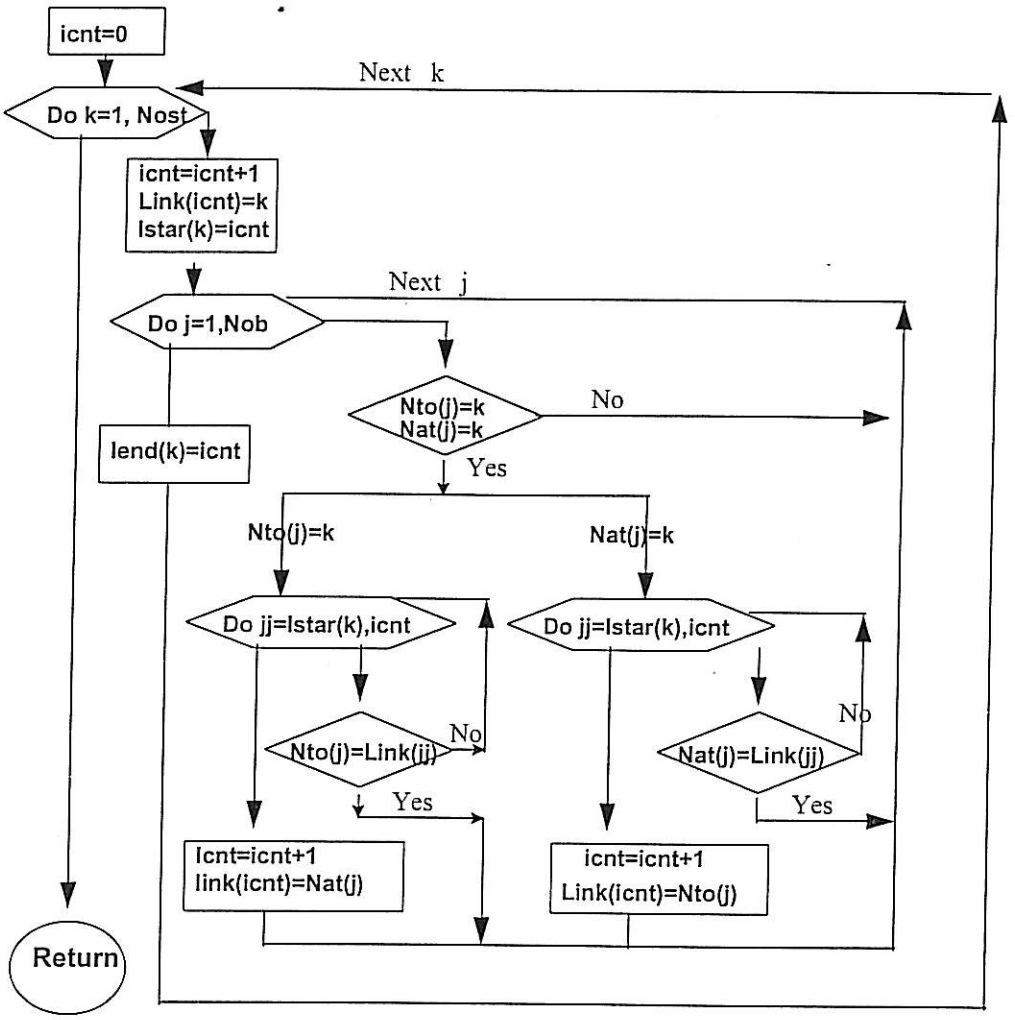


Figure 15: The creation of an adjacency matrix

The control steps for the COMPUTE module are similar to the steps listed in Figure 10. The module consists of four processes (i.e., PROCESS 1,2 ,3 and 4) which require further definitions. The flowchart for PROCESS 2, i.e., the process for distance is given in Figures 16 and 17. The flowchart for other type of

measurements is not shown as they are basically the same, except for the functions for partial differentials.

INPUT ARGUMENTS		OUTPUT ARGUMENTS
D = Measurements		W= Weight
dE = Standard error		L = Computed - Observed
AT = Occupied station		IROW = Number of element for a row
TO = Target station.		ICOL = Column number for each coefficient
N = Provisional Northing		A = Partial differential
E = Provisional Easting		N Coefficients value in vector N

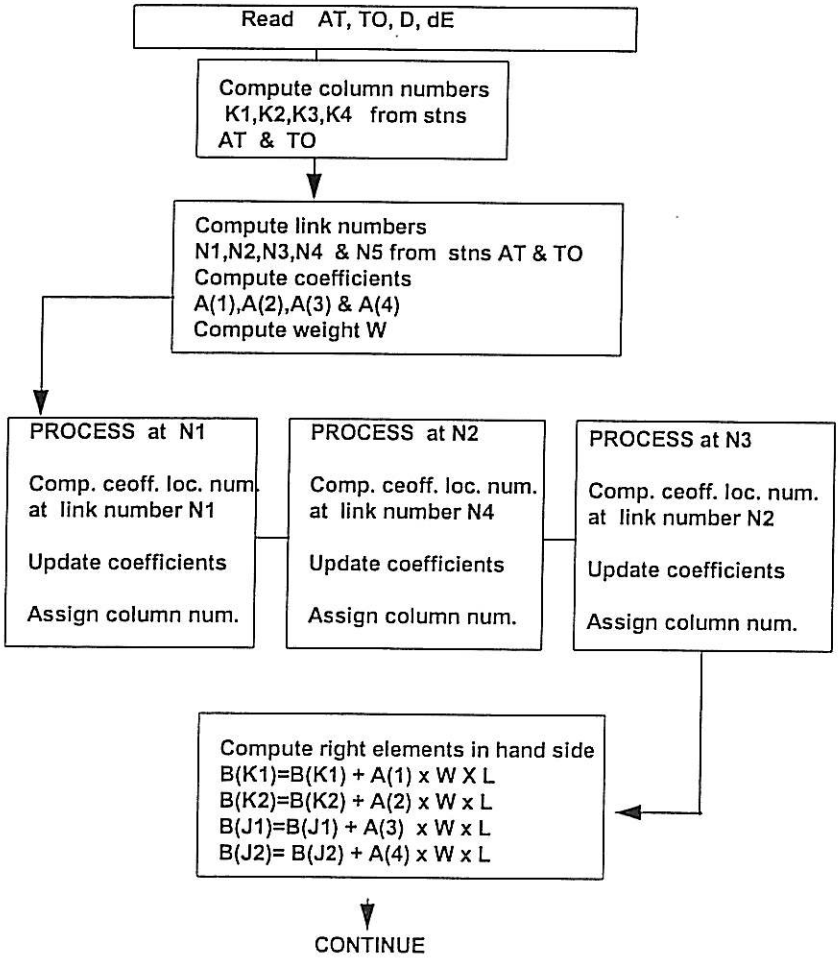


Figure 16: Computing coefficients in vector **n** from a distance

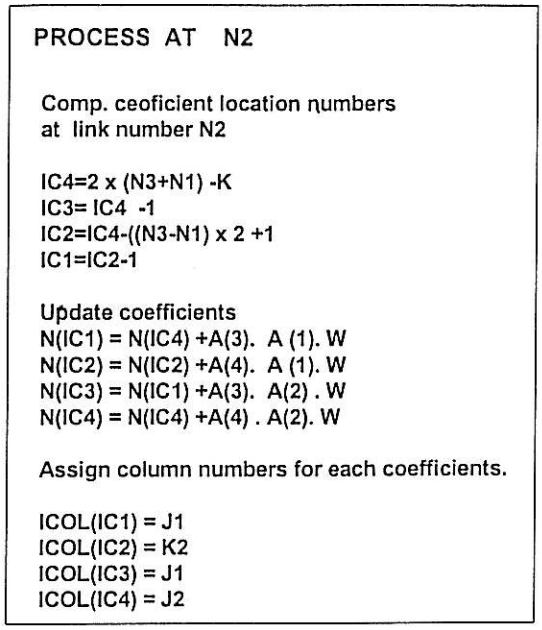


Figure 17: Updating the elements in coefficient vector **n**

5.5.2 Solving normal equations

The flow chart for the CGM for solving normal equations is depicted in Figure 18. The algorithm consists of several standard vector multiplications except for operations involving vector **n** (i.e. **b**= **n** . **p**) which require further definition. This is because **n** is a vector containing only nonzero terms of its original sparse matrix **N** and has been described in detailed in Chapter 4. Vector **n** which contains nonzero terms only in the upper triangular block of matrix N further complicates the computation. The multiplication **b**=**N**. **p** is shown in Figure 19.

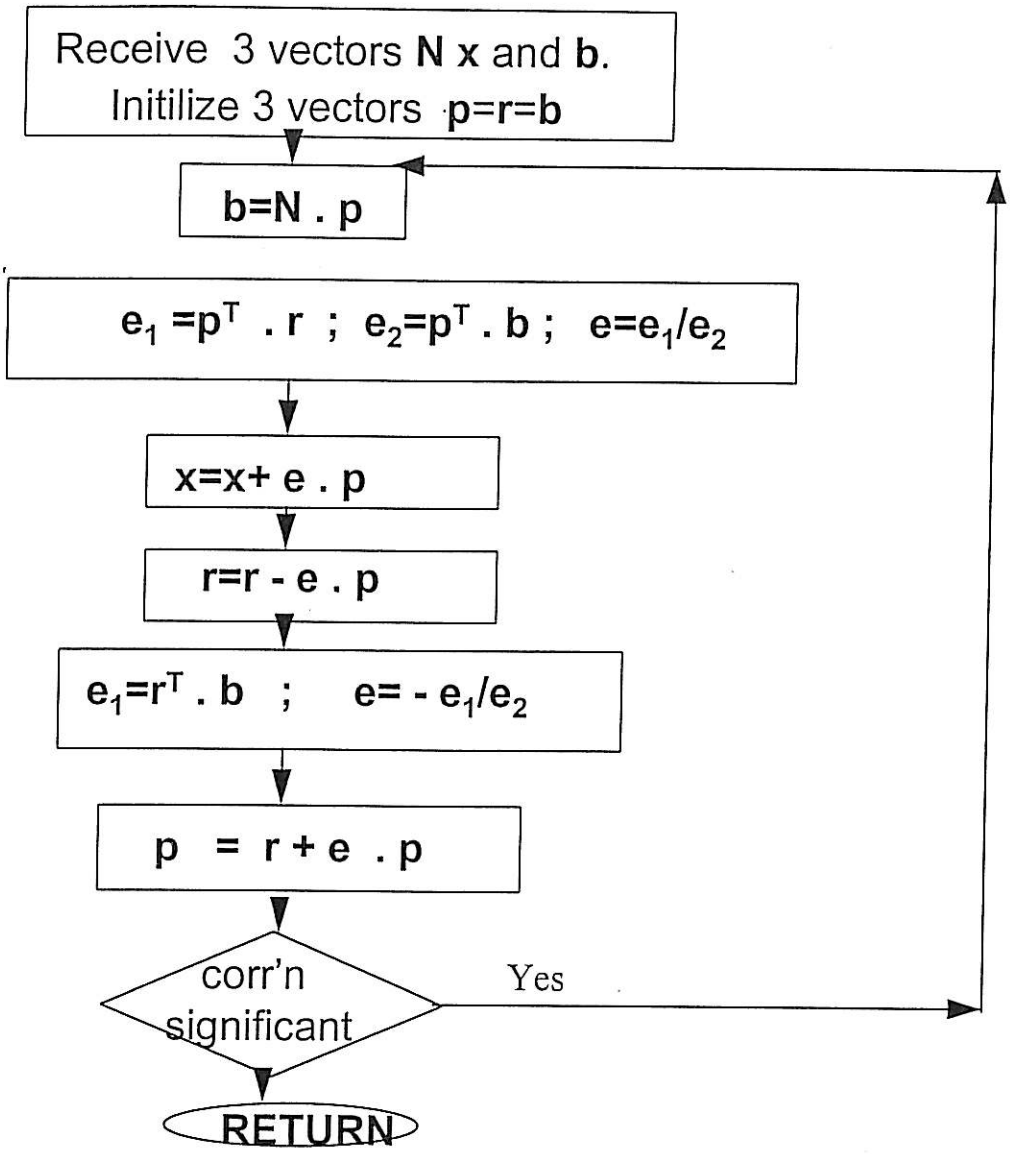


Figure 18: CGM for solving normal equations

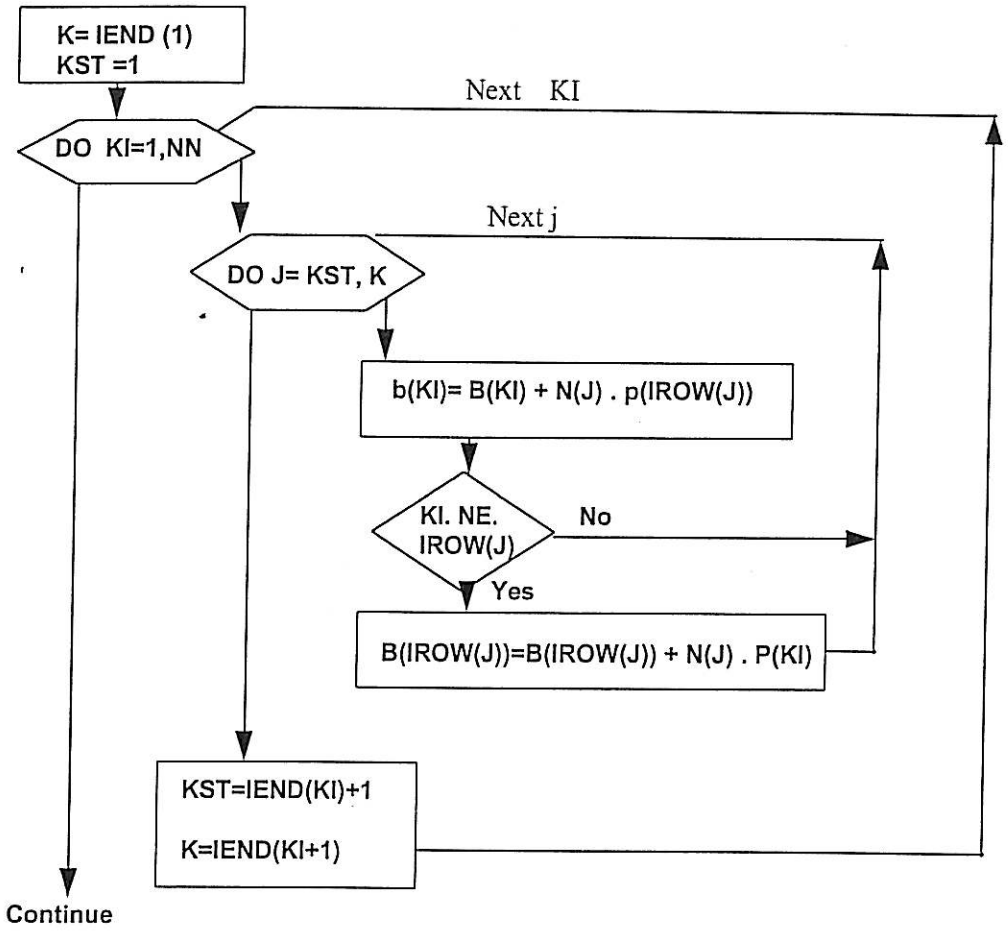


Figure 19: The multiplication of sparse vector b=**n**. p

5.6 Writing and debugging source code

The flowcharts shown in Section 5.5 provide the necessary information about the structure and the algorithm for implementing the COMPUTE module of the cadastral computing program. Based on this information, the writing of the program is only a mechanical transformation of the design into the desired program. The translation involves the process of writing the program by converting each

block within the flowchart into one or more FORTRAN statements. These FORTRAN statements become the source code of the program.

