PROBABILITY DISTRIBUTION OF RETURNS IN HESTON MODEL FOR INDEX PRICES OF FTSE BURSA MALAYSIA KLCI

TEY SEAH YING

UNIVERSITI TEKNOLOGI MALAYSIA

PROBABILITY DISTRIBUTION OF RETURNS IN HESTON MODEL FOR INDEX PRICES OF FTSE BURSA MALAYSIA KLCI

TEY SEAH YING

The report is submitted in partial fulfilment of requirements for the award of the degree of Master of Science (Mathematics)

Faculty of Science
Universiti Teknologi Malaysia

DECEMBER 2012

To my beloved father, mother and husband, for their love and support.

ACKNOWLEDGEMENT

First of all, I would like to express my sincere gratitude to my supervisor, Dr. Arifah Bahar who had greatly assisted me in completing this study in the time frame given. Under her supervision, many aspects of this study had been explored. Her efforts in patiently guiding, supporting and giving constructive suggestions are very much appreciated.

Besides, I also would like to express my sincere appreciation to my beloved family for their advice and moral support, and friends who had kindly provided valuable and helpful comments in the preparation of the thesis. Moreover, I also would like to thank those who have been involved directly and indirectly in the preparation of this thesis. Without their encouragement and support, this research would have been difficult at best.

ABSTRACT

Financial investment is one of the most lucrative opportunities to earn money if the investors can make smart decision in investing their resources. However, investments involve certain risks, it may generate positive or negative returns, which means it may increase or decrease the investors' capital. In this study, the probability distribution of returns for the index prices of FTSE Bursa Malaysia KLCI based on the Heston Model with stochastic variance is constructed to analyse its capability in describing the returns for the index prices of FTSE Bursa Malaysia KLCI. The values of parameters in Heston Model of the index prices of FTSE Bursa Malaysia KLCI have been estimated based on Simulated Maximum Likelihood method using SDE toolbox in Matlab. The solutions of the stochastic differential equation were approximated by Euler-Maruyama method. It is found that for complete set of FTSE Bursa Malaysia KLCI data, the probability distribution of log returns for closing prices of FTSE Bursa Malaysia KLCI fitted the theoretical curve (Dragulescu and Yakovenko, 2002) better at time lag, $t = 1$. However, for truncated data of FTSE Bursa Malaysia KLCI, the probability distribution of log returns for closing prices of FTSE Bursa Malaysia KLCI fitted the theoretical curve (Dragulescu and Yakovenko, 2002) better at time lag, $t =$ 1, 5 and 20.

ABSTRAK

Pelaburan kewangan merupakan salah satu cara untuk memperoleh pendapatan jika pelabur bijak membuat palaburan. Sungguhpun begitu, pelaburan mengandungi risiko tertentu, ia mungkin menghasilkan pulangan positif atau negative dan juga dapat dikatakan ia mungkin menambah atau mengurangkan modal pelabur. Dalam kajian ini, taburan kebarangkalian pulangan bagi harga indeks FTSE Bursa Malaysia KLCI daripada dasar model Heston yang dikaitkan dengan varians stokastik telah dibina untuk menganalisis kemampuan taburan kebarangkalian tersebut dalam penggambaran pulangan bagi harga indeks FTSE Bursa Malaysia KLCI. Nilai-nilai parameter dalam model Heston bagi harga indeks FTSE Bursa Malaysia KLCI telah dianggarkan daripada *SDE toolbox* dalam Matlab dengan menggunakan cara *Simulated Maximum Likelihood*. Penyelesaian bagi persamaan pembezaan stokastik dapat diperolehi dengan menggunakan cara Euler-Maruyama. Didapati bahawa taburan kebarangkalian pulangan bagi data harga indeks FTSE Bursa Malaysia KLCI yang lengkap hampir sepadan dengan lengkung theoretic (Dragulescu and Yakovenko, 2002) pada sela masa, $t = 1$. Walau bagaimanapun, taburan kebarangkalian pulangan bagi data harga indeks FTSE Bursa Malaysia KLCI yang telah dipendekkan didapati hampir sepadan dengan lengkung theoretic (Dragulescu and Yakovenko, 2002) pada sela masa, $t = 1$, 5 dan 20.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF SYMBOLS

