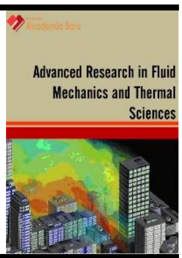
Penerbit Akademia Baru

Journal of Advanced Research in Fluid Mechanics and Thermal Sciences

Journal homepage: www.akademiabaru.com/arfmts.html
ISSN: 2289-7879



Unsteady Free Convection Flow of Nanofluid with Dissipation Effect over a Non-Isothermal Vertical Cone

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ARTICLE INFO

Article history:
Received 5 April 2020
Received in revised form 8 July 2020
Accepted 10 July 2020
Available online 3 September 2020

Keywords:
Vertical Cone; Nanofluids; Viscous Dissipation

ABSTRACT

This paper investigated unsteady free convection flow of nanofluid with dissipation effect over a non-isothermal vertical cone. The dimensional governing equations that consists of continuity, energy and momentum equations are reduced by using appropriate dimensionless variables along with variable wall temperature as its initial and boundary conditions. The case when water is the base fluid has been considered and the effects of the solid volume fraction on the flow and heat transfer characteristics are determined for Silver (Ag), Copper (Cu), Alumina (Al_2O_3) and Titanium oxide (TiO_2) nanofluids. The purpose of the study is to investigate numerically the mathematical model by using the Crank-Nicolson method. The discretization equations were computed, and numerical results were plotted using MATLAB software. It has been shown that when the nanoparticles volume fraction increases, the Nu_X increases and the velocity profile decreases. Moreover, for Silver (Ag), Copper (Cu) and Titanium oxide (TiO_2) nanoparticles, the thermal boundary layer decreases at first but later started to increase at certain values as the nanoparticles volume fraction increases. However, for Alumina (Al_2O_3) nanoparticles, the temperature profile increases as the nanoparticles volume fraction increases. It has also been found in this problem that the Alumina (Al_2O_3) nanoparticles have the highest heating performance while Silver (Ag) nanoparticles have the highest cooling performance.

1. Introduction

The fluid flow over a cone has received much attention due to the various applications involving heat transfer. It is encountered in various industrial applications, as well as in many natural circumstances such as hospitality and health care systems, energy storage systems, aeronautical sciences, geological sciences, astrophysics, micro-inverter chips, space technology, controlling system of engine oil and in nuclear power management system [1]. In the presence studies, free

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https://doi.org/10.37934/arfmts.75.1.111

convection flow over a vertical cone is one of the most interesting topics discussed by researchers under fluid flow area. The angle parameter of cone placed vertically gives challenges to researchers in formulation of the problem. Merk and Prins [2-3] showed the general relations for similar solutions on a process taking place at fixed temperature axi-symmetric forms and showed that the vertical cone has such a solution. Kuiken [4] analyzed the transpiration velocity with free convection effects over a vertical cone at constant temperature. Besides, Sparrow *et al.,* [5] presented a prominent attribute of the movement of heat with magnetic and radiation effects which is liable to change surface condition. Hossain and Paul [6] investigated the transpiration velocity with free convection effects of a vertical cone at constant temperature. Moreover, the heat transfer effects on a vertical cone embedded in a tri-disperse holely medium was analyzed in Cheng [7].

A literature search reveals that many researchers have studied the free convection in nanofluid flow over a vertical cone theoretically for steady and unsteady cases. However, relatively few works had been done on the problem of unsteady free convection flow over vertical cone with variable wall temperature. In this present analysis, the dissipation effects are considered along with the vertical cone. Various studies have been conducted on dissipation effects on free convection flow. This research is an extension of Sambath [8] with influence of nanofluid. It has been detected that significant dissipation due to syrupy fluid may occur in free convection in various equipment due to a large amount of deceleration or which function with high rotational speed. The influence of viscous dissipation is seen much stronger in gravitational fields and in many industrial applications such as electronic cooling situations, to remove the moisture from and make dry process, polymer process streaming in such situation the temperature rises importantly. Braun *et al.,* [9] have obtained answers for problems of well-arranged natural/squeeze transfer of heat through a fluid stimulated by molecular motion.

The influence of viscous cannot be disregarded in the layer of the plane indicating the limit. The momentum gradient and the sticky boundary shear increases right when fluid passes the leading edge of the cone vertically. Gebhart [10] was the first person to discuss the state of difficulty that needs to be resolved by taking sticky nature fluid with breaking up and scattering by dispersion characteristic fluid into account. Further, Hering and Grosh [11] have obtained answers for problems of well-arranged natural/squeeze transfer of heat through a fluid stimulated by molecular motion. The boundary layer flow and heat transfer on the viscoelastic fluid had been studied analytically in Wahid *et al.,* [12] where the magnetohydrodynamic (MHD) slip Darcy flow of viscoelastic fluid over a stretching surface with the presence of thermal radiation and viscous dissipation in a porous medium were examined.