The FORTRAN language statements can be obtained from many textbooks in computer language such as Monro (1982). Briefly there are four types of statements used in FORTRAN programming:

1. Control statements determine the sequence in which operations are performed. Operations such as choosing alternatives and branching to another part of the program are governed by control statements.
2. Arithmetic statements direct the computer to perform computations.
3. Input/output statements instruct the computer to read data from or write to an I/O device.
4. Specification statements describe the arrangement of data read from an input device, or information written to an output device.

The most critical part of writing the cadastral computation program is the creation of normal equations and the observation equations. The nonzero terms have to be correctly arranged and connected to their true positions so that they are accepted by the CGM algorithm. This consumes most of the time used for developing the program.

5.7 Compiler : Microsoft FORTRAN PowerStation

The program is compiled using Microsoft FORTRAN PowerStation compiler. The compiler is a tool for building and debugging MS-DOS-based, 32-bit FORTRAN applications in an integrated Windows environment. The FORTRAN PowerStation compiler produces 32-bit, MS-DOS-extended applications. A special version of the Phar Lap MS-DOS extender environment called DOSXNT.EXE is used to host the compiler and its targets.

The compiler has a very desirable feature, in that, it can produce executable programs which can cope with any volume of data. They are only limited by the hardware that runs the program. The programs can take full advantage of all the disk space on a machine that is available as virtual memory. Other features of the compiler includes the availability of options that can improve execution speed and the consistency of floating-point calculations.

In addition to the standard ANSI FORTRAN 77 syntax and a set of Microsoft proprietary extension, the compiler supports most of the DEC VAX FORTRAN extensions and a large suite for IBM mainframe FORTRAN extensions. Thus mini-computer and mainframe FORTRAN users can easily download their existing codes to FORTRAN PowerStation.

5.8 Compilation process

After the program has been written, it is submitted to computer for compilation. The compiler translates the high-level language codes of the source codes into machine codes. The compilation process detects syntactical errors, such as incorrect spelling and incorrect punctuation, while semantic errors will be discussed in Chapter VI. Errors in programs are called bugs, and the process of locating, isolating, and eliminating bugs is called debugging. Debugging is part of any programming processes. The amount of time that must be spent in debugging depends on the quality of the program. The newly cadastral completed program required a lot of debugging work. To help debugging a program, the compiler provides a listing of all compiler-detected errors.

Apart from this, there are various techniques and tools which can help in debugging, like printing out intermediate results, using cross-reference listings, checking for subscript ranges and other dynamic debugging tools. Every violation of the syntax rules of the language should be detected by the computer during loading the program. The manner in which detection is announced and the amount of explanation provided depends upon the compiler used. In many cases the flaw is obvious and the correction is straight-forwards. However in some situations, the actual error in the program is located far from the statements where its presence is detected. Finally, after a meticulous checking, the program is shown to be free from those syntactical bugs.

5.9 The compiled program

The CGM program is a user-friendly program that is easy to use, easy to learn and handles data intelligently. The program is self contained and can be run interactively in a flexible menu-driven mode. There are two computer programs, CGOBS for observation equations approach and CGNOM for normal equations approach. When each program is successfully loaded, it will display the main menu, and from here the user can select the desired options. In principle, options and commands can be applied repeatedly and in any order, although, of course, surveying logic and principles will restrict the permitted order of use.

The program has extensive error trapping facilities, so the user will be informed if he tries to break any computing rules or surveying rules. In principle any input will be processed in a reasonable manner. At any level the user is informed what action has been taken and messages are displayed when necessary. These messages are classified into three types: notes, warnings and errors. A note is displayed when the program has successfully performed a user request. If the program encounters an error but the program can continue to the next step, a warning message is given. The error message is displayed when the program encounters an error that has to be fixed before further processing can be made.

The program is carefully organized and structured so that further development and extension can be done with ease. In other words, the existing structure and routines need not be altered or deleted when it is further developed. It is written in

the standard FORTRAN 77 with 17 self contained subroutines. The program is compiled using PowerStation FORTRAN which has a facility to use the extended memory of the computer. This means that it does not limit the memory to 640K RAM only, as an ordinary FORTRAN compiler does.

There are no external system library subroutines adopted, except one FORTRAN routine to display the processing time. This short system routine is clearly highlighted in the source code so that it can be easily erased or freeze when necessary.

CHAPTER VI

TESTS AND EVALUATION OF THE CGM PROGRAM

6.1 Introduction

Conjugate gradient is the proposed method for solving cadastral least squares computations. This method should be rigorously tested to determine if it is a better alternative to the conventional Cholesky Decomposition method. There are five objectives in which the tests are carried out in this research:

(1) to prove the validity of setting up the coefficient matrix of the system of observation equations which contains the only non zero elements,

(2) to prove the validity of setting up the coefficient matrix of the system of normal equations which contain only the nonzero elements,

(3) to prove the validity of the least squares computation using the CGM,

(4) to evaluate the utilization of computer memory needed for CGM solution, and

(5) to evaluate the speed performance of the CGM program.

The first three objectives focus on the aspect of validity while the last two concern with performance of the CGM. Validity test is to ensure the method gives right results. The test is fundamental to the iterative CGM because the method is a new tool in cadastral computation and it may also proned to rounding-off error. Furthermore, until the method is proven to give correct results, any evaluation of its performance is meaningless.

Performance evaluation concerns with the question of efficiency of the implemented computer program in terms of utilization of various computer resources. The principal resources are memory and time, each of which contributes to the cost of running a program. The method for minimizing computer storage resources has been described in Chapters III and IV. Memory usage is the amount of variables needed for the execution of a program while time is the execution speed of the program.

This chapter is divided into two main parts. Since the performance test is of no use until the program is proven to be correct and reliable, the first part of this chapter concerns mainly with the tests carried out for the validity of the implemented CGM program. The second part of the chapter concerns with the performance of the method in solving cadastral problems.

6.2 Validity of the CGM approach

The most important question faced after the program is successfully compiled is how to determine whether the program gives the correct results. Confirming the correctness of a program requires a convincing demonstration whether the program actually satisfies the precise requirements of the problems. This testing involves executing the program with input data from simulated and real networks.

Since the CGM program is of an iterative nature, steps should be taken, especially in the early stage, to make sure the iteration process terminates. If there is a bug in the program and the process is not converging, excessive computer time will be used and very little knowledge will be gained. So the initial effort was to ensure the method is correctly translated into a computer program and it converges.

Validity or correctness is a measure of the closeness of a computed value to the true value. The present work uses three methods for testing the correctness of the CGM programs:

(1) the comparisons of the results against some pre-determined values,

(2) the internal numerical check, and

(3) the comparison of the results from a different implementation or approach.

The basis of the first method is solving a preset network whose parameter is already known, such as the sum of internal angles of a triangle is exactly 180

degrees. The method compares the computer-determined outputs with the true parameters of the preset network. The true parameters can be obtained by applying basic mathematical formulae which can be computed by manual calculations.

The second method concerns with internal numerical check within the least squares computational process itself. Theoretically, there exists a vector, whose value should be approaching zero if all arithmetic operations within the process are performed correctly. The vector denoted by **c** is obtained by following formula.

$$\mathbf{c}=\mathbf{A}^{T}\mathbf{W}\mathbf{v} \qquad \text{Eq (5.1)}$$

where **c** is the check vector

A is the design matrix,

W is the weight of measurements, and

v is the residual of measurements.

The proof of this numerical check vector can be found in Cross (1983). This check ensures correct computation and solution of the normal equations and also ensure correct computation of the residuals.

The third kind of validity test is based on comparisons of the results obtained by CGM with their corresponding values obtained from other computer programs. There are three programs used to test the results, namely

(1) STAR*NET-PLUS,

(2) CHOLES, and

(3) CGOBS.

STAR*NET-PLUS is a commercial least squares program which uses the Cholesky decomposition method. STAR*NET-PLUS is an upgraded program of the STAR*NET program, and it can perform at least twice as fast as the standard STAR*-NET program (Starplus Software Inc, 1993). Unfortunately, the program can only displays results to four decimal places and as such it is considered insufficient for the purpose of comparing of results. Validity test requires greater number of decimal places in order to cope with rounding off error and to investigate the rate of convergence. As an alternative a new program (CHOLES) which uses the Cholesky decomposition method is created for this purpose. Since this research investigates CGM in two different approaches (observation equations and normal equations) there are two programs CGOBS and CGNOM implemented using the method. These two programs compliment each other.

The above methods have been tested for three types of networks:

(1) network 1: a simulated network with no redundant measurement,

(2) network2: a simulated network with redundant measurement, and

(3) network 3: a real cadastral network.

The preceeding sections will describe the features of these networks and the testing procedures that were carried out to determine the validity of the CGM.

6.2.1 Test network 1

The network is made up of four stations (i.e. stations 1,2,3,and 4) two of which (i.e. stations 1 and 2) are known points. The coordinate values of the known points and the measurements are shown in Figure 20.

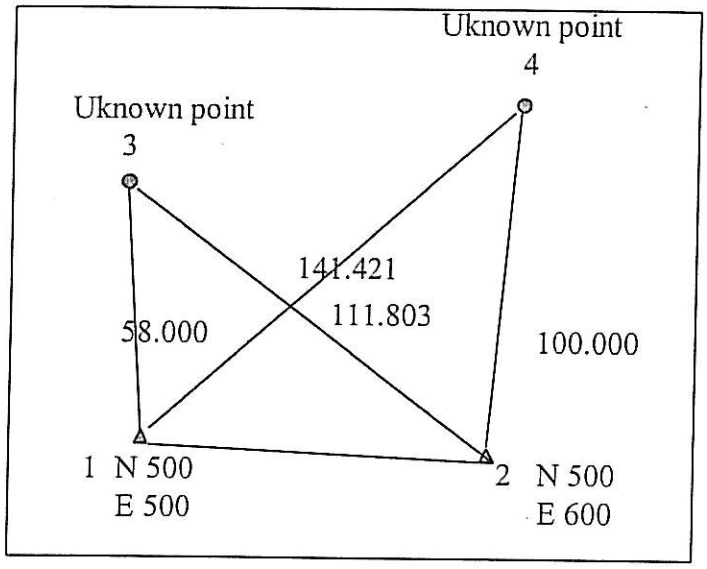
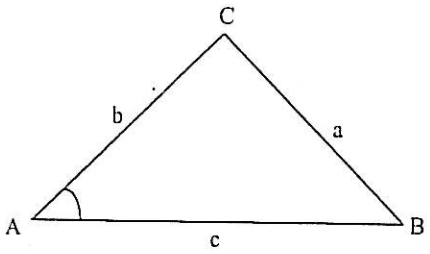


Figure 20: Test network 1.

The network consists of four distance measurements. To suppress the effect of the weights of each and individual measurements, the variance of each measurement is set to unity (i.e., weight equals one). This kind of network is useful for monitoring rate of convergence for iterative method and the effect of rounding-off errors. Networks with different measurement accuracy is described in section 6.2.3.

The true coordinates of the unknown stations (3 and 4) of such network can be computed directly by using standard trigonometric formulae, such as the side formula. In a triangle ABC, the side formula is given by:



$$\sin\frac{1}{2}A = \left[\frac{(s-b)(s-c)}{bc}\right]^{\frac{1}{2}}$$

where a, b, and c are sides of the triangle ABC

$$s = (a + b + c)/2$$

Knowing the angle A, bearing A to C can be computed. Using bearing AC and distance b, coordinates can be determined by:

$$E_C = E_A + \text{Sin}_{AC}\ b \qquad \text{eq(5.2)}$$

$$N_C = N_A + \text{Cos}_{AC}\ b \qquad \text{eq(5.3)}$$

Using equations (5.2) and (5.3), the coordinate values of the unknown stations 3 and 4 were computed to twelve decimal, see Table 22. These coordinates can be considered as true values and they serve as reference values for testing the CGM.

Table 22: The reference coordinate for test network 1

Station	Northing	Easting
3	557.838 860 178 516	504.320 445 955 000
4	599. 999 999 998 731	599.999 496 205 041

The test on this network proceeds to determine whether the CGM converges and the resulting parameters are correct, based on the preset coordinates. To carry out the tests, the coordinates of stations 3 and 4 were shifted to a new set of values as shown in Table 23. The shifts are not systematic, with a combination of positive and negative values with varying magnitude. Two of the shifted coordinates are exceedingly large as initial coordinates in a practical cadastral computation. They are purposely introduced for the above tests.

Table 23: Initial coordinates for test network 1

Stn.	Initial Northing(m)	Initial Easting(m)	Shift in Northing	Shift in Easting
3	558.7	498.5	0.8	-5.8
4	622.0	589.1	+22.0	-10.8

The test network 1 was solved by CGM program CGOBS and CGNOM. Being an iterative method, the CGM program can be terminated at any specified tolerance by user. The level of accuracy selected for the test was set to 12 decimal places. This level of accuracy is considered sufficient for the research in the behaviour of convergence and the effect of rounding-off error.

