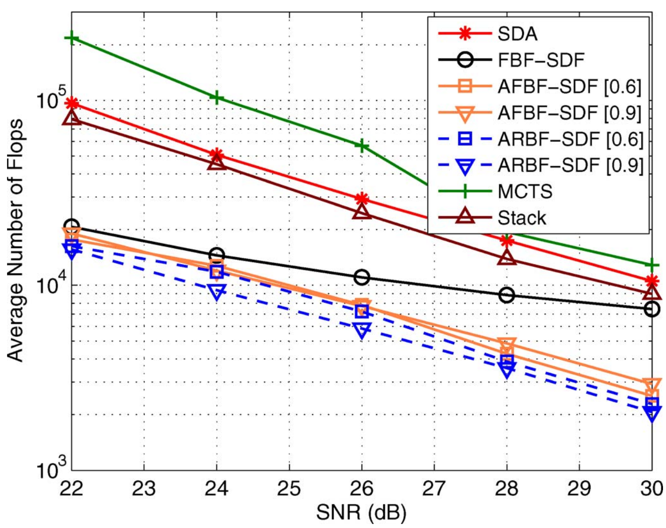

Fig. 4. Comparison of the average flops of various MIMO detection algorithms for the case of 6 × 6, 64-QAM. The AFBF-SDF curves for $P_{t,N} = 0.7$ and 0.8 (which are not shown to avoid cluttering) fall between those for $P_{t,N} = 0.6$ and 0.9, which are same for the ARBF-SDF curves. Note that FCSD, which is not shown in the figure to clearly display the other curves, takes 660 415 flops for any SNR.

the value of $P_{t,N}$ used.) Note that the performance is close to those of the ML detector and FCSD. Figs. 3 and 4 show the corresponding average complexity, respectively. Observe that the average complexity of AFBF-SDF is lower than that of SDA, FBF-SDF, MCTS, the stack algorithm, and FCSD in the entire SNR range considered.

VI. Conclusion

Tree search algorithms that precede the DF search by BF expansion to improve the efficiency of the DF search have been described. They include ML detectors, i.e., FBF- and AFBF-SDF, and a near-ML detector which is referred to as ARBF-SDF. AFBF-SDF effectively switches between the SDA and FBF-SDF modes based on the channel conditions, thereby achieving a much lower complexity—in terms of both the average and the worst-case flops—than SDA in many scenarios. This comes with a moderate increase in the memory requirement (which is still linear in n_t).

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Linear Precoded Cooperative Transmission Protocol for Wireless Broadcast Channels

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***Abstract*—In multiple-input–single-output (MISO) broadcast channels, the achievable diversity gain is limited by the number of source transmitter antennas. Cooperative transmission is able to provide another dimension of diversity gain improvement. In this paper, we introduce a spectrally efficient cooperative transmission protocol in MISO broadcast channels using linear precoding and nonorthogonal relaying. The proposed protocol can achieve maximum diversity gain expressed as the sum of the number of source transmitter antennas and the number of available relays. The diversity and multiplexing tradeoff of the proposed protocol outperforms the noncooperative scheme, even when only a single relay is used. For a large number of relay candidates, the diversity and multiplexing tradeoff of the proposed protocol completely surpasses the existing noncooperative scheme.**

***Index Terms*—Cochannel interference, diversity and multiplexing tradeoff, multiple-input–single-output, relay, zero-forcing.**

I. Introduction

The achievable diversity gain of the multiple-input–single-output (MISO) broadcast channels is constrained by the number of source antennas. Cooperative transmission is able to provide another dimension

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of diversity gain improvement by employing distributed nodes acting as relays to form virtual antennas that forward the information broadcast by the source to the destinations [2]. In MISO broadcast channels supporting space-division multiple access (SDMA), relay/relays equipped with multiple antennas can be utilized to provide another dimension of diversity gain improvement. A multiantenna relay can be an infrastructure relay with centralized antennas [3] or a cluster of single-antenna users [4] who are in idle mode.[1] These relays are carefully placed or selected to ensure that they have superior link quality.

Initial work on cooperative broadcast channels (CBCs) focuses on single-antenna configuration. A dynamic decode-and-forward (DF) strategy is proposed in [5], and it is shown to achieve full diversity gain. However, multiantenna spatial multiplexing is not considered. To bridge the gap, we consider the CBCs supporting spatial multiplexing in MISO broadcast channels. Introducing relays into the network might not be beneficial when the base station and the relays share common cellular bandwidth, as evident in multihop relaying schemes, which require an orthogonal channel for each relay transmission hop [6]. However, by allowing the relay and the base station to simultaneously access the channel, i.e., nonorthogonal relaying, we can prevent sacrificing the common bandwidth.

In this paper, we propose a linear precoded CBC transmission protocol for the MISO broadcast channels that consists of a multiantenna source, M single-antenna destinations, and one or more multiantenna relays. The proposed protocol implements a nonorthogonal relaying strategy, which allows the source and the relays to simultaneously access shared bandwidth. Linear precoding is performed at the source and relays to eliminate cochannel interference. The achievable diversity and multiplexing tradeoff of the proposed protocol dominates the comparable noncooperative zero-forcing beamforming (ZFBF) scheme [7], even when only a single relay is used. Monte Carlo simulations verify that the outage probability of the proposed scheme outperforms the noncooperative scheme.

II. Description of the Cooperative Broadcast Channel Protocol

Consider a broadcast scenario with one multiantenna source, M active[2] single-antenna users, and L multiantenna relays. The source and each relay are equipped with M antennas to support full spatial multiplexing. The relays use the DF relaying strategy. An example of the cooperative broadcast scenario with $M = 2$ and $L = 1$ is shown in Fig. 1. The network is assumed to be symmetrical,[3] where each channel is independent identically distributed with the same channel statistics. The channels are modeled as frequency-nonselective quasi-static Rayleigh fading. The noise observed at each receiver is circularly symmetric complex Gaussian distributed, i.e., $n \sim \mathcal{CN}(0, \sigma_n^2)$. Practical half-duplex constraint is imposed on all nodes.