The free convection flow over vertical plate is analyzed by numerous studies. However, since the thermal conductivity of the conventional heat transfer fluids such as ethylene glycol, water and oil are poor heat transfer fluids, an innovative technique for heat transfer which used ultrafine solid particles in the fluids has been applied tremendously to improve this during the last several years. These types of fluids that suspend nanoparticles in the base fluid are called nanofluid as introduced by Choi [13]. Moreover, Thandapani *et al.,* [14] analyzed the movement of nanofluids over a vertical cone in non Darcian holly medium and Khanafer *et al.,* [15] investigated the heat transfer enhancement in a two-dimensional enclosure utilizing nanofluids.

2. Mathematical Formulation

Consider unsteady free convection flow of nanofluid with influence of dissipation over a non-isothermal vertical cone. A system that uses coordinates to establish position is visualized in Figure 1 in such a way that x-axis is taken on the surface of the cone from the vertex $x = 0$ and y denotes the distance taken vertical to x. Then the nanofluid flow is governed by the following equations along with the usual Boussinesqs approximations as presented by Sambath [8]:

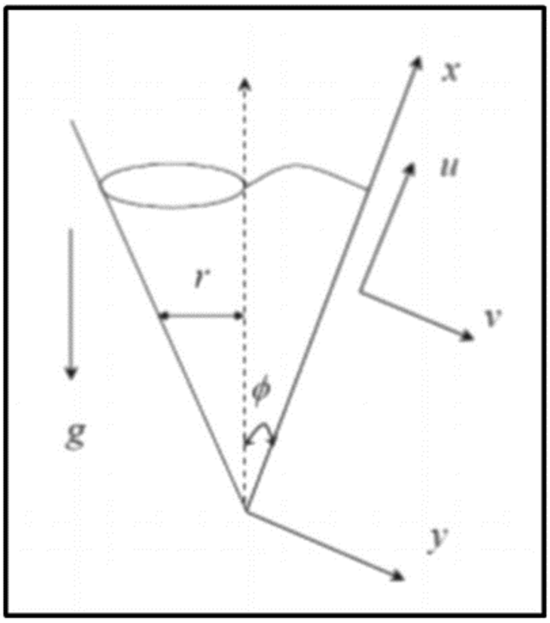


Fig. 1. Physical model with coordinate system

Equation on continuity

$$\frac{\partial}{\partial x}(ur)+\frac{\partial}{\partial y}(vr)=0 \tag{1}$$

Equation on momentum

$$\rho_{nf}\left(\frac{\partial u}{\partial t'}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\mu_{nf}\frac{\partial^2 u}{\partial y^2}+g(\rho\beta)_{nf}\cos\theta(T'-T'_{\infty}) \tag{2}$$

Equation on energy

$$(\rho C_p)_{nf}\left(\frac{\partial T'}{\partial t'}+u\frac{\partial T'}{\partial x}+v\frac{\partial T'}{\partial y}\right)=k_{nf}\frac{\partial^2 T'}{\partial y^2}+\mu_{nf}\left(\frac{\partial u}{\partial y}\right)^2 \tag{3}$$

where, u = velocity component in x direction, v = velocity component in y direction, r = local radius of the cone, t' = time, T' = temperature, x = spatial coordinate along the cone generator, y = spatial coordinate along the normal to the cone generator, ρ = density, μ =Dynamic Viscosity, g = Gravitational force, β =Thermal expansion, C_p =Specific heat at constant pressure, k =Thermal conductivity and nf =nanofluid. Subject to the initial and boundary conditions.

$$
\begin{aligned}
&t' \le 0 : u = 0, v = 0, T' = T'_\infty \text{ for all } x \text{ and } y \\
&t' > 0 : u = 0, v = 0, T'_w(x) = T'_\infty(x) + ax^n \text{ at } y = 0 \\
&u = 0, T' = T'_\infty \text{ at } x = 0 \\
&u \to 0, T' \to T'_\infty \text{ as } y \to \infty
\end{aligned} \tag{4}
$$