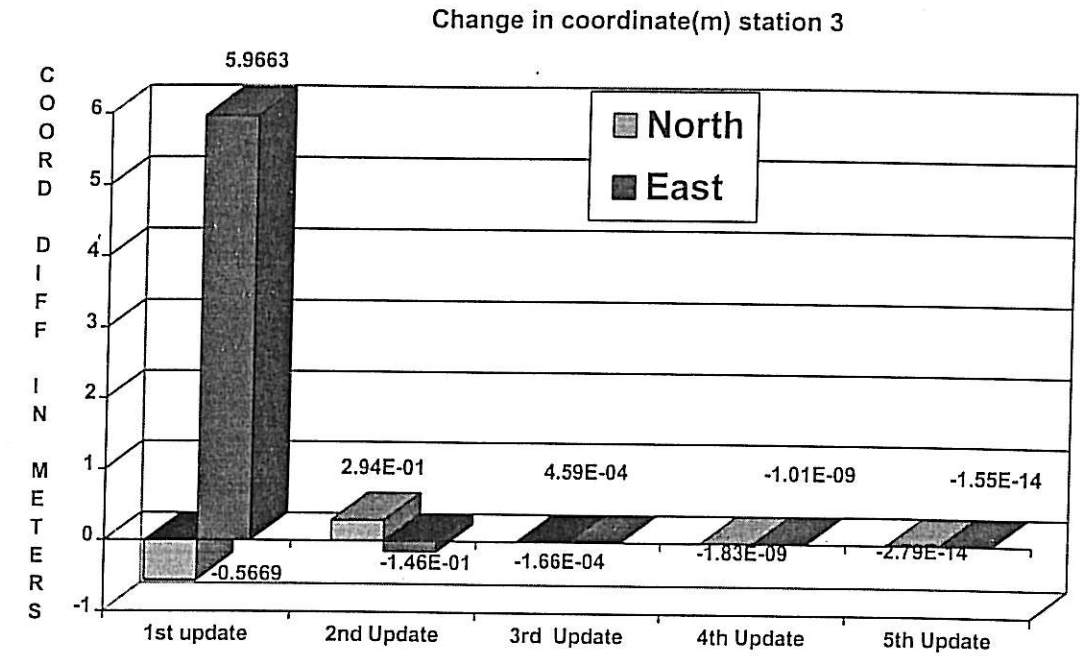
The computation started with the initial coordinate values taken from Table 23. The coordinates of the network converged to the true values after four times of updating the coefficient matrix. The resulting coordinates of the network from both

programs CGOBS and CGNOM were the same and the values agreed with the corresponding reference coordinates to at least twelve decimal places. The computed coordinates and the corresponding reference values are shown in Table 24.

The rates of convergence in the coordinates for both CGOBS and CGNOM are rapid. After the first update, the starting coordinates were moved to their preset values within a few meters. The second updates brought the coordinates nearer to their true values, that is, within tenths of millimeters. After the third update, the coordinates are correct to one thousandth of a millimeter and the correction in the forth update is negligible. The rate of change of corrections to coordinates for CGOBS and CGNOM are shown in Figure 21 and Figure 22 respectively.

Table 24: CGM coordinates and the reference coordinates.

	Northing station 3	Easting station 3
Reference coordinates	557.838 860 178 516	504.320 445 955 000
CGOBS	557.838 860 178 516	504.320 445 955 000
CGNOM	557.838 860 178 516	504.320 445 955 000
Difference	0.000 000 000 000	0.000 000 000 000
	Northing station 4	Easting station 4
Reference coordinates	599.999 999 998 731	599.999 496 205 000
CGOBS	599.999 999 998 731	599.999 496 205 000
CGNOM	599.999 999 998 731	599.999 496 205 000
Difference	0.000 000 000 000	0.000 000 000 000



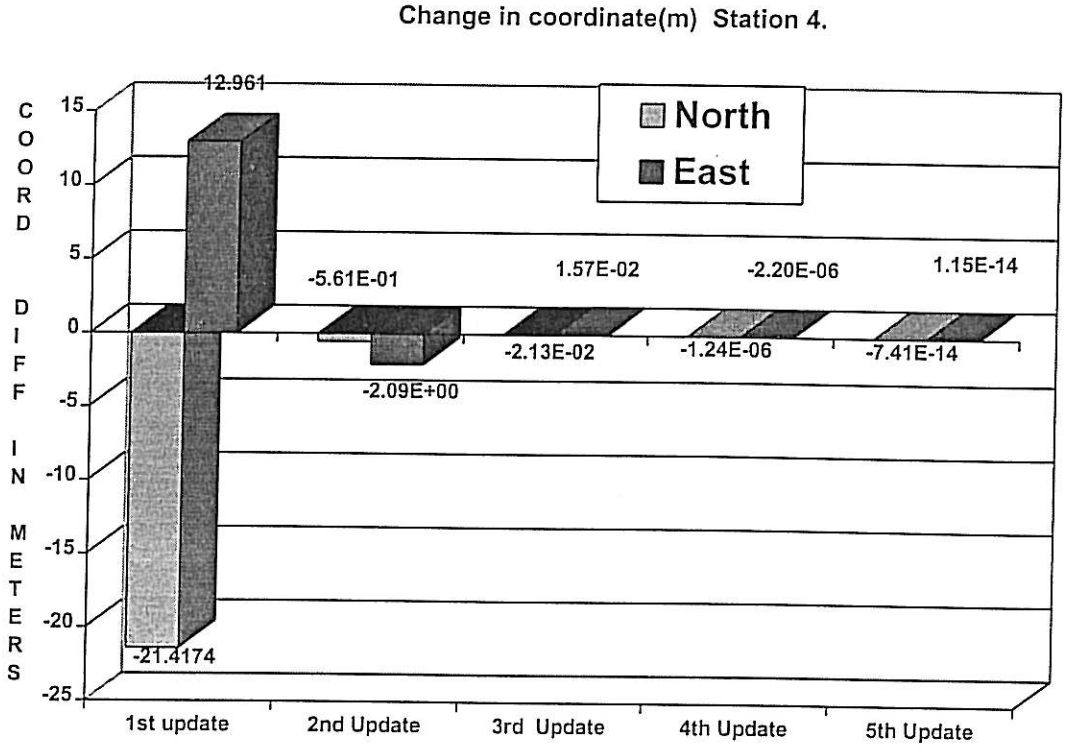
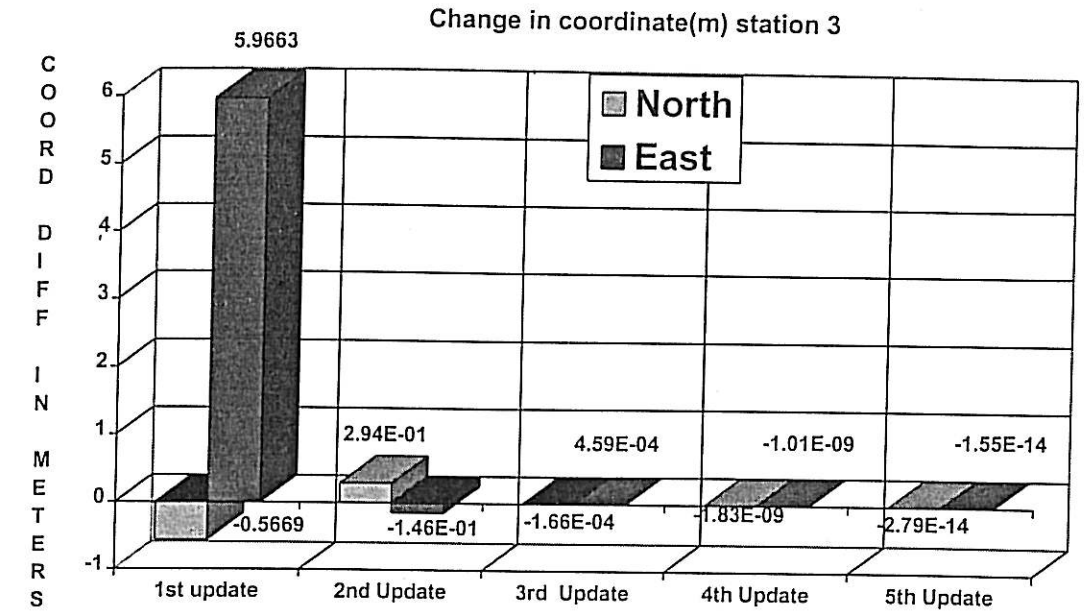


Figure 21: Rates of change of coordinate in CGOBS



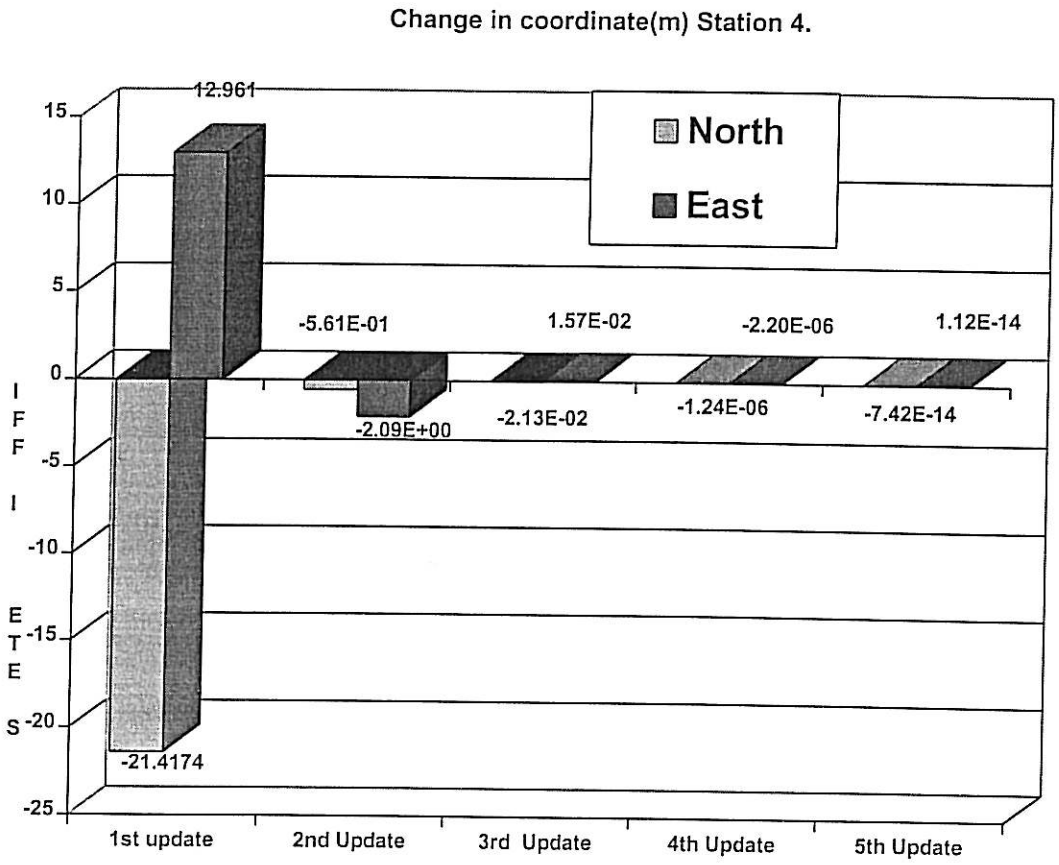


Figure 22: Rates of change of coordinate in CGNOM

Considering the results produced by the CGM program, the following conclusions can be made:

(1) the CGM is correctly translated in the computer program.

(2) the CGM converged and the answer is correct to at least twelve decimal places.

(3) the rounding off error is negligible in CGM.

(4) the rates of convergence in CGM is rapid.

6.2.2 Test network 2

The configuration of the second test network is similar to the first test network, except that it contains four new bearings in addition to the existing distances. The network has four redundant measurements and it becomes an over-determined network which requires the use of least squares method to determine its coordinates. The network is depicted in Figure 23.

The measurements were derived from the true coordinates obtained from the first test and therefore all the measurements (including the distances) fit exactly with geometry of the network. Such a network has three distinct features:

(1) all residuals should be of zero value,

(2) unit variance should be of zero value, and

(3) coordinates should converge to the preset values.

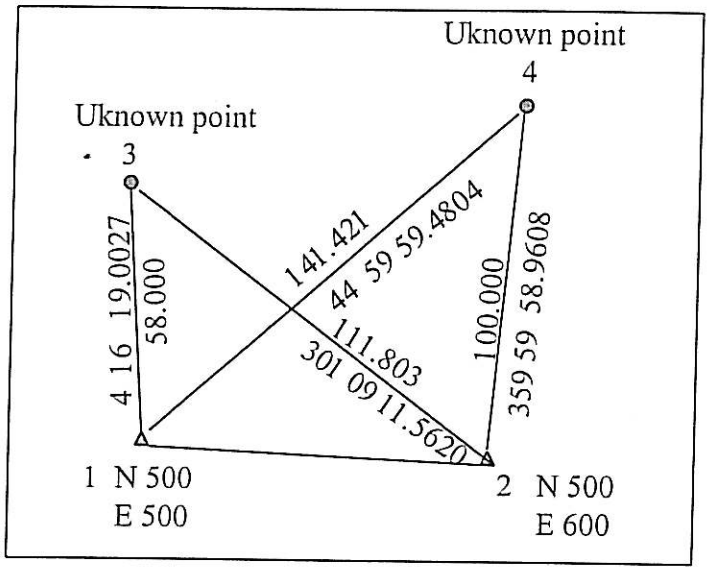


Figure 23: Test network 2.

The network is computed by CGOBS, CGNOM and CHOLES. The initial coordinates of network 1 (as listed in Table 23) were used as the starting coordinates. The solution converged to their preset coordinates, and they are similar to the results of the first network, see Table 24. The residuals and unit variance converged closed to zero value, that is to six decimal places as shown in Table 25.

Since network 2 has redundant measurements, the computations involved a system of over-determined equations. Solving over-determined equations require the use of the least squares method. The computation involved manipulations and

operations of equations that were set up and optimized by the method as being proposed in this research. Based on the least squares parameters derived from the programs, the following conclusions can be made:

(1) The methodology in setting up the observation equations is correct.

(2) The methodology in setting up the normal equations is correct.

(3) The computation of least squares parameters (coordinates, residuals and unit variance) are correct.

