HOMOTOPY CONTINUATION METHOD IN AVOIDING THE PROBLEM OF DIVERGENCE OF TRADITIONAL NEWTON'S METHOD

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Dedicated to my beloved mother and father, Azizah bt. Mahmood and Mat Nor @ Zakaria b. Mat Deris, to my beloved husband, Azmir b. Ayub, to my beloved sons and daughter, Mirza, Harith, Irdina and all my friends.

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ABSTRAK

Kaedah Newton adalah satu kaedah yang terkenal bagi menyelesaikan masalah pengoptimuman bagi fungsi tidak linear. Ini adalah kerana kecekapan kaedah ini dalam kelajuan penumpuan. Namun, Kaedah Newton selalunya akan menghasilkan pencapahan terutamanya apabila nilai awal jauh daripada penyelesaian sebenar. Dalam situasi lain, pencapahan juga berlaku apabila terbitan peringkat ke dua dalam rumus lelaran berangka Kaedah Newton bernilai sifar atau menghampiri sifar. Kaedah Lanjutan Homotopi (Homotopy Continuation Method) mempunyai kebolehan untuk mengatasi masalah ini. Tujuan kajian ini dibuat adalah untuk mengkaji langkah yang diambil dalam Kaedah Lanjutan Homotopi (Homotopy Continuation Method) bagi mengatasi masalah pencapahan dalam Kaedah Newton. Kaedah Lanjutan Homotopi (Homotopy Continuation Method) adalah sejenis kaedah pertubasi yang dapat menjamin jawapan dengan adanya jalan yang tertentu jika kita memilih fungsi homotopi tambahan. Kaedah ini mengubah situasi yang rumit menjadi sederhana yang bertujuan memudahkan penyelesaian dan secara berperingkat mengubah masalah sederhana kepada masalah yang asal dengan mengira nilai-nilai optimum bagi masalah yang diubah dan akhirnya kaedah ini berakhir dengan penghasilan nilai extremum bagi masalah asal. Untuk menguatkan lagi penemuan, kajian ini menyediakan penerangan mengenai kod MATLAB bagi melaksanakan Kaedah Lanjutan Homotopi (Homotopy Continuation Method) dan Kaedah Newton untuk menyelesaikan masalah pengoptimuman. Kajian ini berjaya dalam mengatasi masalah pencapahan dalam Kaedah Newton dan dapat menjamin dalam mendapatkan jawapan.

ABSTRACT

The traditional Newton's Method is known as a popular method for solving optimization problem of non-linear functions. It is derived from the efficiency in the convergence speed. However, Newton's Method usually will yield divergence especially when the initial value is far away from the exact solution. In another situation, divergence also occur when the second derivative in the numerical iteration formula of Newton's Method is equal to zero or tends to zero. Homotopy Continuation Method has the ability to overcome this problem. The purpose of this research is to probe the step taken in Homotopy Continuation Method in avoiding the problem of divergence in Traditional Newton's Method. Homotopy Continuation Method is a kind of perturbation method that can guarantee the answer by a certain path if we choose the auxiliary homotopy function. This method transforms a complicated situation into a simpler one that is easy to solve and gradually deform the simpler problem into the original one by computing the extremizers of the intervening problems and eventually ending with an extremum of the original problem. To strengthen the findings, this thesis presents a description of a MATLAB code that implements the Homotopy Continuation Method and Newton's Method for solving the optimization problem. This study succeeded in avoiding the problem of divergence of traditional Newton Method and can guarantee the answer.

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LIST OF SYMBOLS AND ABREVIATIONS

f	Function
H	Homotopy Function
Δ	Difference
$\rightarrow$	Tends to
ε	Epsilon
x^*	Optimum point
$\mathbb{R}$	Real number

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CHAPTER 1

INTRODUCTION

1.1 Introduction

In the real world, optimization can be used in business transactions, engineering design and in our daily routine. In business, they use optimization to maximize their profit and minimize the cost. In our daily routine, for example when we plan our holiday, we want to minimize the cost but maximize the enjoyment. Therefore, optimization study are both of scientific interest and practical implications and subsequently the methodology will have many applications (Yang, 2008).

Judd (1999) said, in economic, economists assume that firms minimize costs and maximize profits, consumers maximize utility, payers maximize payoffs, and social planners maximize social welfare. They also use optimization methods

in least squares, method of moments, and maximum likelihood estimation procedures.

Generally, optimization consists of three steps. First, understanding the system. Secondly, finding the measure of system effectiveness and finally finding the degrees of freedom analysis and choosing a proper technique to solve the problem. Whatever the real world problem is, it is possible to formulate the optimization problem in a generic form.