We use the baseline scheme, i.e., ZFBF with semi-orthogonal user scheduling [7], for direct transmission between the source and destinations to isolate the benefits of using cooperative relays. The ZFBF with user scheduling scheme [7] allows the source to schedule

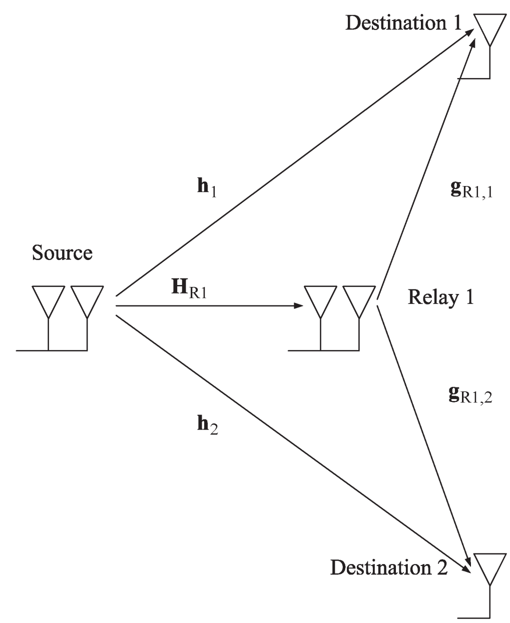

Fig. 1. Cooperative broadcast scenario when $M = 2$ and $L = 1$. Two-antenna source, two-antenna relay, and two single-antenna users.

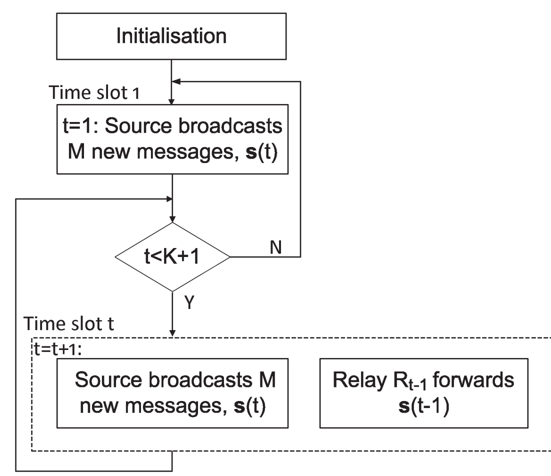

Fig. 2. Flowchart of the proposed CBC protocol.

M active destinations, which have good channel gains and channel directions matching the beam directions of the ZFBF. For convenience of analysis, we assume that an asymptotically large number of users are available to achieve perfectly orthogonal source–destination channels. All nodes assume perfect knowledge of local channel state information (CSI) to enable linear precoding at the source and relays and coherent detection at the relays and destinations. The CSI is obtained in the initialization phase. The flow chart in Fig. 2 summarizes the proposed protocol. The proposed protocol is described in time-slotting fashion.

1) First Time Slot: The source simultaneously broadcasts M messages, such that $\mathbf{x}(1) = \mathbf{W}\mathbf{s}(1)$, where $\mathbf{s}(1) = [s_1(1) \quad \cdots \quad s_M(1)]^T$ with $[.]^T$ denoting transposition; the scalar $s_m(t) \in \mathbb{C}$ is the message transmitted at time slot t to the mth destination; and $\mathbf{W} \in \mathbb{C}^{M\times M} = [\mathbf{w}_1 \quad \cdots \quad \mathbf{w}_M]$ is the precoding weighting matrix, with $\mathbf{w}_m \in \mathbb{C}^{M\times 1}$ as the precoding weighting vector of destination m. The signal received by destination m $\forall m \in \{1,\ldots,M\}$ can be expressed as

$$y_m(1) = \sqrt{E_s}\mathbf{h}_m^T\mathbf{w}_m s_m(1) + n_m(1) \quad (1)$$

[1]The bandwidth cost of clustering (i.e., information exchange overhead between members in a cluster) to the cellular system can be avoided by using secondary radio resources available at the mobile users, such as WIFI and Bluetooth.

[2]Active users are the users served by the source at a particular time. In cellular networks using SDMA, the total number of users is usually much larger than the number of antennas at the base station. Scheduling is performed to enable time sharing between groups of users.

[3]Symmetric assumption simplifies analytical development without affecting the applicability of the CBC scheme in an asymmetric network.

where $\mathbf{h}_m \in \mathbb{C}^{M\times 1}$ is the channel from the source to destination m, E_s is the average transmit power for each message, and $n_m(t)$ is the noise observed at the mth destination at time slot t. Since M parallel MISO channels can be created using user scheduling in an asymptotically large network, we can model the ZFBF with semi-orthogonal user scheduling as a maximal ratio transmission beamformer[4] at each parallel channel, such that $\mathbf{w}_m = (\mathbf{h}_m^*/\|\mathbf{h}_m\|)$, with $[.]^*$ denoting complex conjugation and $\|.\|$ representing the Euclidean norm. The normalizing factor $\|\mathbf{h}_m\|$ is to ensure unit transmission power for each stream, i.e., $\mathbf{w}_m^H\mathbf{w}_m = 1$. Meanwhile, the signal received by relay k is

$$\mathbf{r}_k(1) = \sqrt{E_s}\mathbf{H}_{R_k}\mathbf{W}\mathbf{s}(1) + \mathbf{n}_k(1) \tag{2}$$

where $\mathbf{H}_{R_k} \in \mathbb{C}^{M\times M}$ is the channel from the source to relay k, and $\mathbf{n}_k(t) \in \mathbb{C}^{M\times 1}$ is the receiver noise vector.