Table 25: The computed residuals for test network 2

Observation	at	to	CGOBS	CGNOM	CHOLES
(1) Northing	1		0.000000 m	0.000000 m	0.000000 m
(2) Easting	1		0.000000 m	0.000000 m	0.000000m
(3)Northing	2		0.000000 m	0.000000 m	0.000000 m
(4)Easting	2		0.000000 m	0.000000 m	0.000000 m
(5) Distance	1	3	0.000001 m	0.000001 m	0.000001 m
(6) Distance	1	4	0.000000 m	0.000000 m	0.000000 m
(7) Distance	2	3	0.000000 m	0.000000 m	0.000000 m
(8) Distance	2	4	0.000000 m	0.000000 m	0.000000 m
(9) Bearing	1	3	0.00000 sec	0.00000 sec	0.000000 sec
(10) Bearing	1	4	0.00000 sec	0.00000 sec	0.000000 sec
(12) Bearing	2	3	0.00000 sec	0.00000 sec	0.000000 sec
(13) Bearing	2	4	0.00000 sec	0.00000 sec	0.000000 sec
(14) Bearing	3	4	0.00000 sec	0.00000 sec	0.000000 sec
	Unit variance= 0.000 000 016				

6.2.3 Test network 3

This network is a real and typical cadastral network. It is a section of a certified plan PA01-045948 of Mukim Pulai in the State of Johore, Malaysia, see Figure 24. The network consists of 25 stations, two of which are known stations. It consists of 78 measurements, i.e. 4 coordinate values (i.e. northing and easting of the two known coordinates), 37 distances and 37 bearings. The number of unknowns is 50 and, therefore, the network has 32 redundant measurements.

Unlike the previous test networks, the third network has no preset parameters. Since the network is a real network with redundant measurements measured to various accuracy levels, the coordinates of the stations can no longer can be determined in advance by simple non-rigorous trigonometric methods. The coordinates must be estimated by the rigorous least squares computation process. In this case, there are two ways to test the correctness of the CGM implementation:
(a) comparing results from another independent and established method, and
(b) verify the value of the numerical check vector $\mathbf{c}$ (i.e equation 5.1).

The network was computed using the three least squares programs: CGNOM, CHOLES, and STAR*NET-PLUS. CGOBS was not used since previous tests proved that the result will be similar to CGNOM. The results obtained from the three programs CGNOM, CHOLES and STAR*NET-PLUS were almost identical. The output of CGNOM and CHOLES agreed to twelve decimal places. However, comparison with STAR*NET-PLUS was limited to four decimal places

since the program can only displays to four decimal places. The resulting coordinates are listed in Tables 26 and 27 for northing and easting respectively.

The residuals of all 78 measurements obtained from the three programs agree to at least twelve decimal places. However, the comparison with STAR*NET-PLUS is limited to two decimal places for bearings and four decimal places for distances. The resulting residuals are listed in Table 28. [Note that only 25 residuals out of the 78 measurements are shown].

The numerical check vector $\mathbf{c}= \mathbf{A}^T\mathbf{W}\mathbf{v}$ as in eq(5.1) was computed in CGNOM. All the 50 elements are less than 0.0000004; see Table 29. Note that only 25 elements of the 50 elements of the vector are presented.

The program has been checked and tested using real and simulated data and was confirmed to be properly implemented. The above results provide a conclusive evidence that the CGM is a valid method for solving least squares equations in cadastral computation. The results from the above tests clearly proved that the effect of rounding-off error in the iterative CGM is insignificant.

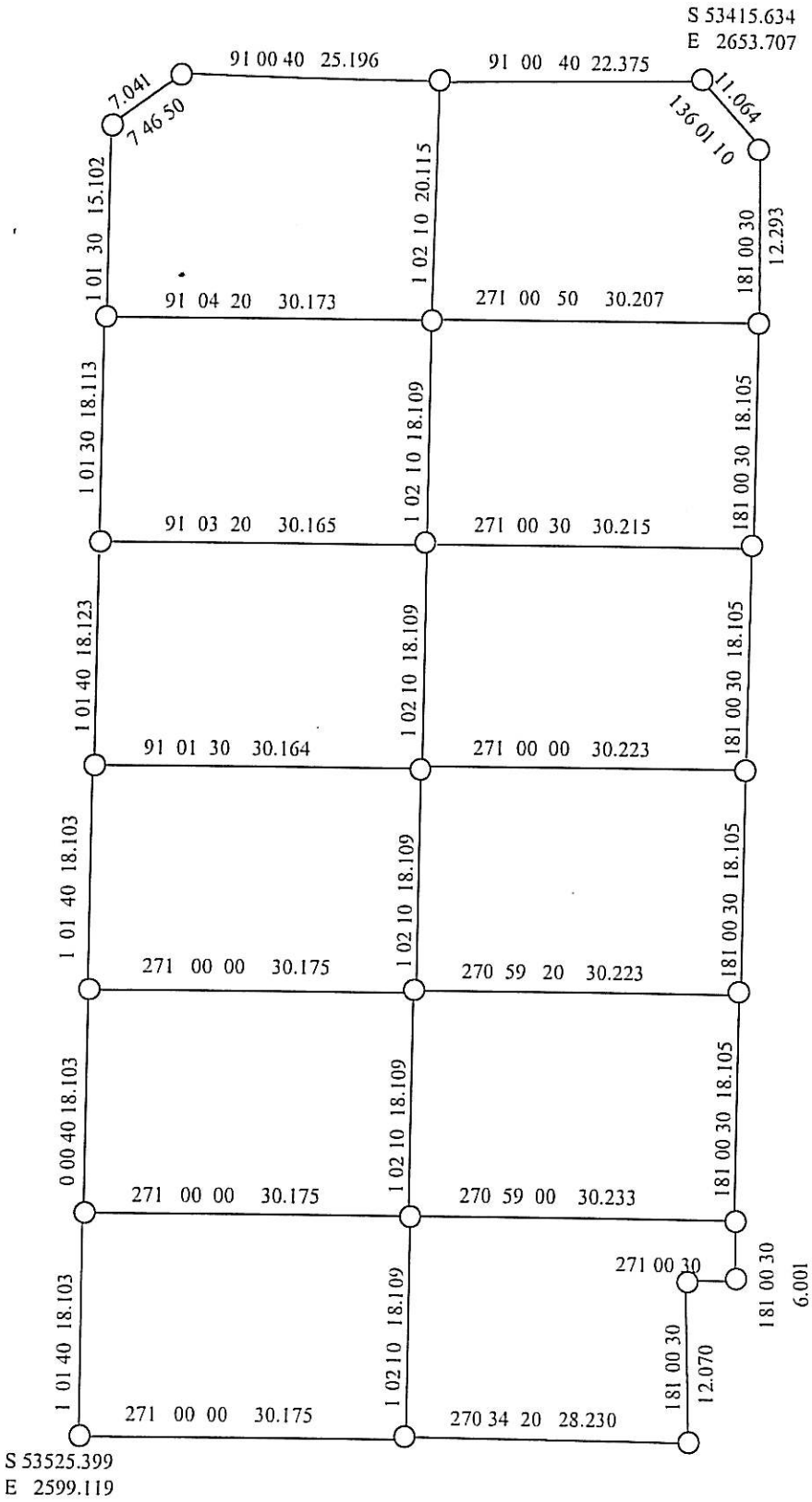


Figure 24: Test network 3

Table 26: Northings for test network 3.

Stn	CGNOM	CHOLES	STAR*NET-PLUS
	Northing(A)	Northing(B)	Norhing(C)
1	-53507.275116780490	-53507.275116780490	-53507.2751
2	-53489.170736119920	-53489.170736119920	-53489.1707
3	-53471.046558993190	-53471.046558993190	-53471.0466
4	-53452.922964093630	-53452.922964093630	-53452.9230
5	-53434.809230423300	-53434.809230423300	-53434.8092
6	-53419.707559556720	-53419.707559556720	-53419.7076
7	-53414.816498587670	-53414.816498587670	-53414.8165
8	-53415.260019481590	-53415.260019481590	-53415.2600
9	-53415.653626367670	-53415.653626367670	-53415.6536
10	-53423.613496015040	-53423.613496015040	-53423.6135
11	-53435.906576688630	-53435.906576688630	-53435.9066
12	-53453.010003498280	-53453.010003498280	-53453.0100
13	-53472.112752317560	-53472.112752317560	-53472.1128
14	-53490.214898566440	-53490.214898566440	-53490.2149
15	-53508.317679942900	-53508.317679942900	-53508.3177
16	-53507.798899740890	-53507.798899740890	-53507.7989
17	-53489.692751319080	-53489.692751319080	-53489.6928
18	-53471.585770739070	-53471.585770739070	-53471.5858
19	-53453.478811304310	-53453.478811304310	-53453.4788
20	-53435.372651371250	-53435.372651371250	-53435.3727
21	-53514.318008317750	-53514.318008317750	-53514.3180
22	-53514.282697550610	-53514.282697550610	-53514.2827
23	-53526.351231112700	-53526.351231112700	-53526.3512
24	-53525.905348903040	-53525.905348903040	-53525.9053
25	-53525.379373632330	-53525.379373632330	-53525.3794

Table 27: Eastings for test network 3.

STN	CGNOM	CHOLES	STAR*NET-PLUS
	Easting (A)	Easting(B)	Easting (C)
1	2599.470093677401	2599.470093677401	2599.4701
2	2559.797984651758	2559.797984651758	2559.7980
3	2600.125892159478	2600.125892159478	2600.1259
4	2600.452490510777	2600.452490510777	2600.4525
5	2600.776910413995	2600.776910413995	2600.7769
6	2601.047418378889	2601.047418378889	2601.0474
7	2606.114663735923	2606.114663735923	2606.1147
8	2631.308397002805	2631.308397002805	2631.3084
9	2653.682806981984	2653.682806981984	2653.6828
10	2661.362909214741	2661.362909214741	2661.3629
11	2661.146148997452	2661.146148997452	2661.1461
12	2660.826562108034	2660.826562108034	2660.8266
13	2660.506938472450	2660.506938472450	2660.5069
14	2660.186989846703	2660.186989846703	2660.1870
15	2659.868081267265	2659.868081267265	2659.8681
16	2629.638986012726	2629.638986012726	2629.6390
17	2629.974710706294	2629.974710706294	2629.9747
18	2630.289743222151	2630.289743222151	2630.2897
19	2630.616407465746	2630.616407465746	2630.6164
20	2630.944226830066	2630.944226830066	2630.9442
21	2659.762473660395	2659.762473660395	2659.7625
22	2657.750801191611	2657.750801191611	2657.7508
23	2657.538494366237	2657.538494366237	2657.5384
24	2629.311964186716	2629.311964186716	2629.3120
25	2599.143193018016	2599.143193018016	2599.1432

Table 28: Residuals for test network 3.

	CGNOM	CHOLES	STAR*NET
Measurement	Resid: (A)	Resid.(B)	Resid.C)
1.East (m)	.024193018015	.024193018015	.0242
2.North(m)	.019626367669	.019626367669	.0196
3.East(m)	-.024193018015	-.024193018015	-.0242
4.North(m)	-.019626636676	-.019626636676	-.0196.
5.Dist (m)	.004349656811	.004349656811	.0043
6.Brg(sec)	35.282429405150	35.282429405150	35.28
7. Dist (m)	.004143289607	.004143289607	.0041
8. Brg(sec)	31.391464977695	31.391464977695	31.39
9. Dist (m)	0.00353730251	0.00353730251	0.0035
10. Brg(sec)	16.616690991622	16.616690991622	16.62
11.Dist (m)	.003638653886	.003638653886	.0036
12. Brg(sec)	3.841149763650	3.841149763650	3.84
13. Dist (m)	.0020933392837	.0020933392837	.0021
14. Brg(sec)	4.313504988777	4.313504988777	4.31
15. Dist (m)	.001687903872	.001687903872	.0017
16 Brg(sec)	-1.101636894619	-1.101636894619	-1.10
17. Dist (m)	.001636927025	.001636927025	.0016
18. Brg(sec)	-9.204142830295	-9.204142830295	-9.20
19. Dist (m)	.002871844685	.002871844685	.0029
20. Brg(sec)	-11.798199425729	-11.798199425729	-11.80
21. Dist (m)	-.003100619356	-.003100619356	-.0031
22. Brg(sec)	19.266084362017	19.266084362017	19.27
23. Dist (m)	.001991559134	.001991559134	.0020
24. Brg(sec)	6.62881946360	6.62881946360	6.63
25. Dist (m)	.001247486218	.001247486218	.0012

Table 29: Numerical check vector for test network 3

Measurement	CGNOM A^TWv
1	-.000000008
2	.000000475
3	-.000000014
4	.000000441
5	-.00000044
6	.000000468
7	-.000000022
8	.000000080
9	.000000005
10	-.000000334
11	.000000133
12	.000000020
13	.000000140
14	-.000000383
15	-.000000034
16	.0000000365
17	.0000000067
18	.0000000432
19	-.0000000175
20	-.0000000158
21	-.0000000137
22	-.0000000791
23	-.0000000188
24	-.0000000608
25	-.0000000017

6.3 Performance of the CGM

Performance of a computer method is based on the efficiency of managing computer resources. Two principal resources are memory and time, each of which contributes to the cost of running a computer program. Time is the speed of executing the program and the memory is the amount of storage used during the computation process. The performance in solving cadastral least squares computation depends primarily on the size and geometry of a network. The network size is the number of stations while the geometry is the shape and the amount of connections found in the network.

To evaluate the performance of the CGM, eight simulated networks were created, i.e networks 4, 5, 6, 7, 8, 9, 10, and 11. A computer program called 'DATA' was written to generate these networks. The geometry of each of the networks 4, 5, 6 and 7 is in the form of squares interconnected with bearings and distances, as shown in Figure 25. The other four networks are also in the form of squares with additional measurements along the diagonals, as shown in Figure 26. Additional data for the networks, whose sizes vary between 3000 to 20000 stations are given in Table 30.