The properties of nanofluid are given by Oztopand and Abu-Nada [16], Vajravelu *et al.,* [17] and Sheikholeslami *et al.,* [18] as follows

$$
\begin{aligned}
&\rho_{nf} = (1-\Phi)\rho_f + \Phi\rho_s \\
&(\rho\beta)_{nf} = (1-\Phi)(\rho\beta)_f + \Phi(\rho\beta)_s \\
&(\rho Cp)_{nf} = (1-\Phi)(\rho Cp)_f + \Phi(\rho Cp)_s \\
&\mu_{nf} = \frac{\mu_f}{(1-\Phi)^{2.5}} \\
&\frac{k_{nf}}{k_f} = \frac{(k_s + 2k_f) + 2\Phi(k_f - k_s)}{(k_s + 2k_f) - \Phi(k_f - k_s)}
\end{aligned} \tag{5}
$$

The thermo-physical properties of water and nanoparticles are listed as Table 1

Table 1
Thermo-physical properties of water and nanoparticles

Fluid	ρ (kg/m^3)	Cp (J/kgK)	k (W/mK)	β×10^5 (K−1)
Pure Water	997.1	4179	0.613	21
Copper (Cu)	8933	385	401	1.67
Silver (Ag)	10500	235	429	1.89
Alumina (Al_2O_3)	3970	765	40	0.85
Titanium Oxide (TiO_2)	4250	686.2	8.9538	0.9

The appropriate non-dimensional parameters will be used.

$$
\begin{aligned}
&X = \frac{x}{L}, Y = \frac{y}{L}(Gr_L)^{\frac{1}{4}}, R = \frac{r}{L} \; where \; r = x\sin\theta \\
&V = \frac{vL}{\text{v}_f}(Gr_L)^{\frac{1}{4}}, U = \frac{uL}{\text{v}_f}(Gr_L)^{\frac{1}{2}}, t = \frac{\text{v}_f t'}{L^2}(Gr_L)^{\frac{1}{2}} \\
&\Pr = \frac{\text{v}}{\alpha}, T = \frac{T' - T'_\infty}{T'_w - T'_\infty}, Gr_L = \frac{g\beta_f (T'_w - T'_\infty) L^3 \cos\theta}{\text{v}_f^2}
\end{aligned} \tag{6}
$$

Eq. (1) - (3) are then reduced to the following non dimensional form.

Equation on continuity

$$
\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} + \frac{U}{X} = 0 \tag{7}
$$

Equation on momentum

$$\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=\phi_1\frac{\partial^2 U}{\partial Y^2}+\phi_2 T\cos\theta \tag{8}$$

Equation on energy

$$\frac{\partial T}{\partial t}+U\frac{\partial T}{\partial X}+V\frac{\partial T}{\partial Y}=\phi_3\frac{1}{\Pr}\frac{\partial^2 T}{\partial Y^2}+\phi_4\varepsilon(\frac{\partial U}{\partial Y})^2 \tag{9}$$

where, U = dimensionless velocity in X direction, V = dimensionless velocity in Y direction, X = dimensionless spatial coordinate along the cone generator, Y = dimensionless spatial coordinate along the normal to the cone generator, t = dimensionless time, T = dimensionless temperature, Pr = Prandtl number, $\varphi_1, \varphi_2, \varphi_3, \varphi_4$ = constants with nanofluid effects and viscous dissipation parameter, $\varepsilon = \frac{g\beta_f L}{(Cp)_f}$ as depicted in Gebhart [10] and Jordan [19]. The corresponding non-dimensional initial and boundary conditions are

$$\begin{aligned} &t\le 0: U=0, V=0,\ T=0 \text{ for all X and Y}\\ &t>0: U=0, V=0,\ T=X^n \text{ at } Y=0\\ &\qquad U=0, \qquad T=0 \quad \text{at } X=0\\ &\qquad U\to 0, \qquad T\to 0 \quad \text{as } Y\to\infty \end{aligned} \tag{10}$$

and the local non-dimensional heat transfer rate are given by

$$Nu_x = Gr_L^{\frac{1}{4}}\frac{X}{T_{Y=0}}\left(-\frac{\partial \mathrm{T}}{\partial Y}\right)_{Y=0} \tag{11}$$

2.1 Numerical Procedure of Crank-Nicolson

All non-dimensional equations are discretized for numerical evaluation and implementation on digital computers. In order to formulate the problems in programming, the equations need to have a process of transferring continuous functions, variables, models, and equations into discrete. After the non-dimensional equations is discretized and iteration *i* and *j* is applied, it is then converted to the system of tri-diagonal equations. The mesh diagram for Crank-Nicolson method is visualized in Figure 2.

The finite difference equation equivalent to the Eq. (7) - (9) are specified as follows.