Out of a total of L available relays, there are K qualified relays that manage to successfully decode all messages from the source. Since a symmetrical system is considered, each user is allocated with the same data rate, i.e., R bit/s/Hz. Assuming Gaussian coding with long codewords and a zero-forcing receiver [8], the qualification criterion for relay k can be expressed as

$$\log_2\left(1 + \frac{\gamma_0}{\left[\left(\mathbf{W}^H\mathbf{H}_{R_k}^H\mathbf{H}_{R_k}\mathbf{W}\right)^{-1}\right]_{m,m}}\right) > R. \tag{3}$$

$\forall\, m \in \{1, 2, \ldots, M\}$, where $[\mathbf{V}]_{m,m}$ represents the mth diagonal element of matrix $\mathbf{V}$, and $\gamma_0 = (E_s/\sigma_n^2)$ is the mean signal-to-noise ratio (SNR). All K qualified relays decode and store the messages in memory.

2) Subsequent Time Slots: In subsequent time slot t, where $t = 2, \ldots, K+1$, the source concurrently broadcasts M new messages, such that $\mathbf{x}(t) = \mathbf{W}\mathbf{s}(t)$. Denote $\mathbf{g}_{R_k,m} \in \mathbb{C}^{M\times 1}$ as the channel between relay R_k and the mth destination. All K qualified relays are scheduled to transmit in a round-robin fashion. Only one relay transmits at each cooperative time slot. Specifically, relay k $\forall\, k \in \{1, \ldots, K\}$ forwards the precoded messages $\mathbf{x}_k = \mathbf{Q}_{R_k}\mathbf{s}(t-1)$, where $k = t-1$, and the precoding matrix $\mathbf{Q}_{R_k} \in \mathbb{C}^{M\times M} = [\mathbf{p}_{R_k,1} \quad \cdots \quad \mathbf{p}_{R_k,m} \quad \cdots \quad \mathbf{p}_{R_k,M}]$, with $\mathbf{p}_{R_k,m} \in \mathbb{C}^{M\times 1}$ as the precoding vector for destination m. Denote $\mathbf{A}_{R_k,m} \in \mathbb{C}^{(M-1)\times M}$ as a matrix that contains all the interfering channels of relay R_k (all channels other than its targeted mth destination), i.e., $\mathbf{A}_{R_k,m} = [\mathbf{g}_{R_k,1} \quad \cdots \quad \mathbf{g}_{R_k,m-1}\ \mathbf{g}_{R_k,m+1} \quad \cdots \quad \mathbf{g}_{R_k,M}]^T$. To nullify the cochannel interference, $\mathbf{p}_{R_k,m}$ is designed to lie in the null space of interference, i.e., $\mathbf{p}_{R_k,m} \in null(\mathbf{A}_{R_k,m})$, so that $\mathbf{g}_{R_k,i}^T\mathbf{p}_{R_k,j} = 0$ for any $i \neq j$. Vector $\mathbf{p}_{R_k,m}$ has unit power, i.e., $\mathbf{p}_{R_k,m}^H\mathbf{p}_{R_k,m} = 1$. At the mth destination, the observation is

$$y_m(t) = \sqrt{E_s}\|\mathbf{h}_m\|s_m(t) + \sqrt{E_s}\mathbf{g}_{R_k,m}^T\mathbf{p}_{R_k,m}s_m(t-1) + n_m(t). \tag{4}$$

Although the remaining relays also receive a mixture of the messages, they will be able to decode the source messages using simple successive decoding since they satisfy the qualification criterion in (3).

The relaying process continues until time slot $t = K+1$, where all K qualified relays have been used. Due to half-duplex constraint, each relay can only be used once in each cooperative transmission phase.[5] The use of multiple relays, i.e., $K > 1$, is beneficial in terms of the diversity and multiplexing tradeoff, which will be discussed in Sections III and IV. Letting $\mathbf{s}_m = [s_m(1) \quad \cdots \quad s_m(K+1)]^T$, the channel matrix

$$\mathbf{H}_m = \begin{bmatrix} \|\mathbf{h}_m\| & 0 & \cdots & 0 \\ \mathbf{g}_{R_1,m}^T\mathbf{p}_{R_1,m} & \|\mathbf{h}_m\| & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & \mathbf{g}_{R_K,m}^T\mathbf{p}_{R_K,m} & \|\mathbf{h}_m\| \end{bmatrix}$$

and noise vector $\mathbf{n} = [n_m(1) \quad \cdots \quad n_m(K+1)]^T$, the signal model after stacking $K+1$ time slots is

$$\mathbf{y}_m = \mathbf{H}_m\mathbf{s}_m + \mathbf{n}. \tag{5}$$

The elements on the main diagonal of the effective channel matrix $\mathbf{H}_m$ represent the direct transmission links (source to destination), whereas the elements on the lower subdiagonal represent the cooperative links (relays to destination). Assuming that Gaussian coding and long codewords are used to convey messages, the mutual information at the mth destination node can be expressed as

$$\mathcal{I}_K = \frac{1}{K+1}\log_2\det\left(\mathbf{I}_{K+1} + \gamma_0\mathbf{H}_m\mathbf{H}_m^H\right) \tag{6}$$

where $[.]^H$ represents Hermitian transposition, and the prelog reflects the total number of channel uses. Mutual information of the baseline scheme, i.e., noncooperative ZFBF with user scheduling [7], can be derived by omitting the cooperative links (subdiagonal elements) from the channel matrix $\mathbf{H}_m$.