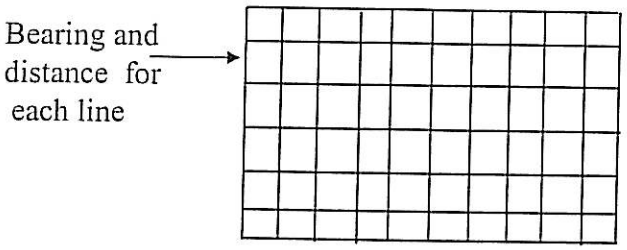


Figure 25: Geometry of Network 4,5,6, & 7

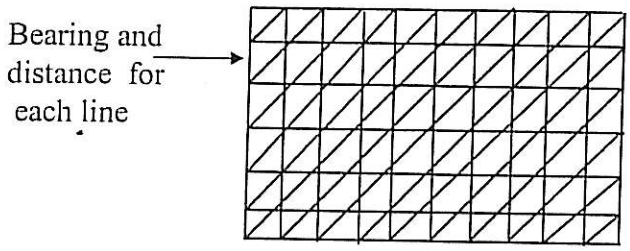


Figure 26: Geometry of Networks 8,9,10 & 11

Table 30: List of network for testing the performance of CGM

	Number of Stations	Number of measurements	Network geometry
Network 4	3000	11744	squares
Network 5	9000	35624	squares
Network 6	15000	59504	squares
Network 7	20000	79404	squares
Network 8	3000	17486	squares + diagonals
Network 9	9000	53246	squares + diagonals
Network 10	15000	89008	squares + diagonals
Network 11	20000	118808	squares + diagonals

6.4 Measuring storage space

The first aspect of the performance is the evaluation of computer memory storage, used by the computer to process the computation. The amount of memory storage will depend on the number of arrays assigned for the program. The arrays in least squares computation can be divided into two groups:

(a) the arrays for processing and presenting the least squares results.

(b) the arrays for storing and solving the linear equations.

The arrays within the first group are for processing and presenting the least squares method. These arrays are one-dimensional filled arrays, that is, they contain no zero terms, and thus they cannot be compressed or restructured to reduce their dimensions.

The second group of arrays are those for storing and processing the linear equations $\mathbf{Ax} = \mathbf{b}$ (as in the case of observation equations) and $\mathbf{Nx} = \mathbf{d}$ (for the normal equations). The bulk of the storage lies in the coefficient matrix **A** or **N**.

6.4.1 Number of elements in observation equations

The approach for constructing observation equation presented in Chapter 3 is found to save tremendous amount of storage. The total elements in vector **a** is less than one percent of the elements in the original matrix **A**, for the networks 4 to 11.

For example, the square grid network 4, consisting of 3000 stations (6000 unknowns) with 11744 measurements would have in matrix A 70.464 million elements (i.e. 11744 x 6000). The same network only generated 46976 elements (i.e. 4 x 11744) using eq.(3.9). This figure represents less than 0.07 percent of all the locations in matrix A, and thus a saving of 99.93 percent. A saving of at least 74 percent can be obtained compared to half-bandwidth optimization scheme (say, bandwidth is 0.5% of unknowns) which would need 180,000 elements (i.e. 30 x 6000).

Figure 27a summarizes the saving of the elements in the observation equations compared to the banded optimization scheme. For networks with greater number of connections and irregular bearing and distance measurements, such as networks 8 to 11, the minimum bandwidth would become much wider and thus generate larger elements within the band, see Figure 27b.

6.4.2 Storage of normal equations

The second approach of CGM is the processing of normal equations $\mathbf{Nx} = \mathbf{d}$. The bulk of the storage lies in the coefficient matrix **N** of the equation. However, it has been substantially reduced to vector **n** containing only the nonzero coefficients within the upper triangle of matrix **N**. Vector **x** and **d** are filled arrays.

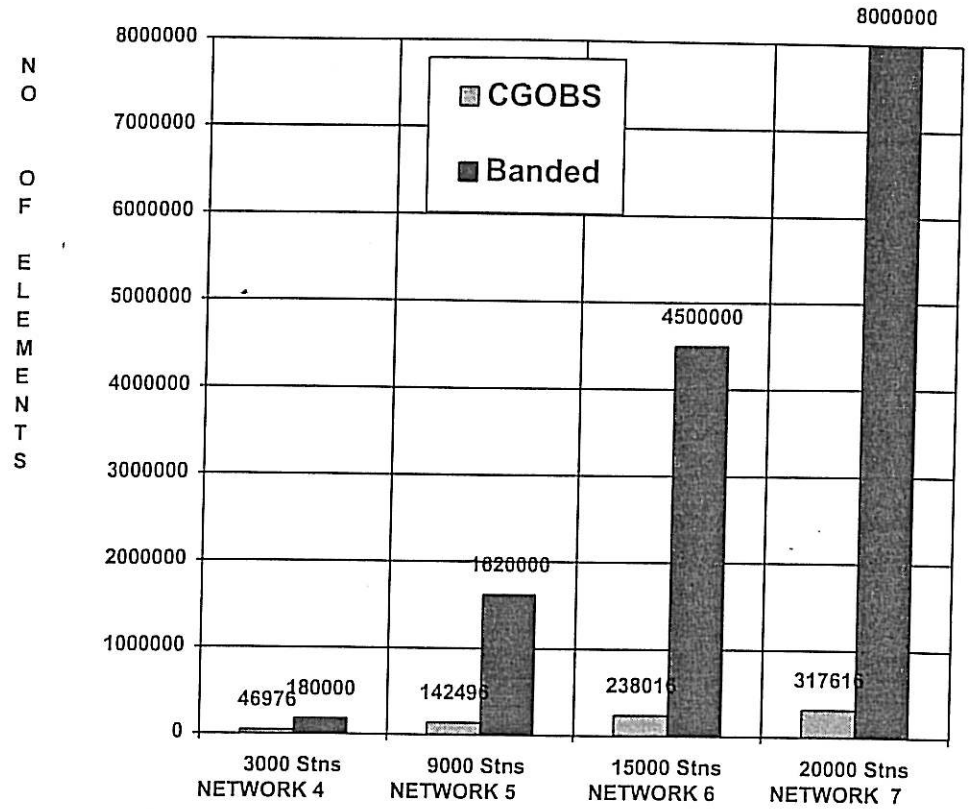


Figure 27a: Number of elements for Networks 4 to 7: Observation equations versus banded scheme

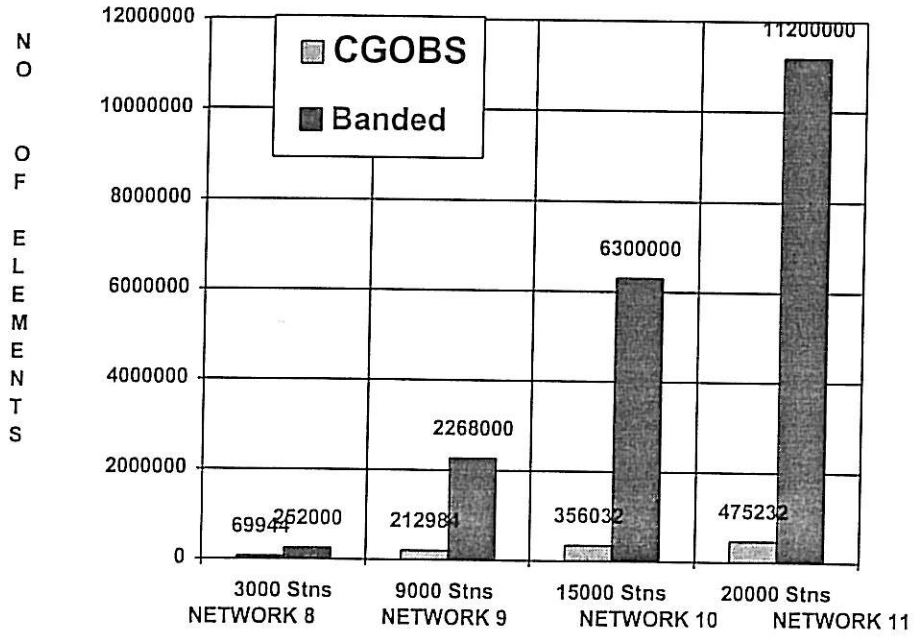


Figure 27b: Number of elements for Networks 8 to 11: Observation equations versus banded scheme

Since matrix **N** in cadastral application is very sparse, the number of elements in coefficient vector **n** is relatively small compared to the full matrix **N**. For example, a rectangular grid network with 3000 stations (i.e., 6000 unknowns) would have 36 million elements (i.e. 6000 x 6000) in matrix **N**, whereas the same network with 9640 connections (i.e. adjacency number) would generate 32560 elements (i.e. 4 x 9640 - 6000) using eq. (4.6). This figure represents less than 0.09 percent of the 36 million locations in matrix **N**, and thus a saving of 99.94 percent. The further saving of 82 percent can be obtained if compared to half-bandwidth optimization scheme (say, bandwidth is 0.5% of unknowns) which would need 180,000 elements (i.e. 30 x 6000). Figures 28a & 28b summarize the saving of the above approach as compared to the bandwidth optimization scheme.

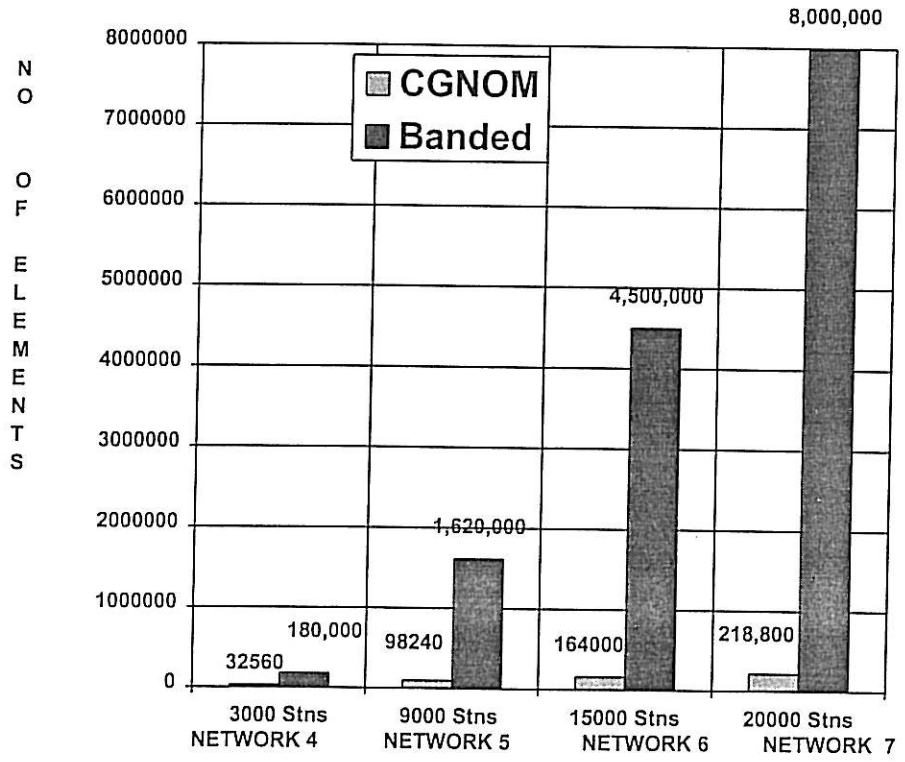


Figure 28a: Number of elements of Networks 4 to 7 : Normal equations versus banded scheme

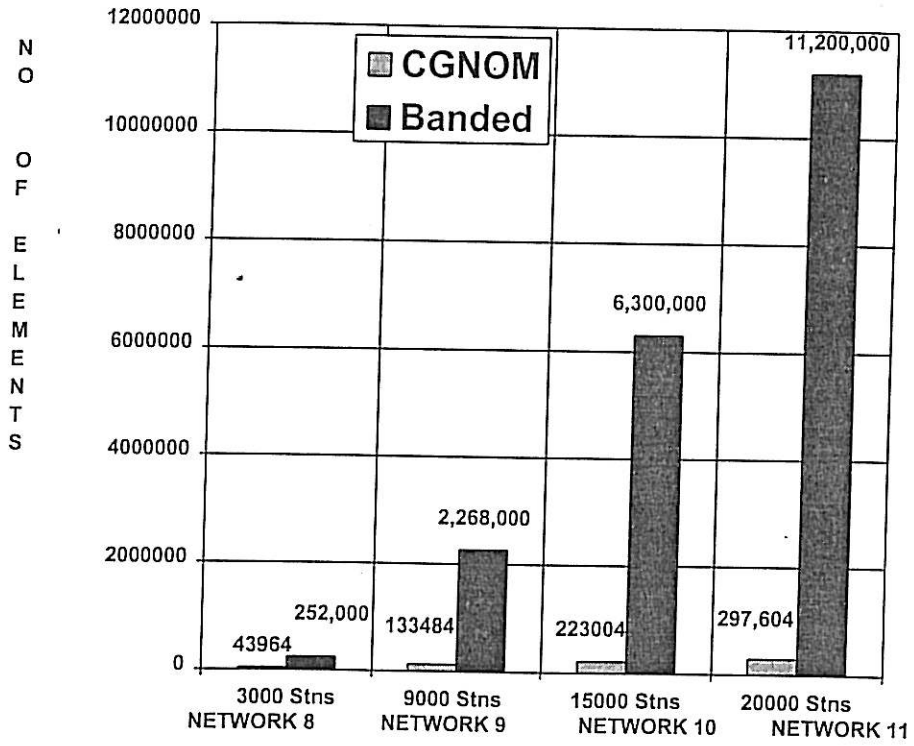


Figure 28b: Number of elements for Networks 8 to 11: Normal equation versus banded scheme

6.5 Speed of execution

The execution speed of a program depends mainly on the algorithm of the method and the computer hardware. There is not much can be done with the computer hardware except getting access to a much powerful computer when they are available. However, the algorithms within a program can be manipulated to minimize the execution time. The actual time for processing a survey network depends upon several factors which include network size and geometry, the number of interconnections, the type of measurements, and the quality of results.

The test networks 4 to 11 were computed using CGOBS, CGNOM and STAR*NET-PLUS. The performance of CGOBS and CGNOM can be compared directly since both programs have similar features. Unfortunately, the output of STAR*NET-PLUS cannot be compared directly with those from the CGM programs, because the two programs were designed for different purposes. STAR*NET-PLUS computes a variety of geodetic values, besides the fundamental parameters (coordinates, residuals and unit variances) derived from the CGOBS or CGNOM. However, these additional computations are not time-consuming. The CGM program produces results in a format suitable for GIS applications. For instance, the CGNOM results were written in a format acceptable by a GIS software MAPINFO.

In addition, STAR*NET-PLUS imposes limits on the size of a network to 10000 stations only. CGOBS and CGNOM can process network virtually of any size, and the only constraint is the limitation of computer memory. Computations on networks up to 40000 stations, on PC based computer with Pentium processor has been successfully carried out, although it took nearly one and a half hour to complete the solution. For small networks (less than 3000 stations), the three programs performed almost of the same speed. However, as the size of a network increases, the CGM shows better performance. Generally, CGOBS and CGNOM computes survey network at least one and a half times faster than the optimized STAR*NET-PLUS, as indicated in Figure 29a. This is a pessimistic figure because banded solutions are very good at regular squared networks. The time difference is longer for networks with larger numbers of connections, as shown in Figure 29b.