Equation on continuity

$$\frac{U_{i,j}^{k+1}-U_{i-1,j}^{k+1}+U_{i,j}^{k}-U_{i-1,j}^{k}}{2\Delta \mathrm{X}}+\frac{V_{i,j+1}^{k+1}-V_{i,j-1}^{k+1}+V_{i,j+1}^{k}-V_{i,j-1}^{k}}{4\Delta Y}+\frac{1}{X_i}\frac{U_{i,j}^{k+1}+U_{i,j}^{k}}{2}=0 \tag{12}$$

Equation on momentum

$$\left(\frac{U_{i,j}^{k+1}-U_{i,j}^{k}}{\Delta t}\right)+U_{i,j}^{k}\left(\frac{U_{i,j}^{k+1}-U_{i-1,j}^{k+1}+U_{i,j}^{k}-U_{i-1,j}^{k}}{2\Delta X}\right)+V_{i,j}^{k}\left(\frac{U_{i,j+1}^{k+1}-U_{i,j-1}^{k+1}+U_{i,j+1}^{k}-U_{i,j-1}^{k}}{4\Delta Y}\right)$$
$$=\phi_{1}\left(\frac{U_{i,j+1}^{k+1}-2U_{i,j}^{k+1}+U_{i,j-1}^{k+1}+U_{i,j+1}^{k}-2U_{i,j}^{k}+U_{i,j-1}^{k}}{2(\Delta Y)^{2}}\right)+\phi_{2}T_{i,j}^{k}\cos\theta \tag{13}$$

Equation on energy

$$\left(\frac{T_{i,j}^{k+1}-T_{i,j}^{k}}{\Delta t}\right)+U_{i,j}^{k}\left(\frac{T_{i,j}^{k+1}-T_{i-1,j}^{k+1}+T_{i,j}^{k}-T_{i-1,j}^{k}}{2\Delta X}\right)+V_{i,j}^{k}\left(\frac{T_{i,j+1}^{k+1}-T_{i,j-1}^{k+1}+T_{i,j+1}^{k}-T_{i,j-1}^{k}}{4\Delta Y}\right)$$
$$=\phi_{3}\frac{1}{\Pr}\left(\frac{T_{i,j+1}^{k+1}-2T_{i,j}^{k+1}+T_{i,j-1}^{k+1}+T_{i,j+1}^{k}-2T_{i,j}^{k}+T_{i,j-1}^{k}}{2(\Delta Y)^{2}}\right)+\phi_{4}\varepsilon\left(\frac{U_{i,j+1}^{k+1}-U_{i,j-1}^{k+1}+U_{i,j+1}^{k}-U_{i,j-1}^{k}}{4\Delta Y}\right)^{2} \tag{14}$$