III. Analysis of Outage Probability and the Diversity and Multiplexing Tradeoff

This section provides performance evaluation of the proposed CBC protocol using outage probability and the diversity and multiplexing tradeoff. The diversity and multiplexing tradeoff is derived using outage formulation, which is usually used for non-ergodic fading channels, i.e., quasi-static channels. Recall that the diversity gain d and the multiplexing gain r of a multiple-input–multiple-output system can be defined as [9]

$$d \triangleq \lim_{\gamma_0\to\infty} -\frac{\log P_e(\gamma_0)}{\log\gamma_0} \text{ and } r \triangleq \lim_{\gamma_0\to\infty}\frac{R(\gamma_0)}{\log\gamma_0} \tag{7}$$

where P_e is the maximum-likelihood (ML) probability of detection error, R is the target data rate in bits per second per hertz, and γ_0 is the mean SNR. Since the ML error probability is tightly bounded by the outage probability at high SNR, the outage probability can be used to derive the diversity and multiplexing tradeoff. The use of ZFBF at the source and relay enables the decomposition of the MISO broadcast channels into parallel MISO channels, i.e., a set of point-to-point MISO channels that are orthogonal to each other. Recall the symmetrical network assumption. This results in each of the parallel MISO channels to exhibit statistically identical outage behaviors.

[4]When the user channels are orthogonal to each other, the channel inversion operation in ZFBF reduces to rotation operation, where the beam directions directly match the channel directions.

[5]When a relay is transmitting, it misses the new messages broadcast by the source, due to half-duplex constraint. As a result, it will not be able to perform successive decoding to decode the new messages broadcast by the source node in the subsequent time slot.

Hence, the analysis of the individual point-to-point MISO channel is sufficient to access the performance of the proposed protocol.

Define the outage event experienced by each user of the proposed CBC protocol as

$$\mathcal{O} \triangleq \bigcup_{K=\{0,1,\ldots,L\}} \mathcal{O}_K \tag{8}$$

where $\mathcal{O}_K$ is the event in which mutual information (when there are K qualified relays) lies below the individual target data rate, i.e., $\mathcal{I}_K \leq R$. The single-user outage probability can be expressed as

$$P(\mathcal{O}) = \sum_{K=0}^{L} P(\mathcal{O}_K)P(K) \tag{9}$$

where $P(K)$ is the probability that K relays are qualified to participate in the cooperation, and $P(\mathcal{O}_{\mathrm{K}})$ is the single-user outage probability when K relays are qualified. Given a total of L available relays, we are able to determine the number of qualified relays K using the following lemma.

Lemma 1: Using high SNR approximation, the probability that K out of L relays satisfy (3) can be approximated as

$$P(K) \approx \frac{L!}{(L-K)!K!}(M\gamma)^{L-K} \tag{10}$$

where $\gamma = ((2^R - 1)/(\gamma_0 \|\mathbf{w}_k\|^2))$, and $K \leq L$.

Proof: See the Appendix. ■

We proceed to determine the worst case outage performance of the proposed protocol when K relays are qualified. The upper bound of $P(\mathcal{O}_{\mathrm{K}})$ is expressed in the following lemma.

Lemma 2: The outage probability for the event that K relays are qualified at high SNR is

$$P(\mathcal{O}_{\mathrm{K}}) \leq \frac{\left(\frac{\gamma_0^r - 1}{\gamma_0}\right)^M}{M!\gamma_0^K}\left((-1)^K - \gamma_0^{(K+1)r}\sum_{i=0}^{K-1}\frac{\left(\ln\gamma_0^{(K+1)r}\right)^i}{i!(-1)^{K-i}}\right) \tag{11}$$

where multiplexing gain r is as defined in (7).

Proof: See the Appendix. ■

Following Lemma 1 and Lemma 2, the single-user outage probability as defined in (9) can be obtained. Using the single-user outage probability formulation, the following diversity and multiplexing tradeoff experienced by each user can be established.

Theorem 1: The achievable diversity and multiplexing tradeoff of the proposed CBC protocol is

$$d_{\mathrm{CBC}}(r) = M(1-r) + [L - (L+1)r]^+ \tag{12}$$

for any multiplexing gain $0 \leq r \leq 1$, where notation $[x]^+$ denotes $\max\{0, x\}$.

Proof: See the Appendix. ■

As a comparison, the single-user diversity and multiplexing tradeoff achievable by the noncooperative ZFBF broadcast scheme with scheduling [7], assuming full spatial multiplexing, can be expressed as

$$d_{\mathrm{ZFBF}}(r) = M(1-r) \tag{13}$$

for any $0 \leq r \leq 1$. The maximum diversity gain $d(r = 0)$ characterizes the rate of decay of the outage probability with SNR, when the data rate is fixed. By employing relays, the proposed scheme achieves an additional diversity gain of L, as compared with baseline

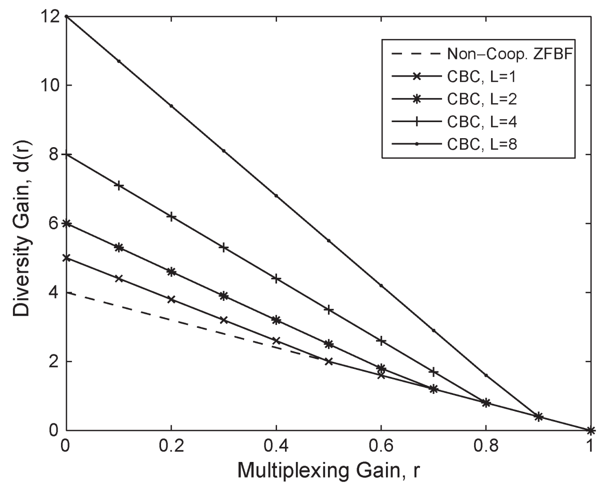

Fig. 3. Diversity and multiplexing tradeoff of the noncooperative ZFBF scheme and the proposed CBC scheme for various numbers of available relays L. Fixed parameter: $M = 4$.

noncooperative scheme. On the other extreme, the maximum multiplexing gain $d(r) = 0$ determines the rate of increase of the data rate with SNR, when the outage probability is fixed. The proposed scheme maintains the same maximum multiplexing gain $r = 1$ as the noncooperative scheme. This justifies that no extra bandwidth is consumed for cooperative relaying. This benefit comes from the use of nonorthogonal relaying in the proposed protocol.