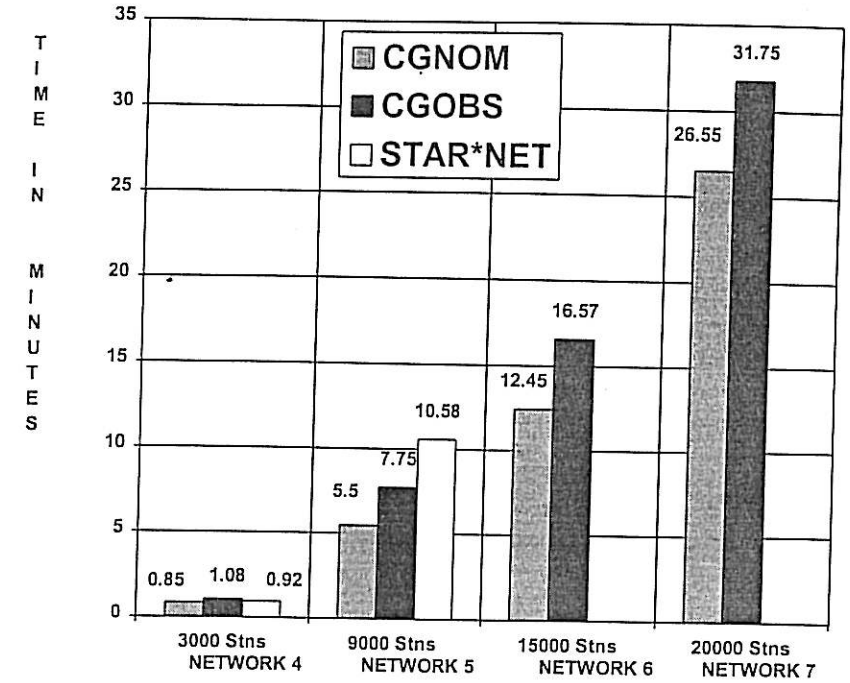


Figure 29a: Execution time for Networks 4 to 7: CGNOM versus CGOBS verses STARNET

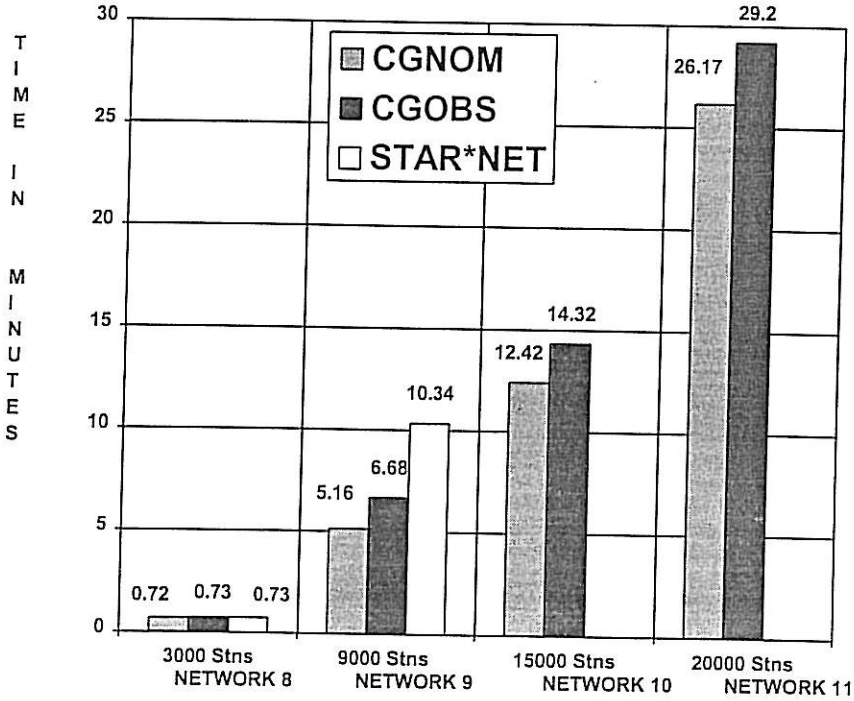


Figure 29b: Execution time for Networks 8 to 11: CGNOM versus CGOBS versus STARNET

6.6 Validity and performance of CGOBS and CGNOM

Both programs CGOBS (solving the observation equations) and CGNOM (solving the normal equations) have shown to produce valid results. The solution of all networks (1 to 11) using the two approaches agreed to 12 decimal places. The output from the test networks 1,2 and 3 are shown in Tables 24-29, but the output of the networks 4 to 11 whose total stations ranges from 3000 to 20000 are not presented, to save space. All the results from the two program CGOBS and CGNOM proved that the effect of rounding-off error in CGM is insignificant.

The performance of CGOBS and CGNOM are almost the same for small networks (say less than 3000 stations), but for larger networks, CGNOM showed to be more efficient than CGOBS in term of computer storage and execution time. GNOM stores smaller amount of nonzero elements than the CGOBS. Therefore, the length of vector **n** which stores the elements of the coefficient matrix of CGNOM is longer than the vector **a** which stores the elements of CGNOBS. The length of these two vectors for networks 4 to 11 are shown in Figures 30a & 30b. On average, the length of vector **n** is 35% less than the length of vector **a.**

In terms of speed, CGNOM is one and a half times faster than CGOBS, as indicated in Figures 29a & 29b. The number of iterations in CGOBS being less than that of CGNOM, see Figures 31a & 31b indicates that the time to complete an iteration in CGOBS is longer than the time taken by CGNOM. This is because, as

per iteration, the number of arithmetic operations in CGOBS is greater than in CGNOM. CGOBS has two major matrix operations (see Table 8) while CGNOM has only one matrix operation (see Table 20) to complete an iteration. On the other hand, the process of creating the system of linear equations from the measurements is simpler in CGOBS. CGNOM has to transform the observation equations into normal equations before CGM is used to solve for the unknowns. However, the transformation process is shorter as compared to the additional matrix operations carried out in the observation equations approach.

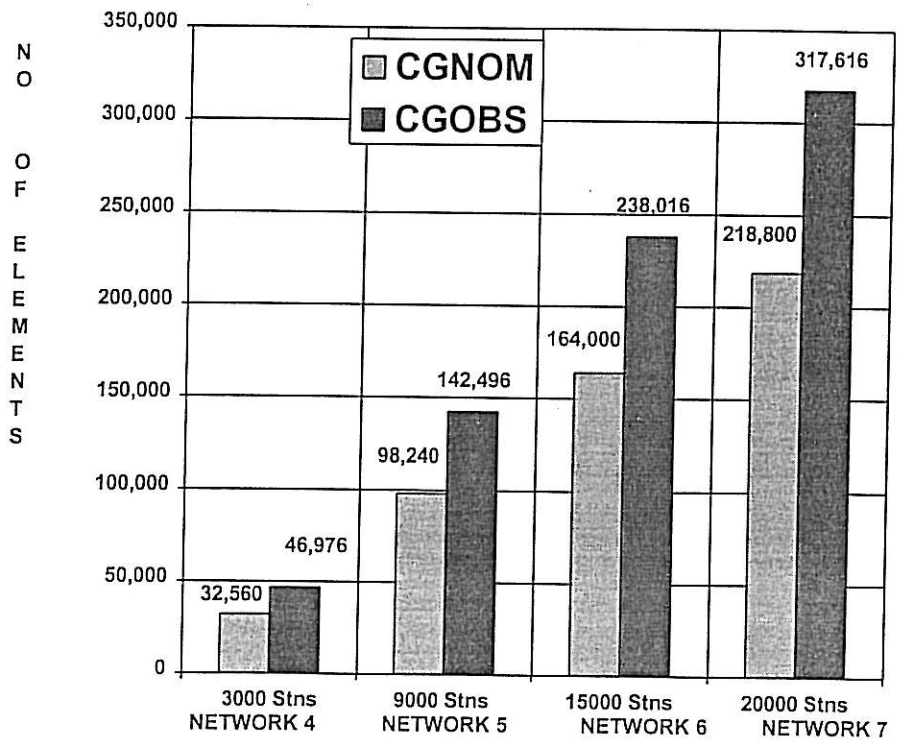


Figure 30a: Number of elements for Netwroks 4 to 7: CGNOM versus CGOBS

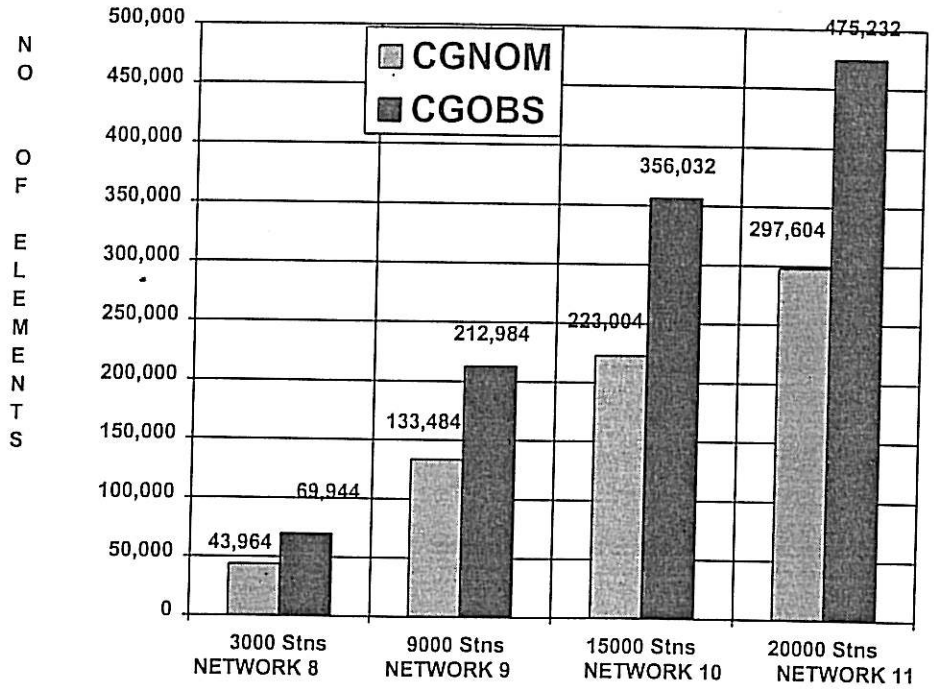


Figure 30b: Number of elements for Networks 8 to 11: CGNOM versus CGOBS

Although the approach of solving the observation equations was found to be less efficient than that of solving normal equations, this is only true for the case of cadastral networks. Note that the system of equations in cadastral networks are actually non-linear systems. They are linearized in the form of the observation equations and later transformed to normal equations. The approach of directly solving the observation equations using the CGM is a new and unique approach, and it is still has some potentials for optimizing special survey networks particularly which generate linear systems. Such a network is found in levelling survey, and recently in modern GPS survey which produces baselines vectors (DX, DY, DZ). Since GPS is becoming a common tool for today's survey operations and in the near future, the solution of the observation equation by the CGM can be an important tool towards total network optimization.

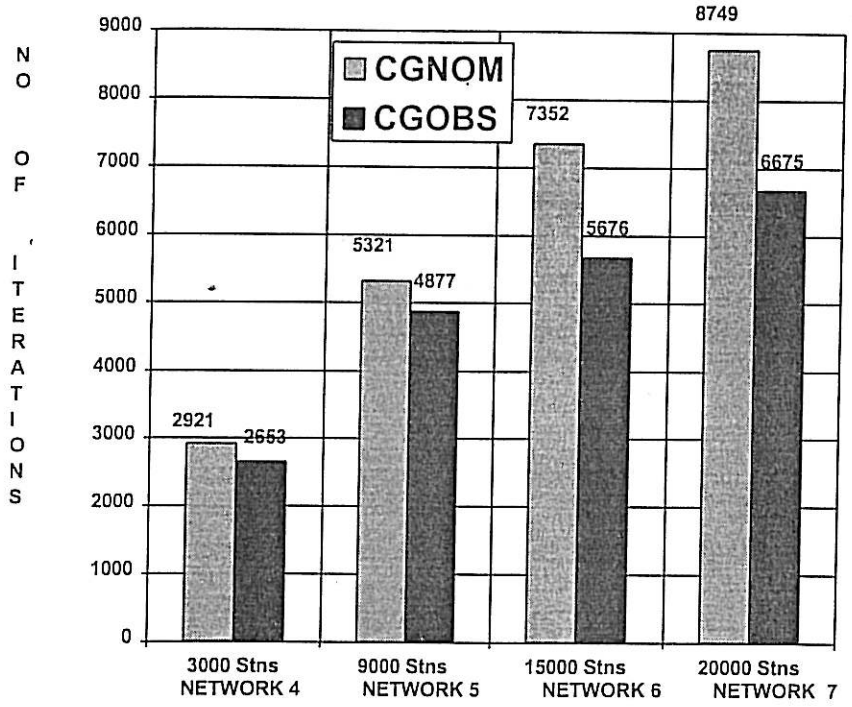


Figure 31b: Number of iterations for Networks 4 to 7: CGNOM versus CGOBS

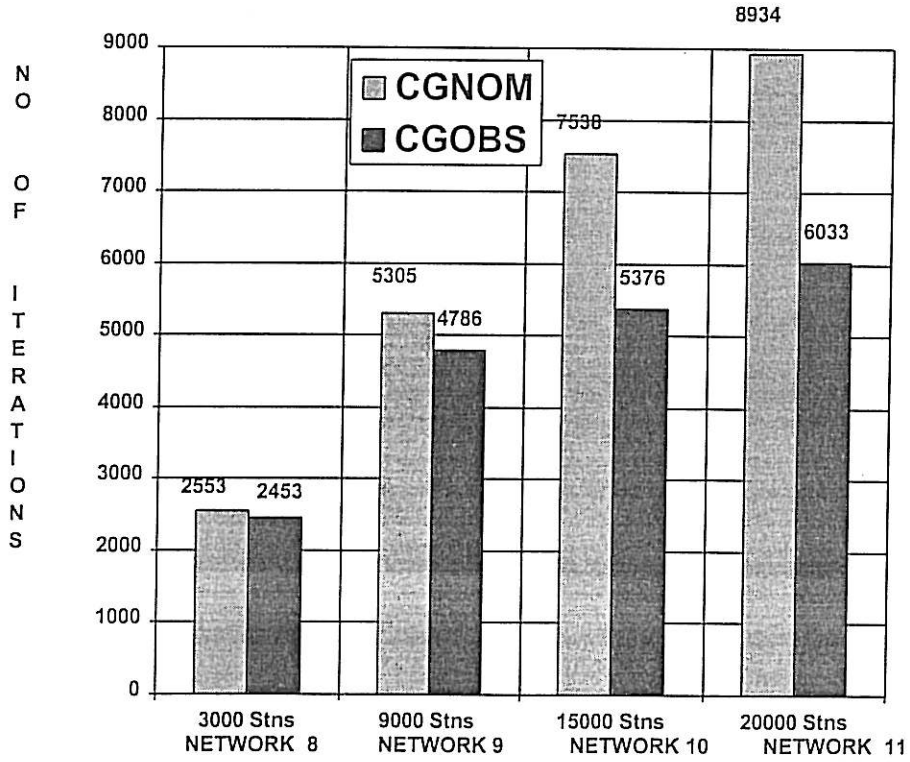


Figure 31b: Number of iterations for Networks 8 to 11: CGNOM versus CGOBS

REFERENCES

Abdullah Daud (1989). Combined processing of satellite and terrestrial observations, M. Phil thesis, University of Newcastle Upon Tyne.