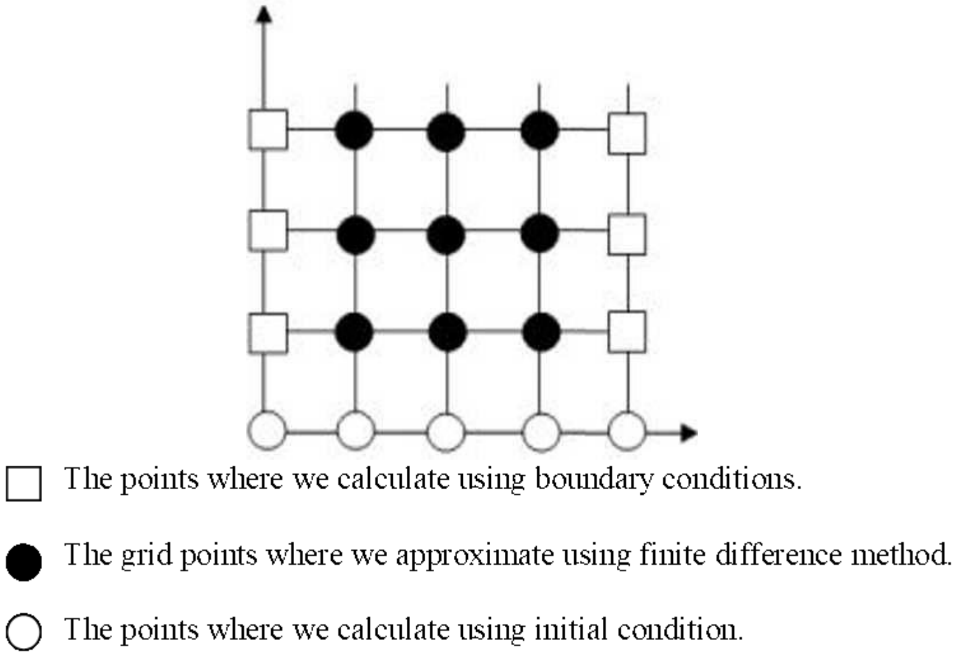


Fig. 2. Mesh diagram for Crank-Nicolson method

3. Results

This transient, non-linear non-dimensional coupled PDE is solved by using Crank-Nicholson method. The system of equations was then solved by using well known tri-diagonal matrix algorithm, which is Thomas algorithm in the period of time, t. The integral area is treated as a square or with X_{max}=1 and Y_{max}=10. The value of Y_{max} is compatible to Y_{∞} and it is located outside both the velocity and temperature boundary layers. The value for Y is taken to be 10 by analyzing in detail consideration in order to satisfy the ultimate and penultimate conditions of Eq. (10) with accuracy up to 10^{-4}. The meshing size has been mended as ΔX = 0.05, ΔY =0.05 and the step size is Δt = 0.01. The shortness ignorance is $O(\Delta t^2 + \Delta Y^2 + \Delta X)$ approaches to null value as Δt, ΔY and ΔX approaching the quantity of null. Based on the above calculations, approximations and computations, it can be concluded that an elaborate and systematic plan of action shows a solution can be able to exist and perform in harmonious as explained by Bapuji *et al.,* [20-21] and Thandapani *et al.,* [14]. The computations are carried for various values of the nanoparticles volume fraction for four types of nanoparticles with water as the base fluid. Throughout this study, Silver (Ag), Copper (Cu), Alumina (Al_2O_3) and Titanium oxide (TiO_2) nanofluids are used and nanoparticles volume fraction (Φ) varied from 0 to 0.4. In order to verify the accuracy of the present method, the results are compared with those of Hering [22] and Sambath [8] in the absence of the nanoparticles. The local heat transfer rate with different values of *Pr* and n when *X* = 1.0 and *ε* = 0 are examined, and the similarities of the answers are noted as in Table 2.