S_t	-	Stock prices
t	-	Time dependence
μ	-	Drift parameter
σ_t	-	Time dependence volatility
v_t	-	Variance
θ	-	Long time mean of variance
γ	-	Rate of relaxation to θ
κ	-	Variance noise
r_t	-	log return
W_t	-	Standard Wiener process
ρ	-	Correlation coefficient
h	-	Stepsize
$\mathcal{N}$	-	Normal distribution
$P_t(.\vert.)$	-	Transition probability function
$\delta(.)$	-	Delta function
$\Pi_t(v)$	-	Probability distribution of variance
$P_t(.)$	-	Probability density function
$K_1(.)$	-	Fist order modified Bessel function
$l_n(.)$	-	Log-likelihood function

LIST OF APPENDICES

CHAPTER 1

INTRODUCTION

1.1 Background of the Study

Kuala Lumpur Stock Exchange started to have a downward trend in July 1997 as it fell below its psychological level of 1000 points. According to Hughes (1999), the contagious effects of speculative activity on the Thai Baht had led to the crisis in 1997. Based on the economic report 1997/98, in September 1997, the KLCI touched its lowest level since 20th April 1993 of 675.15. In September 1998, the KLCI fell sharply to as low as 262.7 points from 1077.3 points in June 1997 followed the implementation of exchange control on 1st September 1998 (Hughes, 1999). According to Goh and Lim (2010), the Dot Com bubble that burst in March 2000 had adversely affected the growth of Malaysia's economy. The KLCI which was recovering from the 1997 crash took another hit and went down. From 2003 to 2007, the index point of KLCI started to pick up and eventually surpassed the 1500 points in the early of 2008. Unfortunately, in 2008, the severe global recession crisis that began with the burst of the U.S. housing bubble is transmitted to Malaysia (Goh and Lim, 2010). KLCI fell below 1000 points markedly in September 2008. According to The World Bank (2011), the Malaysian economy completed the rebound from the downturn but the growth momentum is volatile.

Nowadays, one of the best businesses to earn money is financial investment if the investors can make smart decision when doing investments. There are many types of financial investments in Malaysia, such as stocks, shares, commodities, unit trusts and bonds. However, since the growth momentum in Malaysia is volatile, these investments involve certain risks, it may generate positive or negative returns, which means it may increase or decrease the investors' capital.

Volatility plays an important role in various area of finance, for example, risk management, derivatives pricing and portfolio optimization. Hence, many studies have been conducted in the area of volatility modelling and estimation.

Stochastic differential equations (SDE) are the mathematical tools that can capture the volatility of the growth momentum. Stochastic differential equations are used in financial field and act as a basis of many stochastic volatility models, such as Hull-White model and Heston model (Remer and Mahnke, 2004).

Geometric Brownian Motion is an example of stochastic process that satisfies a stochastic differential equation. According to Duplantier (2005), a French mathematician, Louis Bachelier mentioned in his PhD thesis in 1900 that the stock price dynamics follows Brownian Motion. More than half a century later, Samuelson modified Bachelier model in 1965 and discussed that the return rates, instead of the stock prices, follow Geometric Brownian Motion which can avoid negative value (Piasecki, 2006).

Time series of stock prices, interest rate and exchange rate fluctuations are some of the common examples of stochastic processes. According to Hull (2009), Geometric Brownian Motion is the most used stochastic process as it is always positive because it follows log-normal distribution. In 1993, Steven Heston proposed a model for the stock price dynamics using Geometric Brownian Motion with stochastic volatility. The model is known as Heston model. This model assumes that the volatility of the asset is not constant but follows a random process.

1.2 Statement of the Problem

Index prices behave randomly so it is not accurate to be represented by initial value problem with ordinary differential equation. The Heston model, which is governed by Geometric Brownian Motion with stochastic volatility, is always used to describe the evolution of the volatility of an underlying asset since the volatility of the underlying asset is not constant. Hence, the incorporation of stochasticity into index prices is important in determining correctly the probability distribution of log returns for stock indexes.

However, Heston (1993) does not provide the probability distribution of log returns for stock prices. Dragulescu and Yakovenko (2002) had derived the analytical formula for the probability distribution of log returns in the Heston model. According to them, their equations reproduce the probability distribution of returns for time lag 1 and 250 trading days very well by using only four parameters. However, the number of parameters of other similar models can easily go to a few tens. Moreover, Silva and Yakovenko (2003) has tested that the probability distribution of log returns agreed very well with the analytical formula for NASDAQ, S&P 500 and Dow-Jones for the period 1982-1999 via visual inspection.

In this study, we will test wheteher the techniques given in Dragulescu and Yakovenko (2002) will be able to capture the volatility of the index prices for FTSE Bursa Malaysia KLCI (formerly known as KLCI). The test needs to include the estimation of the parameters of the Heston model.