Abdullah Daud (1990) " Combined processing of satellite and terrestrial observations", The Surveyor. The professional Journal of the Institution of Surveyor Malaysia. Vol 23, No 3; 78-81.

Abdullah Daud (1992) " Optimization in configuration of Electronic Distance Measurement (EDM) baseline. Buletin Ukur. Jilid 3, Bil 2; 49-55.

Abdullah Daud (1993). 3DSUITE: Least squares computation in three dimensional computer program, Faculty of Geoinformation Sciences & Eng. UTM, Unpublished.

Abdullah Daud (1994a). CADAS: Cadastral computation computer program, Faculty of Geoinformation Sciences & Eng. UTM, Unpublished.

Abdullah Daud (1994b) 2DTRA : 2D coordinate transformation, Faculty of Geoinformation Sciences & Eng. UTM, Unpublished.

Abdullah Daud (1995a). " Developing digital cadastral databases for GIS: The conjugate gradient method". Proceedings of seminar on the integration of remote sensing and GIS for applications in South East Asia, Kuala Lumpur, March 27-29.

Abdullah Daud (1995b). " A conjugate gradient approach to least squares analysis of cadastral survey data" , Bulletin ukur, Fakulti Ukur dan Harta Tanah,UTM. Jilid6 ; No 2.

Abdullah Daud (1995c). TRANS: 3D coordinate transformation, Faculty of Geoinformation Sciences & Eng. UTM, Unpublished.

Abd. Majid A. Kadir, Ghazali Desa & Tham Siew Kee (1993). "The integration of GPS and GIS" The Surveyor. The Professional Journal of The Institution of Surveyors Malaysia.

Abd. Majid A. Kadir & Ghazali Desa (1994)." Analisis pengukuran GPS untuk penubuhan titik kawalan ukur kadestra". Bulletin Ukur, Fakulti kej. & Sains Geoinformasi. Jld 5 , No 3; 155-168.

Alan, G. & Joseph W.H.Liu (1981). Computer solution of large sparse positive definite systems. Prentice-Hall, Inc, Englewood Cliffs, New Jersey.

Alberda, J.E. (1980). " A review of analysis techniques for engineering survey control schemes." Industrial and engineering survey conference, London U,K.

Alonso, R.D. Barbarra H. G.Malina & A Abad (1988)." Quasi-copies: Effective data storing for information retrieval system" Advances in database technology EDBT '88. Lecture notes in computer sciences No 303. Springer Verlag New York.

Anderson, S. (1980). " Swedish Land Information system. " The Canadian Surveyor. vol.(34); 14 -120.

Aronof, S. (1989) Geographic Information System: A Management Perpective, Ottawa, WDL Publication.

Ashkenazi, V. (1976). " Strength of long lines in terrestrial geodetic control networks. Bulletin Goedesique, 50(2); 169-180.

Baarda, W. (1967). " Statistical Concepts in Geodesy." Publications on Geodesy New Series 2(4). Netherlands Geodetic Commission,Delft, The Netherlands.

Baarda, W. (1968). " A testing procedure for use in Geodetic Networks" Publication on Geodesy New Series 2(5). Netherlands Geodetic Commission, Delft, The Netherlands.

Bhirud, L.L.(1975). Matrix operations on the computer. Oxford & IBH Publication Co. New Delhi

Blakemore, M. (1984) " Generalization and error in spatial databases", Cartographica. Vol 29; 189-202.

Bomford, G. (1980). Geodesy. 4th edition. Clarendon Press, Oxford.

Burgreen, G. W. & Baysal, O. (1994) " Aerodynamic shape optimization using preconditioned conjugate gradient methods". AIAA Journal V 32, No 11; 2145-2152.

Burrough, P.A. (1986). Principles of geographic information systems for land resource assessment, Oxford, Oxford University Press.

Bose, T. & Chen, M. Q. (1995). "Conjugate gradient method in adaptive bilinear filtering", IEEE Transactions on signal processing Vo 43, No 6; 1503-1508.

Caspary, W.F. (1987). " Concepts of network and deformation analysis" . Monograph ii, School of surveying. University of New South Wales, Kensington, N.S.W Australia.

Campbell, G. & Egesborg, P. (1990). " Designing a Property Fabric Information System for Canada Lands" . Proceeding of the XIX FIG Congress, Helsinki, Finland. Jun 10-19, vol 3; 619-629.

Carr, P.E. (1990) " Coordination of the cadastre in New South Wales: A Holistic Approach", The Australian Surveyor. March, Vol 35, No 1.

Chrisman, N.R. (1984). " The role of quality information in the Long-Term Functioning of a Geographic Information System" . Proceedings of the 6th International Symposium on computer Assisted Cartography, Washigton D.C ; 303-312.

Chrisman, N.R.(1989). " Modeling error in overlaid categorical maps" in Accuracy of spatial databases" , edited by M.F. Goodchild and S Gopal, London: Taylor & Francis.

Clark, D. W. (1987). "Cadastral Overlay : Ontological Accuracy." Proceedings of the GIS '87, San Francisco, Oct 26-30, vol.2; 666-677.

Cooper, M.A.R. (1981) Fundamental of survey measurement and analysis, London, Crosby Lockwood Staples.

Cooper, M.A.R. (1987) Control surveys in civil engineering, New York, Nichols Publishing Company.

Cornelis, A. B. , Sherif, A. & Desmond, D. (1996) " GIS and Geointegration", ITC Jounal, Special Habitat issues, Escede, the Netherland.

Cowen, D.J. (1988). " GIS vs CAD vs DBMS : What are the differences" Photogrametric Engineering and Remote Sensing. Vol. 54, no 11.

Crook, C. (1990). " Upgrading the Spatial Accuracy of the New Zealand Digital Cadastral Database" .' Proceedings of the Symposium on Spatial database Accuracy. Melbourne, Australia, Jun 19-20; 84-94.

Cross, P.A. (1981) Geodetic Suite, Working paper no 4, North East London Polytechnic, Department of surveying.

Cross, P.A. (1983)."Advanced least squares applied to position- fixing." Working Paper no.6. North East London Polytechnic.

Cuthill, E. & McKee, J. (1969) " Reducing the bandwidth of sparse symmetric matrices", Proc 24th Nat. Conf. Assoc. Comput. Mach, ACM.

Dale, P.F. (1976). Cadastral surveys within the Commonwealth. London: HM Stationery Office.

Dale, P.F. & Mclaughlin J. D.(1988). Land information Management. New York: Oxford University Press.

Dongarra, J. J. & Van der Vorst H. A. (1992) " Performance for various computers using standard techniques for solving sparse linear equations", Supercomputer Vol 9.

Dangermond, J. (1989). "Entering Digital Data into a GIS" . Proceedings of the AM/FM International Conference XII, New Orleans, Apr 10-13
Durgin P.M (1993) ; 129-137.

Doytser, Y. & Eytan Gelbman (1995) " Rubber-sheeting algorithm for cadastral maps" , Journal of surveying engineering v 121, No.4.

Dragomir, V. C, Ghitau D.N Mihailescu M.S. & Rotaru M.G. (1982) Theory of the earth's shape, Amsterdam, Elsevier

Durgin, P.M. (1993) "Measurement based databases: one approach to the integration of survey and GIS cadastral data," Surveying and Land Information Systems V 53, No1.

Elfick, M. H. (1989),"Management of land Boundary Information." Proceedings of the 31st Australian Survey Congress, Hobart, Australia ; 103-113

Etter, D.M.(1987) Structured Fortran 77 for engineers & scientists 2nd Ed. The Benjamin/Cummings publications, California.

Ethridge, M. M.(1989). " Does the National Geodetic Reference System Need to be Upgraded " P.O.B, Vol 15, No 1.

Fletcher, R. & Reeves C.M.(1959) "Function minimization by conjugate gradient" , Computer Journal 6.

Fisher, P. (1991) " Modelling soil map-unit inclusions by Monte Carlo simulation". International Journal of Geographic Information system, No 5.

Fisher, P. (1993) " Algorithm and implementation uncertainty in viewshed analysis". International Journal of geographic Information system, No 7.

Forsythe, G.E & Moler, C.B. (1967). Computer Solution of linear algebraic Systems. Englewood Cliffs, Prentice-Hall.

Frank, A.U. (1990) "Spatial Concepts, Geometric Data Models and Data Structures". Paper presented at the GIS Design Models and Functionality Conference, Leicester U.K.

Frank, A.U. , Egenhofer, M.J. & Kuhn, W.(1991). "A perspective on GIS technology in the nineties" Photogrametric Engineering and Remote Sensing, vol 57, no 11; 1431-1435.

Friedley, D. (1989)." Digital Cadastral Mapping Maintenance Issues in Local Government LIS/GIS Programs." Paper presented at the URISA Annual Conference, Boston.

Fu, Z. & Dowling, E. M.(1994). " Conjugate gradient projection subspace tracking", Proceedings of Conference on signals, systems & computers Vol 1, IEEE.

Fu, Z. & Dowling, E. M. (1995) " Conjugate gradient eigen structure tracking for adaptive spectral estimation". IEEE Transactions on signal processing Vol 43; 1151-1160.

George, A. & Joseph, L. (1981). Computer solution of large sparse positive definite systems, New Jersey, Prentice-Hall, Inc.

GEOsurv Inc (1993): GeoLab, User Guides. GEOsurv, Ottawa, Canada.

George, E. F., Michael, A.M. & Cleve, B. M (1977). Computer methods for mathematical computations. New Jersey, Prentice-Hall Inc.

Gijzen, V. M. (1995)." Conjugate gradient-like solution algorithms for mixed finite element approximation of the Biharmonic equation, applied to plate bending problem". Computer Methods in Applied Mechanic and Engineering Vol 121, No 1-4; 121-136.

Golub, G. et. all (1996) Recent advances in iterative methods, New York, Springer-Verlag.

Goodchild, M.F. (1989). "Modelling error in objects and fields" Accuracy and spatial databases". Edited by M.F Goodchild and S. Gopal. New York: Taylor & Francis ; 107-113.

Goodchild, M.F & Gopal, S (1989). Accuracy of spatial databases. New York: Taylor & Francis.

Goodchild, M.F. (1991). "Issues of quality and uncertainty" in Accuracy of spatial databases, edited by M.F. Goodchild and P Fisher, Workshop on annual meeting, Association of American Geographers, Miami.

Gregory, A. E. , Guoray, C. & Mark, J. T. (1995) "Estimating spatial data error by inference on a casual network" .Proceedings of the sixth international symposium on spatial data handling, Taylor and Francis

Grundig, L. & Bahndorf, J. (1984)." Accuracy and reliability in geodetic networks-Program system OPTUN," Journal of surveying engineering vol 110, no 2.

Grundig, L. & Bahndorf, J (1985). "OPTUN- A program system for extended preanalysis and adjustment" . Proceeding of the precise engineering and deformation survey workshop, Calgary, Canada.

Hadjiraftis, A.C & Jones, A.C. (1990). "The reconciliation of DCDB-dependent spatial datasets with DCDB upgrades" . Proceeding of the symposium on spatial database accuracy, Melbourne, Australia June 19-20; 95-112.

Hageman, L. A. & Young, D. (1981) Applied iterative methods. New York, Academic press.

Halmos, F. & Kadar, I. (1977)." An attempt to interpret physically the notion-system of geodetic information" . Bulletin Geodesique, no 51; 1-16.

Halmos, F. , Kadar, I. & Karsay, F. (1974)." Local adjustment by least squares filtering" . Bulletin Geodesique, no 111; 21-51.

Halim Setan (1995). Functional and stochastic models for geometrical detection of spatial deformation, Ph.D. thesis, City University London.

Hamilton, A. C. (1982)." Introduction to the concept of a matrix for land information". Proceedings of Centennial Convention, Canadian Institute of Surveying, Ottawa, April ; 485-499.

Harary, F. (1969). Graph Theory, Addison-Wesley Publication Company.

Hebblethwaite, D.H (1989). " Concepts for coping with a shifting cadastral model." The Australian Surveyor, vol 34, no 5; 486-493.

Henderson, J.P.(1989). "Digital Geographic data quality-Two case studies". New Zealand Surveyor, vol 32 no 274; 375-386.

Hestenes, M.R. (1959) "Iterative computational methods", Communications on pure and applied mathematics, No 9 ; 85-95.

Hestenes, M.R.(1980). Conjugate direction methods in optimization, New York, Springer-Verlag.

Hestenes, M.R. & Steifel (1952) " Methods of conjugate gradients for solving linear systems" J. Res. Nat. Bur. Standard 49 ; 409-436.

Henssen , J.L.G & Williamson, I.P. (1990). " Land Registration, Cadastre and its Interrelations: A world perspective." Proceedings of the F.I.G XIX International Congress, Helsinki, Finland, June 10-19 vol 7; 13-43.

Hintz, R.J. , W.J. Blackham, B. M. Dana & J.M. Kang (1988). "Least squares analysis in temporal coordinate and measurement management". Surveying and mapping vol 48, no 3; 173-183.

Hintz, R,J. & Onsrud, W.J. (1990)." Upgrading Real Property Boundary Information in a GIS." URISA Journal, Vol 2, no1; 2-10.