Positive multiplexing gain ($r > 0$) and positive diversity gain ($d > 0$) can be simultaneously obtained, subject to the diversity and multiplexing tradeoff. As in [5], we say that protocol A uniformly dominates protocol B when $d_A(r) \geq d_B(r)$ for any multiplexing gain r. The diversity and multiplexing tradeoff of the CBC protocol uniformly dominates the noncooperative ZFBF protocol, i.e., $d_{\mathrm{CBC}}(r) \geq d_{\mathrm{ZFBF}}(r)$ for any r. When the multiplexing gain $r < (L/(L+1))$, the diversity and multiplexing tradeoff of the proposed protocol is always better than the noncooperative ZFBF, i.e., $d_{\mathrm{CBC}}(r) > d_{\mathrm{ZFBF}}(r)$. When the multiplexing gain $r > (L/(L+1))$, the tradeoff performance of the proposed protocol is identical to that of the noncooperative protocol. This can be explained as follows: Suppose that there is only one relay participating in the cooperation, i.e., $L = 1$. In this case, two messages are broadcast by the source to each user, whereas only one of the messages is repeated by the relay. This indicates that the cooperative link (relay to destination) provided by the relay cannot support multiplexing gain greater than 1/2. Generally, the maximum multiplexing gain supportable by the cooperative links is a linear function of the number of participating relays, i.e., $r = (L/(L+1))$. When L is large enough, i.e., $(L/(L+1)) \approx 1$, the achievable diversity and multiplexing tradeoff of the proposed CBC protocol not only completely outperforms the noncooperative protocol but also approaches the optimal diversity and multiplexing tradeoff of the MISO channels with $M + L$ transmitter antennas. Fig. 3 visualizes the tradeoff curves when $M = 4$ for different L's. It can be observed that the maximum diversity gain of the proposed protocol is a linear function of the number of relays: adding more relays improves the diversity gain. The tradeoff curve of the CBC protocol is a piecewise linear function of two pieces, with a crossing point between the pieces that occurs at $r = (L/(L+1))$. This crossing point indicates the maximum multiplexing gain supportable by the cooperative links, as previously discussed. As the number of relays increases, the crossing point is shifted toward the direction of $r = 1$. This indicates that the cooperative links can support higher multiplexing gain when a larger number of relays are used.

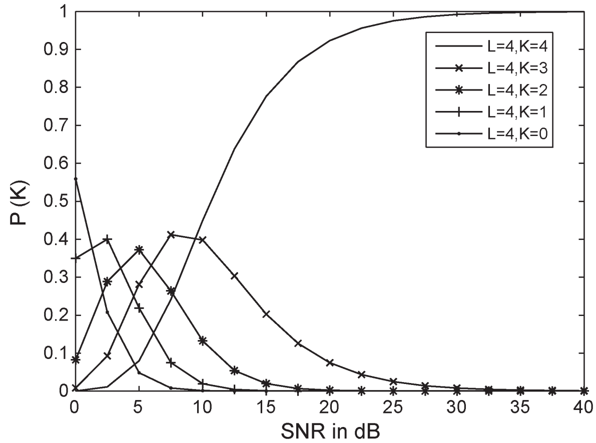

Fig. 4. Probability that K relays out of $L = 4$ available relays are qualified. Fixed parameters: $M = 2$, and $R = 1$ bit/s/Hz.

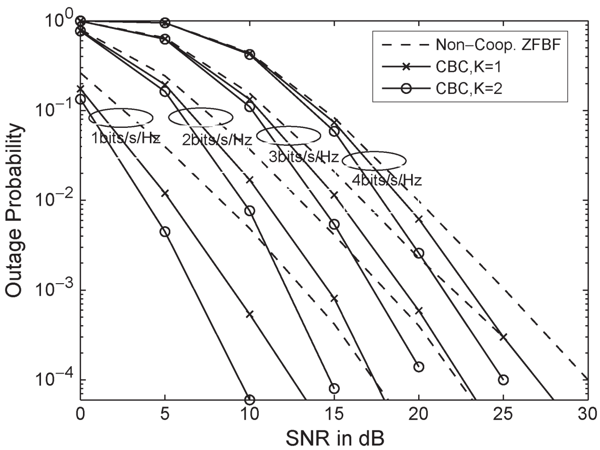

Fig. 5. Outage probability versus SNR of the noncooperative ZFBF scheme and the proposed CBC scheme at various target data rates R. Fixed parameters: $M = 2$, and the number of participating relays $K = 1, 2$.

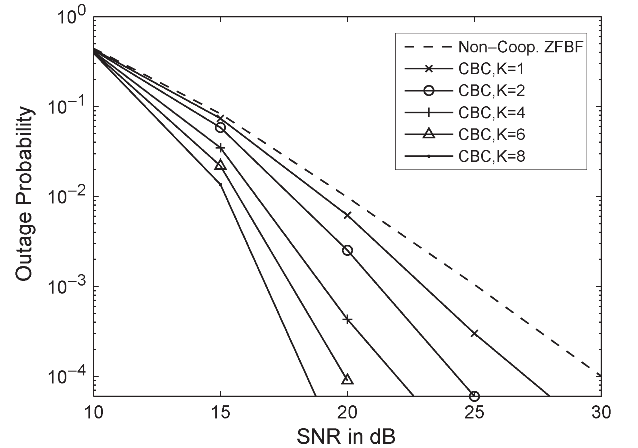

Fig. 6. Outage probability versus SNR of the noncooperative ZFBF scheme and the proposed CBC scheme with various numbers of participating relays K. Fixed parameters: $M = 2$, and $R = 4$ bit/s/Hz.