Table 2
Comparison of Nu_X at X = 1.0

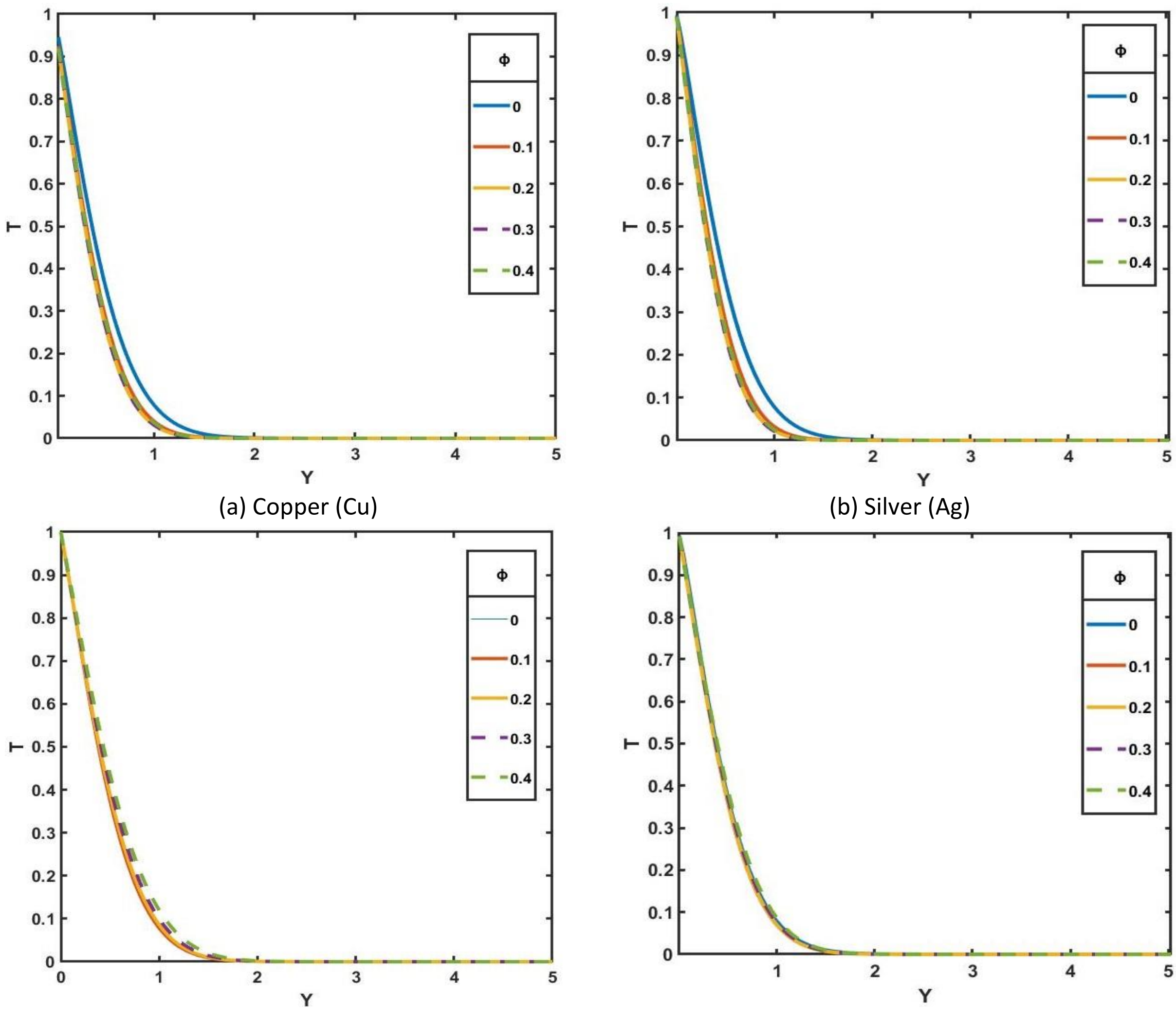
	Pr=0.03			Pr=0.1		
	Hering	Sambath	Present Results	Hering	Sambath	Present Results
n	$-\theta\sqrt{Pr}$	$Nu_x/Gr_L^{\frac{1}{4}}$	$Nu_x/Gr_L^{\frac{1}{4}}$	$-\theta\sqrt{Pr}$	$Nu_x/Gr_L^{\frac{1}{4}}$	$Nu_x/Gr_L^{\frac{1}{4}}$
0	0.1244	0.1243	0.1396	0.2113	0.2115	0.2136
0.2	0.1338	0.1336	0.1476	0.2263	0.2266	0.2286
1	0.16307	0.1622	0.1723	0.2739	0.2727	0.2745
2	0.1886	0.1877	0.1936	0.3136	0.3116	0.3134
4	0.2229	0.2187	0.2217	0.3684	0.3617	0.3637
8	0.2655	0.2535	0.2545	0.4367	0.4183	0.4215
	Pr=0.7			Pr=1.0		
	Hering	Sambath	Present Results	Hering	Sambath	Present Results
n	$-\theta\sqrt{Pr}$	$Nu_x/Gr_L^{\frac{1}{4}}$	$Nu_x/Gr_L^{\frac{1}{4}}$	$-\theta\sqrt{Pr}$	$Nu_x/Gr_L^{\frac{1}{4}}$	$Nu_x/Gr_L^{\frac{1}{4}}$
0	0.4511	0.4529	0.4591	0.5104	0.5125	0.5211
0.2	0.4794	0.4810	0.4884	0.5148	0.5436	0.5537
1	0.567	0.5666	0.5782	0.6389	0.6822	0.6538
2	0.6436	0.6397	0.6555	0.724	0.7195	0.7403
4	0.7484	0.7362	0.7574	0.8406	0.8271	0.8548
8	0.881	0.8480	0.8772	0.9889	0.9523	0.9903

Based on all the above cases, the comparisons are found to be in a good agreement. The accuracy increases with an increase of *n* and *Pr.* Table 3 depict Nu_x for different values of Φ for four types of nanoparticles with *X* = 1.0, *Pr* = 6.2, *ε* = 0.1 and *n* = 0.5. It is obvious that the heat transfer rate increases as the nanoparticles volume fraction increases. Moreover, the changes in Nu_X is found to be higher for higher values of Φ . It is also clear that the Nu_X is higher in the case of Silver (Ag), and next Copper (Cu), Titanium oxide (TiO_2) and lastly, Alumina (Al_2O_3) nanofluids.

Table 3
Valeus of Nu_X for various values of Φ

Φ	Copper (Cu)	Silver (Ag)	Alumina (Al_2O_3)	Titanium Oxide (TiO_2)
0.1	-0.3389	-0.2036	-1.0370	-0.9566
0.15	0.0635	0.2003	-0.6908	-0.6051
0.2	0.3791	0.5166	-0.3961	-0.3153
0.3	0.8959	1.0400	0.0982	0.1535