Hintz, R.J.(1994) Personal communication.

Hintz, R.J., Abdullah D, Wahl J.L, McKay D & Wurm K (1995) "Evaluation of the conjugate gradient approach to solution of large least squares problems in surveying" ACSM proceeding vol 1 ; 92-101.

Hunter, G.H. (1991). Proceedings of the symposium on spatial database accuracy. Melbourne, The University of Melbourne.

Hunter, G.J. & Williamson, I.P. (1990). "The development of a historical digital cadastral database" . International journal of geographic information systems vol 4, no 2 ; 169-179.

Hunter, G.J. & Goodchild, M.F(1995) "Dealing with error in spatial databases: A simple case study" , Photogrammetric Engineering & remote sensing, Vol 61 No 5.

Jacobi, O. (1988)." Error propagation in digital maps". Proceedings of the XVI Congress of international society for photogrammetry and remote sensing, Kyoto, Japan Jul 1-10 vol 3 ; 348-356.

Jayanandran, D. & G.J. Hunter (1990). Proceedings of the national conference on cadastral reform 90. Melbourne, Department of surveying and land information, University of Melbourne.

Jennings, A. (1966)." A compact storage scheme for the solution of symmetric linear simultaneous equations". Computer Journal No 9; 281-285.

John McLaughlin & David Palmer (1996) "Land Registration and development", ITC Journal, Special Habitat issues, Escede, The Netherlands.

Jo Henseen (1996) " Cadastral information, an important land management tool" ITC Journal, Special habitat issues, Escede, The Netherland.

Jorgenson, P.C. K. Kubik P. Frederiksen & Weng (1985). "Ah, Robust estimation" . Australian Journal of Geodesy, Photogrammetry and surveying No 42 ;19-32.

JUPM (1995) Towards the implementation of a coordinated cadastral survey system in Peninsula Malaysia. JUPM, Malaysia Unpublished.

Kjellberg, P.(1990)." Towards a Spatial Data Model for Land Information Systems Supporting Coordinate Re-adjustments and Topographical Map Production." Paper Presented at 3rd Scandinavian Research Conference on Geographical Information Systems, Helsingor,Denmark,Nov 14-16.

Kjerne,D. (1987). Modelling Cadastral Spatial Relationships Using An Object-Oriented Information Structure. Msc. Thesis, Department of Geography, Portland State University, Portland, Oregon.

Klessig, R. & Polak. E. (1972)" Efficient implementations of the Polak-Ribbie conjugate algorithm." SIAM Journal Control 10; 524-549.

Kopak, S. G. (1993) "Cadastral survey's automation related to surveying and land information systems" , Surveying and Land Information System V 53.

Krakiwsky, E.J. (1982)." A synthesis of recent advances in the method of least squares" . Publication no. 10003. Department of surveying engineering, The university of Calgary, Calgary, Canada.

Krakiwsky, E. J. (1985) Papers for the CIS adjustment and analysis seminars, The Canadian Institute of Surveying, July.

Kuang Shanlong (1996) Geodetic network analysis and optimization design: Concept and application, Michigan, Ann Arbor Press, Inc.

Kuhn, W. (1990). " Editing spatial relations" . Proceedings of the 4th international symposium on spatial data handling, Zurich, Jul 23-27 vol 1; 423-432.

Landesman, E. M. & Hestenes, M. R. (1991) " Linear algebra for mathematics, scince and engineering" New Jersey, Prentice Hall.

Leick, A.(1980). " Geometric geodesy, 3D geodesy and conformal mapping" Report no 19. Department of surveying engineering, University of Maine, Orono, USA.

Leick, A.(1990). GPS satellite surveying. New York: John Wiley & Son.

Leung, Y , Goodchild, M.F. & Lin, C.C (1992) "Visualization of fuzzy scenes and probability fields" , Proceedings of the 5th international symposium on spatial data handling, IGU Commission on GIS, Charleston, South Carolina ;480-490.

Loius, G. H. Jr. (1993) " The National Spatial Data Infrastructure is a National Resource" GIS world Sept, USA.

Lowell, K.E (1992) " On the incorporation of error into spatial data systems", Proceedings of the GIS/LIS '92 Annual Conference, San Jose, California; 485-493.

MacLean, E. (1992) " The use of variability diagrams in a GIS to improve the interpretation of Digital Soil Maps" , Proceedings of the 1992 ACSM/ASPRS Annual Conference, Washington, DC ; 66-76.

Maling, D.H. (1992) Coordinate System and map projections. 2nd edition. University of Wales, Pergamon Press.

Masters, E.G. (1988)." Upgrading a digital graphic database." Proceedings of the URPIS 16, Sydney, Australia ; 384-391.

Maurant, G. (1988)." Practical use of the conjugate gradient method on parallel supercomputers." Computer Physics communications v53 n1-3 May 1988.

Marble, D.F. (1989) Geographic information system: an overview, in Peuguet DJ and Marble D.F, London, Taylor & Francis.

Masry, S.E. & Y.C. Lee (1988)."An introduction to digital mapping. Lecture notes no 56", Department of surveying engineering, University of New Brunswick, Fredericton, Canada.

Masters, E.G. (1988). "Upgrading a digital graphical database". Proceedings of the URPIS 16, Sydney, Australia ; 384-391.

Mepham, M.P. & E.J. Krakiwsky (1984) " CANDSN: a computer aided network design and adjustment system" . The Canadian surveyor, vol 38,no 2 ; 99-114.

Mikhail, E.M. (1976) Observation and least squares , New York, Dun Donnelley

Mikhail, E. M. & Gracie, G. (1981) Analysis and adjustment of survey measurements , New York, Van Nostrand Reinhold.

Mikhail, E. M. & Helmering, R. J. (1973) " Recursive methods in photogrametric data reduction". Photogrametric engineering and remote sensing Vol 39 No 9; 983-989.

Monro, D. M. (1982) " Fortran 77", London, Edward Arnold.

Moreno, R.J. & Onsrud, H.J. (1990). "Legally supportable cadastral information system" . Proceedings of the ACSM/ASPRS annual convection, Denver, Colorado, march 18-23 vol 1; 275-279.

NGS (1990). Information flyers of the national geodetic survey. Maryland, National geodetic survey.

Noordin Ahmad & Bong Thai Kwong (1995) "Vectorization of cadastral linework from scanned image" , The surveyor : The professional Journal of the Institution of surveyor Malaysia, 4th quarter.

Nordin Ahmad (1994) "Requirement analysis and success factors of geographic information system" , The surveyor: The professional Journal of the Institution of surveyor Malaysia, Vol 29, No 3.

Nordin Ahmad (1995) "Towards a spatial data sharing" The Surveyor, The professional Journal of the Institution of surveyor Malaysia.4th Quarterly.

NRC (1980). Needs for a multipurpose cadastre. Washigton DC: National geodetic survey, Rockville, Maryland USA.

NRC (1983). Procedures and standards for a multipurpose cadastre. Washington DC: National Academy Press.

Onsrud, H.J. (1990). " Liability concerns for surveyors contributing data to public land information system" . ACSM Bulletin April(20-24).

Ottawa, T(1987). "Accuracy of digitizing: Overlooked factors in GIS operations". Proceedings of the GIS/LIS '87, San Francisco, California vol 1 ; 295-299.

Paul, M. D. (1993)." Measurement-based databases: One approach to the integration of survey and GIS cadastral data." Surveying and land information system, Vol 53 No.1.

Pearson, F.(1990). Map projections: Theory and applications. Boca Raton, CRC Press.

Peterson, J.K. (1989). "An updating procedures for digital map databases to overcome the problem of changing coordinates of geodetic reference points". Proceedings of the GIS/LIS '89. Orlando, Florida Nov 26-30, vol 1 ;134-142.

Peuquet, D.J. (1988). "Representations of geographic space: Towards a conceptual synthesis" Annuals of the association of American geographers vol. 78 no 3; 375-394.

Peuquet, D.J (1989). Introductory readings in Geographic Information System, London, Taylor and Francis.

Pissanetsky, S. (1984) Sparse matrix technology, London, Academic Press.

Polak, E. (1973) " An historical survey of computational methods in optimal control" SIAM Review, vol 15, No 2.

Pope, A.J. (1976). " The statistics of residuals and the detection of outliers". NOAA technical report Nos. 65 NGS 1. Department of commerce, Rockville, Maryland.

Press, W. H., Flannery B P, Teukolsky S A & Vetterling W T(1987) " Numerical recipes: The art of scientific computing" London, Cambridge University Press.

Price, S. (1989). "Modeling the temporal element in land information systems". International journal of geographical information systems, Vol 3 no 3 ;233-243.

Pullar, D.V. (1989). "Data organizational of a digital cadastral database". Proceedings of the ACSM/ASPRS annual conference, Baltimore, Maryland Apr 2-7 vol 5 ; 87-93.

Psimarni, A. (1994) " On the solution of symmetric linear systems by the preconditioned conjugate gradient method". International Journal of Computer Mathematics Vol 52, No 34 ; 225-241.

Renong (1995) " NALIS: National Land Information Infrastructure System". MacDonald Dettwiller & Malaysia Business Association- Unpublished.

Reid, J.K (1971) Large sparse sets of linear equations, Proceedings of the Oxford conference of the Institute of Mathematics and its applications, London, Academic Press.

Robinson, V.B. & Frank, A.U. (1985). " About different kinds of uncertainty in collections of spatial data" . Proceedings of the 7th international symposium on computer assisted cartography, Washington DC, Mar 11-14 ; 440-449.

Schwarz, C. R. (1978) "TRAV10 horizontal networks adjustment program". NOAA technical memorandum NOS NGS 12. National geodetic survey, Rockville.

Schwarz, C.R. (1989). " North American Datum of 1983". Rockville, MD: National geodetic survey.

Scotts, C.M. (1987). " Land base maintenance" . Proceedings of the AM/FM international conference X, Snowmass, Colorado, Jul 20-23; 299-302.

Schemit, L. A. & Lai, Y.C. (1994) "Structural optimization based on preconjugate gradient analysis methods" International Journal for Numerical Methods in Engineering Vol 37, No 6; 943-964.

Shmutter, R. & Doystsher, Y. (1991) " New method for matching digitized cadastral maps" Technical paper 91, ACSM/ASPRS, Bethesde; 201-246.

Shane, A. I. (1996) " Maintaining a GIS: Critical issues". Proceedings AWRA annual symposium, GIS and Water resources, Sept 22-26, Fort Lauderdale, Florida.

Smith, G.L.(1988) "Implementation of a coordinated cadastre for South Australia", The Australian Surveyor, Sept, Vol 4 No 1.

Shariff, M.H.(1995) " A constrained conjugate gradient method and the solution of linear equations". Computer & Mathematics with application, V 30; 25-37.

Snay, R.A. (1976). " Reducing the profile of sparse symmetric matrices". NOAA technical memorandum NOS NGS -4. US Department of Commerce, Rockville, Maryland.

Starplus Software Inc. (1993). STAR*NET-PLUS: Least square survey adjustment programs. Oakland, Canada.

Stefanovic, P.(1980 " Pitfalls in blunder detection techniques". Proceedings of the 14th congress of the international society of photogrametry and remote sensing, Hamburg, Germany, vol 3 ; 687-700.

Suarjana, M. & Lau Kincho (1994) " Successive conjugate gradient methods for structural analysis with multiple load cases", International Journal for Numerical Methods in Engineering vol 37, No 24; 4185-4203.

Taher Buyong & Frank, A.U. (1989) Measurement-based multipurpose cadastre " Proceedings of the ACSM/ASPRS Annual convention, Baltimore. Vol 5 ; 58-66.

Taher Buyong & Hintz, R.J. (1991). " A User Interface for CALS". The Surveyor, The Professional Journal of The Institution of Surveyors Malaysia, Vol 26 ;8-17.

Taher Buyong & Kuhn, W (1990)." Local Adjustment for Measurement-Based Multipurpose cadastral Systems". Proceedings of the ACSM/ASPR Annual Convention, Denver, Colorado, March 18-23, Vol3 ;19-27.

Taher Buyong (1992). Measurement- based multipurpose cadastral system: Ph. D. Thesis. University of Maine.

Taher Buyong (1995). " GIS for Local Authorities" , Bulletin Ukur, Jilid 6, No 1. Fakulti Ukur, UTM.

Tang, Li Q. (1995) " Transient solutions by least squares finite-element method and Jacobi conjugate gradient technique", Numerical Heat Transfer, Part B: Fundamental V 28, No 2; 183-198.

Toh, Swee Eng (1984). ":An introduction to CALS- Computer assisted land survey system" . The surveyor: The professional Journal of the institutional of surveyors Malaysia, vol 19, no 1; 10-14.

UN (1985)." Review of the latest technology and its relationship to policy, Economy and development in cartographic data manipulation." Report of the Ad Hoc Group of experts on cadastral surveying and land information system. United Nations economic and social council, New York.

Vanicek, P. & Krakisky, E.J. (1986) Geodesy: The concepts. Second edition, Holland, North Holand Publishing Company.

Veres, S. A. & Youcai (1987) " Applications of robust estimation in close-range photogrammetry". Photogrammetric engineering and remote sensing Vol 53 No 2; 171-175.

Vincenty, T. (1979) " The HAVAGO three-dimensional adjustment program. NOAA Technical memorandum NGS-17, Rockville.

Weitzman, E. (1989) " Measurement databases for propoerty mapping, URISA Annual conferenee, August 6-10, Boston.

Wells, D.E & Krakiwsky, E.J. (1971). The method of least squares. Dept. of surveying engineering, University of New Brunswick, Fredericton, N.B, Canada.

White, M.S. & Griffin, P. (1985). "Piecewise linear rubber-sheet map transformation" . The American cartographer vol.12, no.2 ;123-131

Williamson, I. P. (1986). "Cadastral and land information system in developing countries". The Australian Surveyor, Vol 33; 27-43.

Wilcox, D.J. (1983). " Classifications, Standard of accuracy, and specifications for cadastral surveying" . Proceeding of the ACSM annual convection, Washington DC Mar 13-18 ; 227-286.

NRC (1980) Needs for a Multipurpose Cadastre. Washington DC: National Academy Press.

Yang, D. Y , Chen, G. S. & Chou, K. P. (1993)." Application of preconditioned conjugate gradient-like methods to reactor kinetics". Annals of Nuclear Energy V 20 N ; 9-33.