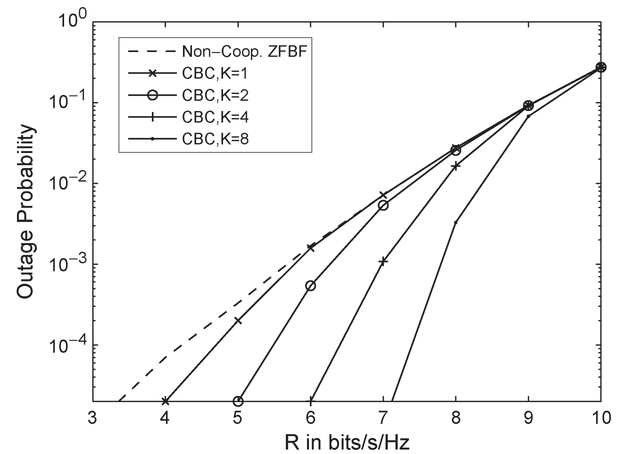

Fig. 7. Outage probability versus target data rate of the noncooperative scheme and the proposed CBC scheme, with various numbers of participating relays K. Fixed parameters: $M = 2$, and SNR = 30 dB.

IV. Numerical Results

In this section, we present several Monte Carlo simulation results to demonstrate the performance gain of the proposed CBC scheme in comparison with the existing noncooperative ZFBF scheme [7].

Fig. 4 shows the simulation of the probability when K relays out of L available relays are qualified, i.e., $P(K)$, without using high SNR approximation. In this simulation, (14) is used. As SNR increases beyond 15 dB, it can be observed that $P(K = L) \to 1$ and $P(K \neq L) \to 0$. This agrees with the result in Lemma 1, stating that, at high SNR, all relays are qualified. In the subsequent experiments, we use the assumption that the number of qualified relays is equal to the size of the available relays, following the fact that almost all relays are qualified at reasonable SNR, i.e., SNR ≥ 15 dB. We use the term "participating relay" to reflect that $P(K = L) = 1$ is assumed.

Fig. 5 shows the outage probability versus SNR under various target data rates R. Recall that the diversity gain characterizes the slope of the outage probability curve. The steeper the slope, the higher the diversity gain. The result shows that the proposed CBC protocol is able to achieve higher diversity gain if compared with the noncooperative scheme, even by using only one relay. Fig. 6 shows the outage probability versus SNR under various K's. The result confirms that the diversity gain of the proposed scheme improves when the number of participating relays increases.

Fig. 7 shows the relationship between the outage probability and the target data rate. In general, the proposed protocol delivers performance gain against the noncooperative protocol, which is evident from the higher data rate supportable by the proposed protocol at fixed outage probability. When more relays participate in the cooperation, the proposed protocol can support even higher data rate for fixed outage probability. This is in line with the result in Theorem 1 that the diversity and multiplexing tradeoff of the proposed protocol improve with increasing number of relays.

V. Conclusion

We have proposed a spectrally efficient cooperative transmission protocol for MISO broadcast channels. The proposed protocol is able to avoid cochannel interference among multiple destinations using practical linear precoding and provide another dimension of improvement. Analytical results have revealed that a maximum diversity gain, which is expressed as the summation of the number of source transmitter antennas and the available relays, can be achieved. Analytical results have also shown that the proposed protocol uniformly dominates the noncooperative protocol. When the number of available relays is large enough, the diversity and multiplexing tradeoff of the proposed protocol completely outperforms the comparable noncooperative protocol. Monte Carlo simulations further verify that the proposed protocol achieves better robustness and rate-and-reliability tradeoff than the comparable scheme.

APPENDIX

Proof of Lemma 1: Recalling (2), the virtual channel matrix $\mathbf{H} = \mathbf{H}_{R_k}\mathbf{W}$, where $\mathbf{H}_{R_k} \sim \mathcal{CN}(0, \mathbf{I}_M \otimes \sigma^2\mathbf{I}_M)$, with symbol $\otimes$ denoting the Kronecker product. Under the zero-forcing criterion, the received signals at the relay can be decomposed into M parallel streams. Denote $\mathbf{h}_k$ as the kth column of $\mathbf{H}$, $\tilde{\mathbf{H}}$ as the remaining columns of $\mathbf{H}$ after removing the kth column and $\mathbf{w}_k$ as the kth column of $\mathbf{W}$. From [10], the instantaneous SNR of the kth stream at the relay receiver can be expressed as $\gamma_k = \gamma_0 \mathbf{h}_k^H \mathbf{Q}\boldsymbol{\Lambda}\mathbf{Q}^H \mathbf{h}_k$, where $\gamma_0 = (E_s/\sigma_n^2)$ is the mean SNR, $\mathbf{Q}$ is the matrix of eigenvectors, $\boldsymbol{\Lambda}$ is the matrix of eigenvalues, and $\mathbf{Q}\boldsymbol{\Lambda}\mathbf{Q}^H = \mathbf{I} - \tilde{\mathbf{H}}(\tilde{\mathbf{H}}^H\tilde{\mathbf{H}})^{-1}\tilde{\mathbf{H}}^H$. Vector $\mathbf{h}_k$ is the weighted sum of complex Gaussian vectors [11], which has distribution $\mathbf{h}_k \sim \mathcal{CN}(0, \sigma^2\|\mathbf{w}_k\|^2\mathbf{I}_M)$. Following similar steps as in [12], the kth stream SNR is an exponential variable with cumulative density function (cdf): $F(\gamma_k) = 1 - \exp(-(\gamma_k/(\gamma_0\sigma^2\|\mathbf{w}_k\|^2))$.