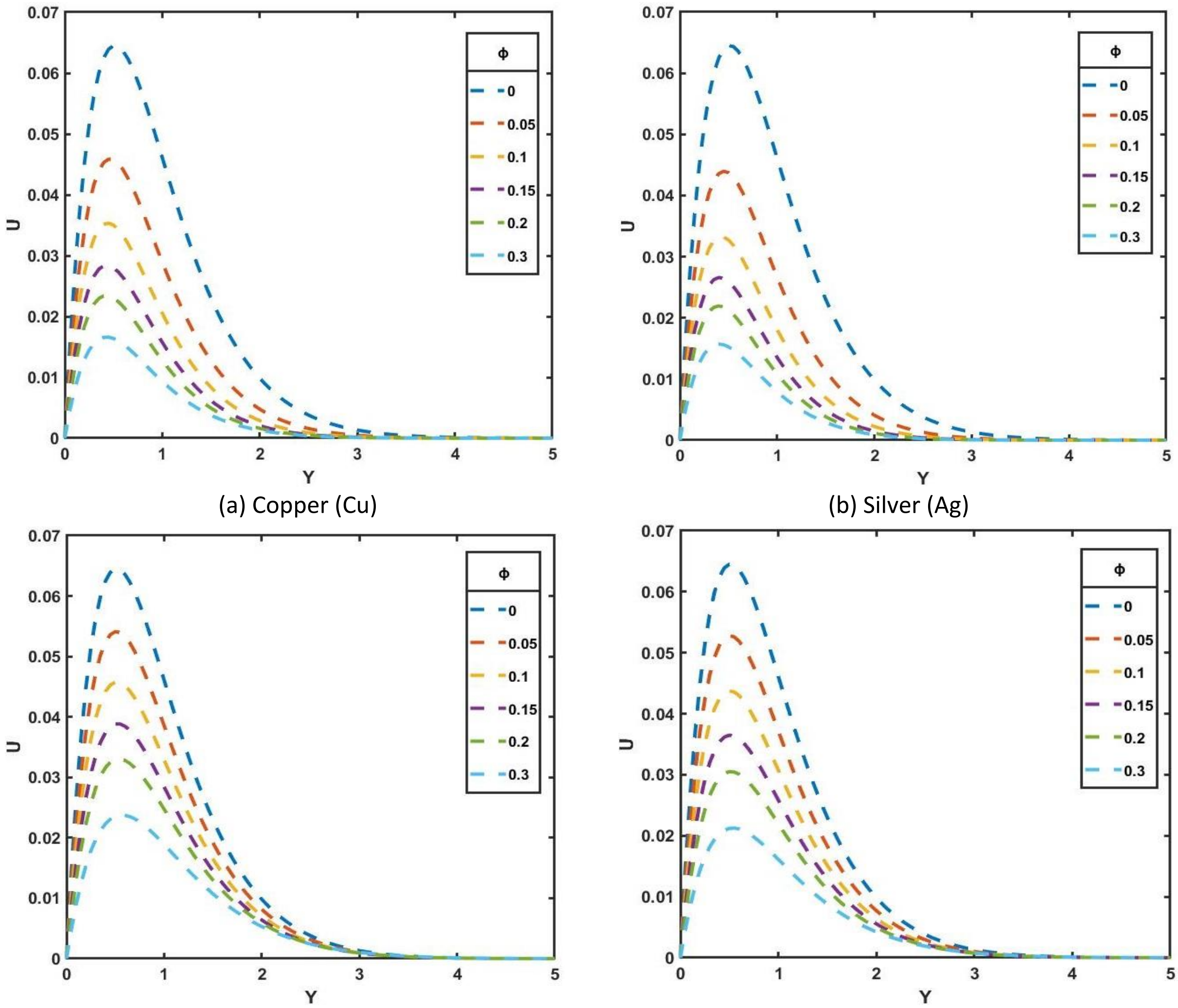
It is established from Figure 3 the influence of Φ on temperature profile in the case when *X = 1.0, Pr = 6.2, ε = 0.1 and n = 0.5.* These figures illustrate the streamline for different values of Φ and it can be observed that when the nanoparticles volume fraction increases from 0 to 0.4, for Silver (Ag), Copper (Cu) and Titanium oxide (TiO_2) nanoparticles, the thermal boundary layer decreases at first but later started to increase at certain value. However, for Alumina (Al_2O_3) nanoparticles, the thermal boundary layer increases as the nanoparticles volume fraction increases. This indicates that using the Alumina (Al_2O_3) nanoparticles is more effective for the enhancement of free convection heat transfer. It has also been clear that at the surface of the cone, the heat transfer rate is the highest for Alumina (Al_2O_3) nanoparticles and the smallest for Silver (Ag) nanoparticles as the nanoparticle volume friction increases. Moreover, Figure 4 shows the influence of Φ on velocity profile in the case when *X = 1.0, Pr = 6.2, ε = 0.1 and n = 0.5*. These figures illustrate the streamline for different values of nanoparticle volume fraction, and it can be observed that when the volume fraction of the nanoparticles increases from 0 to 0.3, the velocity profile decreases. This is because the thickness of the thermal boundary layer rises with an increase in the values of Φ hence consequently reduced the velocity of the nanofluid. Figure 5 displays the character of the different types of nanoparticles on temperature profile and velocity profile when Φ = 0.1. It is well observed on this figure that the values of the temperature and velocity changed when different types of nanofluid were used.