1.3 Objectives of the Study

The objectives of study are:

1.3.1 To estimate the long time mean of variance, the rate of relaxation to this mean, the variance noise and the drift parameter of the Heston model.

1.3.2 To assess the probability distribution of log returns of index prices FTSE Bursa Malaysia KLCI from the Heston model via visual inspection.

1.3.3 To compare the observed probability distribution of log returns of index prices FTSE Bursa Malaysia KLCI with the analytical formula (Dragulescu and Yakovenko, 2002).

1.4 Scope of the Study

This study focuses on estimation of the probability distribution of log returns in the Heston model by using empirical data. In this study, the index prices of FTSE Bursa Malaysia KLCI were collected from Yahoo! Finance website (*http://finance.yahoo.com/q/hp?s=%5EKLSE+Historical+Prices*) from 3rd December 1993 until 4th May 2012 which consisted 4544 data of the index prices of FTSE Bursa Malaysia KLCI. The technique used to estimate the parameters is Simulated Maximum Likelihood method. The Euler-Maruyama method is used to solve the stochastic differential equation.

1.5 Significance of the Study

The graphical representation of the probability distribution of the index prices of FTSE Bursa Malaysia KLCI will guide the investors on how to invest in FTSE Bursa Malaysia KLCI based on the highest probability of log returns. Since FTSE Bursa Malaysia KLCI comprises the largest 30 companies listed on the Main Board, it will more aptly define market activities in Malaysian stock market.

1.6 Organization of the Study

The organization of this study can be divided into 5 chapters. The report starts with the study framework followed by literature review and finally it ends with conclusion and recommendation.

In chapter 1, background of the study and statement of the problem are described. This chapter also contains the objectives of this study, the scope, and the significance of the study.

Chapter 2 is mainly about the literature review of this study. Some remarkable histories such as Brownian Motion, Cox-Ingersoll-Ross process, Simulated Maximum Likelihood method and Euler-Maruyama approximations are briefly described. Besides that, the previous research about Heston model is also discussed in this chapter.

Chapter 3 presents the research methodology. The design of research is illustrated. This chapter also discusses the Heston model and the method used to find the analytical formula for the probability distribution of log-returns in Heston model. Besides that, the algorithm of Euler-Maruyama approximations and the parameters estimation are briefly illustrated. Moreover, the algorithm for calculating the probability distribution of log-returns for KLCI is described in this chapter.

Chapter 4 contains the results and discussions. The estimated parameter values of the Heston model are presented in this chapter. Furthermore, the discussion of the suitability of the analytical formula with the probability distribution for log-returns of index prices is also described in this chapter.

Chapter 5 is the final chapter of the study which consists of the conclusion of the whole study and some recommendations for the future research.

REFERENCES

Aihara, S. and Bagchi, A. (2006). Filtering and identification of Heston's stochastic volatility model and its market risk. *Journal of Economic Dynamics & Control*. Vol 30. 2363 – 2388. Elsevier.

Aldrich, J. (1997). R. A. Fisher and the Making of Maximum Likelihood 1912 – 1922. *Statistical Science*. Vol 12. Number 3. 162-176. Institute of Mathematical Statistics.

Alfonsi, A. (2012). Strong order one convergence of a drift implicit Euler scheme : Application to the CIR process. *Statistics and Probability Letters*. Vol 83. 602–607. Elsevier.

Anissimov, M. (2012). *What is Brownian Motion*. Conjecture Corporation.

Baduraliya, C. H. and Xuerong, Mao. (2012). The Euler–Maruyama approximation for the asset price in the mean-reverting-theta stochastic volatility model. *Computers and Mathematics with Applications.* Vol 64. 2209 – 2223. Elsevier.

Ballestra, L.V., Pacelli, G. and Zirilli, F. (2007). A numerical method to price exotic path-dependent options on an underlying described by the Heston stochastic volatility model. *Journal of Banking & Finance.* Vol 31. 3420–3437. Elsevier.

Bernaschi, M., Torosantucci, L. and Uboldi, A. (2007). Empirical evaluation of the market price of risk using the CIR model. *Physica A.* Vol 376. 543–554. Elsevier.

Cao, W., Liu M. and Fan, Z. (2004). MS-stability of the Euler–Maruyama method for stochastic differential delay equations. *Applied Mathematics and Computation*. Vol 159. 127 – 135. Elsevier.

Carmona, J. and Leon, A. (2007). Investment option under CIR interest rates. *Finance Research Lett*ers. Vol 4. 242–253. Elsevier.