Define A as the event when a relay is qualified. Since the SNR of each stream is independent identically distributed,[6] the probability that a relay is qualified is $P(A) = (P(\gamma_k > 2^R - 1))^M = \exp(-M\gamma)$, where $\gamma = (2^R - 1)/(\gamma_0\sigma^2\|\mathbf{w}_k\|^2)$. Using the approximation $\exp(-x)x \to 0 \approx 1 - x_{x\to 0}$, $P(A)$ can be approximated as $P(A) \approx 1 - M\gamma$ at high SNR, i.e., $\gamma_0 \to \infty$ and $\gamma \to 0$. The probability that K relays out of L available relays ($K \leq L$) can successfully decode the source message has a binomial distribution, which is expressed as

$$P(K) = \frac{L!}{(L-K)!K!} P(A)^K (1 - P(A))^{L-K}. \quad (14)$$

If follows that the high SNR approximation for $P(K)$ is

$$P(K) \approx \frac{L!}{(L-K)!K!} (1 - M\gamma)^K (M\gamma)^{L-K}. \quad (15)$$

Since $M\gamma \ll 1$ as $\gamma_0 \to \infty$, the lemma is proved. Note that, for Rayleigh fading, $\sigma^2 = 1$. ■

Proof of Lemma 2: From (6), the outage probability when K relays are selected can be expressed as

$$P(\mathcal{O}_\mathrm{K}) = P\left(\det\left(\mathbf{I}_{K+1} + \gamma_0\mathbf{H}_m\mathbf{H}_m^H\right) \leq 2^{(K+1)R}\right). \quad (16)$$

It can be seen that the argument of the determinant $\mathbf{I}_{K+1} + \gamma_0\mathbf{H}_m\mathbf{H}_m^H$ has the structure of a tridiagonal matrix. Represent variable $x = \|\mathbf{h}_m\|^2$ as the norm square of the element on the principal diagonal of matrix $\mathbf{H}_m$ and variable $z_i = |\mathbf{g}_{R_i,m}^T\mathbf{p}_{R_i,m}|^2$ as the absolute square of the subdiagonal element of $\mathbf{H}_m$. Following similar steps as in [14] and [15], it is readily shown that the determinant can be lower bounded as follows:

$$\det\left(\mathbf{I}_{K+1} + \gamma_0\mathbf{H}_m\mathbf{H}_m^H\right) \geq (1 + \gamma_0 x)^{K+1} + \prod_{i=1}^{K}(\gamma_0 z_i). \quad (17)$$

[6]Note that the SNR of each stream in the linear receiver is not strictly independent in practice. However, this assumption does not affect the diversity of the system and is used to make the analysis tractable, as discussed in [13].

Since we are interested in the worst-case outage probability, we derive its upper bound by substituting (17) into (16) as

$$P(\mathcal{O}_\mathrm{K}) \leq P\left(\left((1+\gamma_0 x)^{K+1} + \prod_{i=1}^{K}\gamma_0 z_i\right) \leq 2^{(K+1)R}\right)$$
$$\leq P\left((1+\gamma_0 x) \leq 2^R\right) P\left(\prod_{i=1}^{K}\gamma_0 z_i \leq 2^{(K+1)R}\right). \quad (18)$$

Following similar steps as in Lemma 1, the probability density function (pdf) of x is known to be $f(x) = (x^{M-1}\exp(-x/(\sigma^2))/(\sigma^{2M}\Gamma(M)))$. Rewriting $P((1+\gamma_0 x) \leq 2^R) = P(x \leq ((2^R - 1)/\gamma_0))$, the cdf of x is thus

$$P\left(x \leq \frac{2^R-1}{\gamma_0}\right) = 1 - \exp\left(-\frac{2^R-1}{\sigma^2\gamma_0}\right)\sum_{n=0}^{M-1}\frac{1}{n!}\left(\frac{2^R-1}{\sigma^2\gamma_0}\right)^n.$$

Using Taylor expansion, invoking binomial theorem and substituting $R = r\log_2\gamma_0$, we obtain

$$P\left(x \leq \frac{\gamma_0^r - 1}{\gamma_0}\right) = \frac{\left(\frac{\gamma_0^r - 1}{\sigma^2\gamma_0}\right)^M}{M!} \quad (19)$$

which solves the first term of (18). Next, we wish to get the closed-form equation for the second term $P(\prod_{i=1}^K \gamma_0 z_i \leq 2^{(K+1)R})$ in (18). Denote the mutual information $\mathcal{I} = \ln(\prod_i^K \gamma_0 z_i)$ and $f(z_1, \ldots, z_K)$ as the joint density function of $(z_1, \ldots, z_K)$. Following similar steps in [16], we obtain the following expression for the pdf of $\mathcal{I}$:

$$q(\mathcal{I}) = \frac{\exp(\mathcal{I})}{\gamma_0^K}\int_1^{\exp(\mathcal{I})}\int_1^{\psi_1}\cdots \int_1^{\psi_{K-2}} \Psi(\psi_1, \ldots, \psi_{K-1}, \gamma_0)\prod_{i=1}^{K-1}\frac{1}{\psi_i}\,d\psi_i \quad (20)$$

where $\Psi(\psi_1, \ldots, \psi_{K-1}, \gamma_0) = f((\exp(\mathcal{I})/(\gamma_0\psi_1)), (\psi_1/(\gamma_0\psi_2)), \ldots, (\psi_{K-2}/(\gamma_0\psi_{K-1})), (\psi_{K-1}/\gamma_0))$. Omit the subscript of $\mathbf{g}_{R_k,m}$ and $\mathbf{p}_{R_k,m}$ for simplicity. We know that $\mathbf{g} \sim \mathcal{CN}(0, \boldsymbol{\sigma}^2\mathbf{I}_M)$ or, equivalently, $g_i \sim \mathcal{CN}(0, \sigma^2)$ for $i = 1, \ldots, M$, where g_i is the ith element of vector $\mathbf{g}$. We can express z_k as the absolute square of the weighted sum of normal variables, i.e., $z_k = |\sum_{i=1}^{M_t} g_i p_i|^2$. According to [11], z_k is exponentially distributed with pdf $f(z) = (1/\sigma^2)\exp((-z/\sigma^2))$. The joint density function $f(z_1, \ldots, z_K) = \prod_{k=1}^K (f(z))$ since z_k is independently distributed. Using this property, we can determine the density function of $\Psi(\psi_1, \ldots, \psi_{K-1}, \gamma_0)$, which is expressed as