In solving the optimization problem on nonlinear equations, some uncontrollable situation as overflow and divergence might usually arise. This situation occurred because of the bad structure of equations and the initial guess that is far away from the solution.

Homotopy Continuation Method belong to the family of continuation methods. This method was known as early as the 1930s. In 1960s in United States, this method was used by kinematician for solving mechanism synthesis problems. Homotopy Continuation Method gives a set of certain answers and some simple iteration process to obtain our solutions more precisely. Like all other continuation method, they represent a way to find solution to a problem by constructing a new problem that is simpler than the original one, and then gradually transform this simpler problem into the original one. The big advantage of this method is it can guarantee the answer by a certain path if we choose the auxiliary homotopy function. It does not matter if we choose the initial value far away from the solution and if the second derivatives in our iteration is equal to or tends to zero.

1.2 Background of the Problem

Newton's Method have several weakness. Firstly, if the initial value is far away from the solution, then the Newton's Method is not guaranteed to converge to the minimizer. Finding a good initial value x_0 for Newton's method is a crucial problem. Sometimes, this method leads to divergent oscillations that move away from the answer, which it overshoots. Secondly, if the second derivatives in the numerical iteration formula of Newton's Method is equal to zero or tends to zero, the solution has the possibility to diverge.

Homotopy Continuation Method can be used to generate a good starting value. This method can guarantee to converge to the answer if we choose the auxiliary homotopy function well. Homotopy Continuation Method does some simple iteration process to obtain our solutions more precisely. The algorithm of this method is clear and easy as well as its convergence speed is fast.

Wu (2005), presents some useful rules for the choice of the auxiliary homotopy function to avoid the problem of divergence of traditional Newton's Method for finding the roots. In this study, we modify the methods introduce by Wu (2005), and present some useful rules for choosing the auxiliary homotopy function to avoid the problem of divergence of traditional Newton's Method for solving optimization problem.

1.3 Statement of the Problem

Newton's Method usually will yield divergence especially when the initial initial value is far away from the exact solution. The homotopy method can be used to generate a good starting value. In other situation, divergence also occur when the second derivatives of numerical iteration formula of Newton's Method is equal to zero or tends to zero. Homotopy Continuation Method has ability to overcome this problem.

The purpose of this research is to probe the step taken in Homotopy Continuation Method in avoiding the problem of divergence on Traditional Newton's Method.

1.4 Research Question

For this research, there are some questions need to be answered.

i. What is the reason behind the situation of divergence in Newton's Method?
ii. How does Homotopy Continuation Method generate a good starting value to avoid the problem of divergence in Newton's Method?
iii. What is the algorithm of Homotopy Continuation Method for finding extremizer?

iv. What is the result analysis of the ability of Newton's Method and Homotopy Continuation Method in solving optimization problem?

v. What is the result analysis of the trace of initial solution produce by Linear Homotopy and Newton Homotopy.

1.5 Objectives of the Study

Objectives of this study are:

i. To identify the reason behind the situation of divergence in Newton's Method.

ii. To study how Homotopy Continuation Method generate a good starting value .

iii. To develop the algorithm of Homotopy Continuation Method for finding extremizer.

iv. To analyze and compare the ability of Newton's Method and Homotopy Continuation Method in solving optimization problem.

v. To analyze and compare the trace of initial solution produce by Linear Homotopy and Newton Homotopy.

1.6 Scope of the Study

The scope of this study is to apply Homotopy Continuation Method for solving optimization problem. Nonlinear equations on univariate function will be used. Some example will be presented.

We will only focus only in solving optimization problem on univariate function of nonlinear equation for several reason. First, one dimensional problem illustrate the basic techniques in very simple and easy to understand problem. Second, one dimensional problem is important since many of multivariate method reduce to solving a sequence of one-dimensional problems.

1.7 Significance of the Study

To explore as well as to excel the knowledge in the usage of Homotopy Newton's Method to overcome the problem in Newton's Method. This research presents some useful rule of Homotopy Continuation Method to avoid the problem of divergence of traditional Newton's Method. Using the homotopy function, Newton's Method will become more efficient. The new Homotopy Continuation Method can be used in the industry instead of Newton's Method.

This research is good enough to produce the research paper. Research paper will also be sent and published in the national journal. The result of this

research can be used for further research in related areas. Furthermore, this research will enhance contribution from Mathematicians in Malaysia especially in Operational Research area. It is hoped that the work presented here will contribute towards progress in the numerical techniques and other relative fields for scientists or engineers.

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