(a) Copper (Cu) (b) Silver (Ag)

(c) Alumina (Al_2O_3) (d) Titanium Oxide (TiO_2)

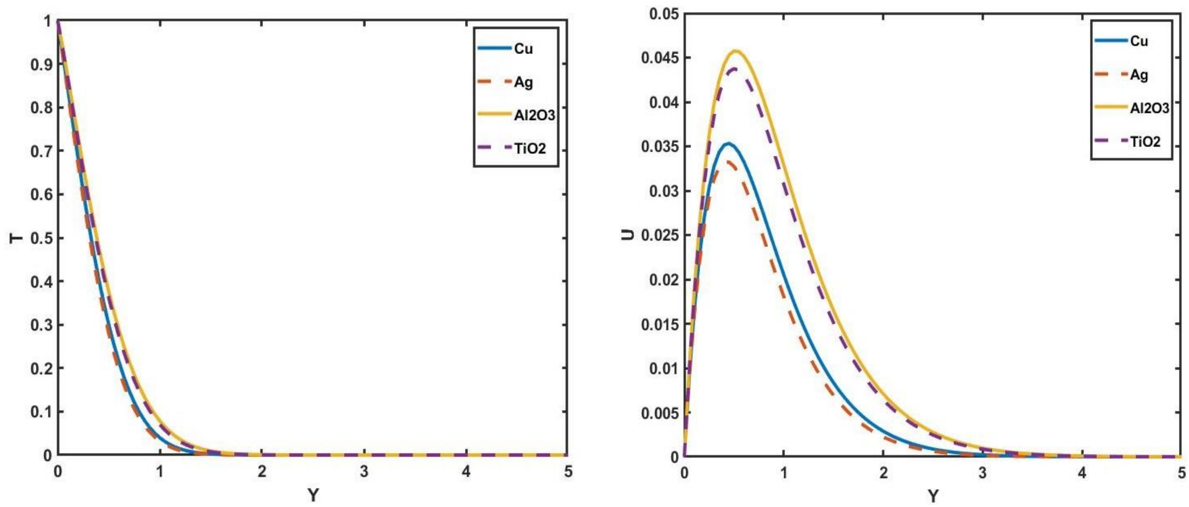
Fig. 3. Influence of Φ on temperature profile at *t* = 1



(a) Copper (Cu) (b) Silver (Ag)

(c) Alumina (Al_2O_3) (d) Titanium Oxide (TiO_2)

Fig. 4. Influence of Φ on velocity profile at *t* = 1



(a) Temperature Profile (b) Velocity Profile

Fig. 5. (a) and (b) for different types of nanofluids at *t* = 1

4. Conclusions

The problem of dissipation effects with free convection flow of nanofluid over a non-isothermal vertical cone has been studied and the case when water is a base fluid has been considered. The formulation and the solutions of the problem has been presented by using MATLAB programming software. Comparison with previously published works is performed and excellent agreement is obtained. The influence of the solid volume fraction Φ on the flow and heat transfer characteristics for Silver (Ag), Copper (Cu), Alumina (Al_2O_3) and Titanium oxide (TiO_2) nanofluids has been determined. It has been shown that when the nanoparticles volume fraction increases, the Nu_X increases and the velocity profile decreases. Moreover, for Silver (Ag), Copper (Cu) and Titanium oxide (TiO_2) nanoparticles, the temperature profile decreases at first but later started to increase at certain values as the nanoparticles volume fraction increases. However, for Alumina (Al_2O_3) nanoparticles, the thermal boundary layer increases as the nanoparticles volume fraction increases. It has also been found in this problem that the values of the temperature and velocity changed when different types of nanofluid were used and Alumina (Al_2O_3) nanoparticles have the highest heating performance while Silver (Ag) nanoparticles have the highest cooling performance.

Acknowledgement

The authors would like to acknowledge Ministry of Education (MOE), UTM Zamalah and Research Management Centre-UTM, Universiti Teknologi Malaysia (UTM) for the financial support through vote numbers 5F004, 5F116, 07G70, 07G72, 07G76, 07G77 and 17J98 for this research.

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