Cox, J. C., Ingersoll, J. E. and Ross, S. A. (1985). A theory of the Term Structure of Interest Rates. *Econometrica*. Volume 53, Issue 2, 385-408. The Econometric Society.

Dragulescu, A. A. and Yakovenko, V. M. (2002). Probability Distribution of Returns in the Heston Model with Stochastic Volatility. *Quantitative Finance*. Vol 2. 443-453. Institute of Physics Publishing.

Duplantier, B. (2005). Brownian Motion, "Diverse and Undulating". *Progress in Mathematical Physics*. Vol. 47. 201 – 293. Poincare Seminar 2005.

Ermogenous, A. (2005). Brownian Motion and Its Applications in The Stock Market. *Department of Applied Mathematics, Illinois Institute of Technology*. Chicago.

Ewald, C. O. and Wang, W. K. (2010). Irreversible investment with Coc-Ingersoll-Ross type mean reversion. *Mathematical Social Sciences*. Vol 59. 314-318. Elsevier.

Fiorentini, G., Leon, A. and Rubio, G. (2002). Estimation and empirical performance of Heston's stochastic volatility model : the case of a thinly traded market. *Journal of Empirical Fina*nce. Vol 9. 225 – 255. Elsevier.

Gardiner, C. W. (1993). *Handbook of Stochastic Methods for Physics, Chemistry and the Natural Sciences.* Berlin : Springer.

Goh, S. K. and Michael Lim, M. H. (2010). The Impact of The Global Financial Crisis : The Case of Malaysia. *TWN Global Economy Series. Third World Network.* Jutaprint.

Grzelak, L. A. and Oosterlee, C. W. (2010). On The Heston Model with Stochastic Interest Rates.

Gushchin, A. A. and Kuchler, U. (2004). On oscillations of the geometric Brownian motion with time-delayed drift. *Statistics & Probability Letters*. Vol 70. 19–24. Elsevier.

Heston, S. L. (1993). A Closed-form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options. *The Review of Financial Studies*. Vol 6. Issue 2. 327-343.

Hughes, H. (1999). Crony Capitalism and The East Asian Currency and Financial 'Crises'. *Policy*. 3-5. Spring.

Hull, J.C. (2009). Options, Futures and Other Derivatives. New Jersey : Prentice Hall.

Iacus, S. M. (2008). *Simulation and Inference for Stochastic Differential Equations*. New York : Springer.

In Jae Myung. (2002). Tutorial on maximum likelihood estimation. *Journal of Mathematical Psychology.* Vol 47. Issue 1. 90–100. USA : Academic Press.

Kanagawa, S. Arimoto, A., and Saisho, Y. (2005). Numerical simulation of multi dimensional reflecting geometrical Brownian motion and its application to mathematical finance. *Nonlinear Analysis*. Vol 63. e2209 – e2222. Elsevier.

Kima, J., Kima, B., Moon, K. S. and Wee, I. S. (2012). Valuation of power options under Heston's stochastic volatility model. *Journal of Economic Dynamics & Control.* Vol 36. 1796 – 1813. Elsevier.

Kristensen, D. and Shin, Y. (2012). Estimation of dynamic models with nonparametric simulated maximum likelihood. *Journal of Econometrics*. Vol 167. 76 – 94. Elsevier.

Kho, C. C. (2011). *Application Of Black-Scholes Option Pricing Model In Financial Investment.* Degree of Bachelor of Science. Universiti Teknologi Malaysia, Skudai.

Ladde, G. S. and Wu, L. (2009). Development of modified Geometric Brownian Motion models by using stock price data and basic statistics. *Nonlinear Analysis.* Vol 71. e1203-e1208. Elsevier.

Lepage, T., Law, S., Tupper, P., and Byrant, D. (2006). Continuous and tractable models for the variation of evolutionary rates. *Mathematical Biosciences.* Vol 199. 216–233. Elsevier.

Li, H., Xiao, L. and Ye, J. (2012). Strong predictor–corrector Euler–Maruyama methods for stochastic differential equations with Markovian switching. *Journal of Computational and Applied Mathematics.* Vol 237. 5 – 17. Elsevier.

Lochowski, R. M. (2011). Truncated variation, upward truncated variation and downward truncated variation of Brownian motion with drift — Their characteristics and applications. *Stochastic Processes and their Applications.* Vol 121. 378–393. Elsevier.

Mao, X., Yuan, C. and Yin, G. (2007). Approximations of Euler–Maruyama type for stochastic differential equations with Markovian switching, under non-Lipschitz conditions. *Journal of Computational and Applied Mathematics.* Vol 205. 936 – 948. Elsevier.