$$\Psi(\psi_1, \ldots, \psi_{K-1}, \gamma_0) = \left(\frac{1}{\sigma^2}\right)^K \exp\left(-\frac{\exp(\mathcal{I})}{\sigma^2\gamma_0\psi_1}\right)$$
$$\times \exp\left(-\frac{\psi_1}{\sigma^2\gamma_0\psi_2}\right) \times \cdots \times \exp\left(-\frac{\psi_{K-2}}{\sigma^2\gamma_0\psi_{K-1}}\right)\exp\left(-\frac{\psi_{K-1}}{\sigma^2\gamma_0}\right)$$

where $\psi_i \in [1, \exp(\mathcal{I})]$ for $i = 1, \ldots, K-1$. At high SNR, $\lim_{\gamma_0\to\infty}\Psi(\psi_1, \ldots, \psi_{K-1}, \gamma_0) = (1/\sigma^2)^K$. Hence, the high SNR approximation of the pdf of mutual information $\mathcal{I}$ is

$$q(\mathcal{I}) \approx \frac{\exp(\mathcal{I})}{(\sigma^2\gamma_0)^K}\int_1^{\exp(\mathcal{I})}\int_1^{\psi_1}\cdots\int_1^{\psi_{K-2}}\prod_{i=1}^{K-1}\frac{1}{\psi_i}\,d\psi_i. \quad (21)$$

Applying change of variable, $\phi_i = \ln \psi_i$, $d\phi_i = (1/\psi_i)d\psi_i$, we get the following:

$$q(\mathcal{I}) \approx \frac{\exp(\mathcal{I})}{(\sigma^2\gamma_0)^K} \int_1^{\exp(\mathcal{I})} \int_1^{\phi_1} \cdots \int_1^{\phi_{K-2}} \prod_{i=1}^{K-1} d\phi_i = \frac{\exp(\mathcal{I})\mathcal{I}^{K-1}}{(\sigma^2\gamma_0)^K (K-1)!}. \tag{22}$$

Given the density of $\mathcal{I}$, the equation for $P(\prod_{i=1}^K \gamma_0 z_i \leq 2^{(K+1)R})$ can be expressed as

$$P(q(\mathcal{I}) \leq \theta) = \int_0^\theta \frac{\exp(\mathcal{I})\mathcal{I}^{K-1}}{(\sigma^2\gamma_0)^K (K-1)!} \, d\mathcal{I}$$

where $\theta = (K+1)R\ln 2$. Combining the results from the table of integral section 3.351.1 in [17] and substituting $R = r\log_2 \gamma_0$, we manage to obtain the closed-form equation for the cdf of $q(\mathcal{I})$

$$P\left(q(\mathcal{I}) \leq (K+1)r\ln\gamma_0\right) = \frac{1}{(\sigma^2\gamma_0)^K} \times \left(\frac{1}{(-1)^K} - \gamma_0^{(K+1)r} \sum_{i=0}^{K-1} \frac{\left(\ln \gamma_0^{(K+1)r}\right)^i}{i!(-1)^{K-i}} \right). \tag{23}$$

Combining (19) and (23), the lemma is proved. Note that, for Rayleigh fading, $\sigma^2 = 1$. ■

Proof of Theorem 1: We use special symbol $\doteq$ to denote exponential equality, i.e., $f(\gamma_0) \doteq \gamma_0^n$ to denote $\lim_{\gamma_0\to\infty}(\log_2 f(\gamma_0)/\log_2 \gamma_0) = n$. Combining the results from Lemma 1 and Lemma 2, the outage probability of the proposed CBC protocol can be expressed as

$$P(\mathcal{O}) = \sum_{K=0}^{L} P(\mathcal{O}_K)P(K) \doteq \gamma_0{}^{-(L-K)(1-r)-[(M+K)(1-r)-r]}.$$

Combining the negative exponent of the γ_0, we have $d = L + M - [L+M+1]r$. More specifically, the diversity and multiplexing gain can be expressed as the following piecewise linear function:

$$d(r) = \begin{cases} M(1-r) + L - (L+1)r, & \text{if } \quad 0 \leq r \leq \frac{L}{L+1} \\ M(1-r), & \text{if } \quad \frac{L}{L+1} < r \leq 1. \end{cases}$$

Hence, the theorem is proved. ■

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Outage and BER Analysis for Ultrawideband-Based WPAN in Nakagami-m Fading Channels

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***Abstract*—This paper presents a performance analysis of multiband orthogonal frequency-division multiplexing (MB-OFDM) in ultra wideband (UWB)-based personal area networks (UPANs). A UPAN consists of devices with different UWB technologies at the physical layer. Approximate expressions for the outage probability and average bit error rate (BER) are derived in closed form for the MB-OFDM target receiver, taking into account multi-user interference (MUI), as well as external interference in the form of time-hopping (TH) and direct-sequence (DS) UWB signals.**

***Index Terms*—Bit error rate (BER), direct-sequence (DS) ultrawideband (UWB), interference, multiband orthogonal frequency-division multiplexing (MB-OFDM), outage, time-hopping (TH) UWB, wireless personal area network (WPAN).**

I. Introduction

Concurrent appearance of two ultrawideband (UWB) standards [1], [2] in the future wireless communications market raises new questions

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