Maruyama, G. (1955). Continuous Markov processes and stochastic equations. *Rendiconti del Circolo Matematico di Palermo.* Vol 4. Issue 1. 48 - 90. Springer-Verlag.

Ministry of Finance Malaysia. (1998). *Economic Report 1997/1998*. Malaysia.

Ohnishi, M. (2003). An Optimal Stopping Problem for a Geometric Brownian Motion with Poissonian Jumps. *Mathematical and Computer Modelling*. Vol 38. 1381-1390. Elsevier.

Pederson, A. R. (2000). Estimating the Nitrous Oxide Emission Rate from the Soil Surface by Means of a Diffusion Model. *Scandinavian Journal of Statistics*. Vol 27. Number 3. 385-403. Blackwell Publishers Ltd.

Piasecki, J. (2006). Centenary of Marian Smoluchowski's Theory of Brownian. *XIX Marian Smoluchowsk Symposium on Statistical Physics.* 14-17 May : Krakow, Poland. Vol 38.

Picchini, U. (2007). SDE Toolbox : Simulation and Estimation of Stochastic Differential Equations with MATLAB. http://sdetoolbox.sourceforge.net

Postali, F. A. S. and Picchetti, P. (2006). Geometric Brownian Motion and structural breaks in oil prices: A quantitative analysis. *Energy Economics.* Vol 28. 506–522. Elsevier.

Poufinas, T. (2008). On the number of deviations of Geometric Brownian Motion with drift from its extreme points with applications to transaction costs. *Statistics and Probability Letters*. Vol 78. 3040-3046. Elsevier.

Remer, R. and Mahnke, R. (2004). Application of Heston model and its solution to German DAX data. *Physica A.* Vol 344. 236–239. Elsevier.

Rollin, S. B., Ferreiro-Castilla, A., and Utzet, F. (2010). On the density of log-spot in the Heston Model. *Stochastic Processes and their Applications*. Vol 120. 2037 - 2063. Elsevier.

Salehi, H. (1967). Review: N. Wiener, A. Siegel, B. Franklin, W. T. Martin, Differential Space, Quantum Systems, and the Prediction. *The Annals of Mathematical Statistics*. Vol 38. Number 6. Institute of Mathematical Statistics.

Sauer, T. (2006). *Numerical Analysis*. Pearson Addison Wesley.

Silva, A. C. and Yakovenko, V. M. (2003). Comparison between the probability distribution of returns in the Heston model and empirical data for stock indexes. *Physica A.* Vol 324. 303 - 310. Elsevier.

Sinkala, W., Leach, P. G. L. and O'Hara, J. G. (2008). An optimal system and group-invariant solutions of the Cox-Ingersoll-Ross pricing equation. *Applied Mathematics and Computation.* Vol 201. 95–107. Elsevier.

The World Bank. (2011). *Malaysia Economic Monitor*. 61483.

Van Haastrecht, A. and Pelsser, A. A. J. (2010). Efficient, Almost Exact Simulation of the Heston Stochastic Volatility Model. *International Journal of Theoretical and Applied Finance (IJTAF).* Vol 31. Number 1. Netspar.

Vicente, R., Toledo, C. M., Leite, V. B. P. and Caticha, N. (2006). Underlying dynamics of typical fluctuations of an emerging market price index : The Heston model from minutes to months. *Physica A.* Vol 361. 272 - 288. Elsevier.

Voit, J. (2003). From Brownian motion to operational risk: Statistical physics and financial markets. *Physica A*. Vol 321. 286 – 299. Elsevier.

Wu, F., Mao, X. and Chen, K. (2009). The Cox–Ingersoll–Ross model with delay and strong convergence of its Euler–Maruyama approximate solutions. *Applied Numerical Mathematics.* Vol 59. 2641 – 2658. Elsevier.

Zhang, X. (2006). Euler–Maruyama approximations for SDEs with non-Lipschitz coefficients and applications. *Journal of mathematical Analysis and Applications.* Vol 316. 447 – 458. Elsevier.

Zhao, G., Song, M. H., and Liu, M. Z. (2010). Exponential stability of Euler–Maruyama solutions for impulsive stochastic differential equations with delay. *Applied Mathematics and Computation.* Vol 215. 3425 – 3432. Elsevier.

Zhu, J. (2003). Testing for Expected Return and Market Price of Risk in Chinese A-B Share Market: A Geometric Brownian Motion and Multivariate GARCH Model Approach. *Social Science Research Network.* 1148149.