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ANALYSIS OF AN URBAN WATER SUPPLY BY LINEAR PROGRAMMING

by

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ABSTRACT

This study investigates the use of linear programming to determine waters distribution schedule from different sources to different water-demanding sectors as well as to determine the scheduled priority use of different types of water resources. For this purpose, the transportation and mixed integer programming algorithms are used. The formulation of both algorithms were analyzed using LINDO program. The method of linear programming (LP) is adopted to optimize water delivery schemes for Irbid City, the second largest city in Jordan. This city as well as the whole country depends mainly upon surface water, and groundwater resources which are scarce and expensive to exploit. The result of the study shows a reasonable argument where all surface water from all sources (Mukhaiba, Wadi Arab and Local supply) is used and none of the groundwater supply is diverted for agricultural purposes due to its higher cost.

INTRODUCTION

Due to the increasing water scarcity around the globe, great attention has been given to water resources planning and management. It involves the concepts of examining ways to meet the short and long term needs and requires an understanding on how to use the available resources to meet the demand. Providing adequate water supply from various limited resources is becoming a problem of serious concern to meet increasing demands. The problem of supplying an adequate amount of water to meet the different water-demanding sectors is of utmost important to a country which has a limited resources like Jordan. It is reported that the current water demand in Jordan is about 660 MCM denominated mainly on irrigation requirement. This demand will triple in the next 20 years with a projected population growth of 3.5%. (Jordan Water Resources Report, 1985).

Remedial action should be taken in order to meet the demand in future in which the present water production cannot meet peak demands without using storage water. The response to this shortage has been developed by identifying various possible resources to meet future demands.

OBJECTIVE

To furnish water planner a quick reference of the idea on how the existing water supply is distributed, a linear programming method is adopted to optimize water delivery schemes for Irbid, the second largest city in Jordan. This city as well as the whole country depends mainly upon surface water and groundwater resources which are scarce and expensive to exploit.

Taking the advantage of mixed integer programming (MIP), this problem is first formulated using this algorithm to determine the priority of use of different type of water resources from different cities (sources). The MIP is used rather than pure integer programming (IP) or normally is referred to integer linear programming because the IP algorithm uses the integer value of zero and one to determine the 'yes-or-no' decision (Hillier & and Lreberman, 1990). In such decisions, the only two possible choices are either yes or no. In this study some of the variables are required to have integer value and therefore MIP algorithm is adopted. The determination of priority of use can be applied using the MIP in order to first evaluate the sources of water supply to be delivered to Irbid City as well as scheduling of those sources.

Then the transportation algorithm is formulated to optimize the delivery of the available water supply from different parts of resources to satisfy the demand of various water users. The optimizing objective is to minimize costs and these parameters are required by water planners to assist their planning and decision making. An early action is necessary for water planners to avoid an inadequate water supply for the city to meet a future fast growing demands.

STUDY AREA

The city of Irbid is located at the northernmost part of the Country of Jordan. According to the 1985 population census, this city has a population of 750,000 and expected to have an annual increase of 3.5% population growth. The pattern of settlement in most part of the country is influenced by water availability and it happened that Irbid has become a focus of population concentration settlement due to its adequate water supply and water availability. The water resources for this city depend our surface and groundwater systems. The groundwater supply to the city comes from Wadi Aqib and Samaya.

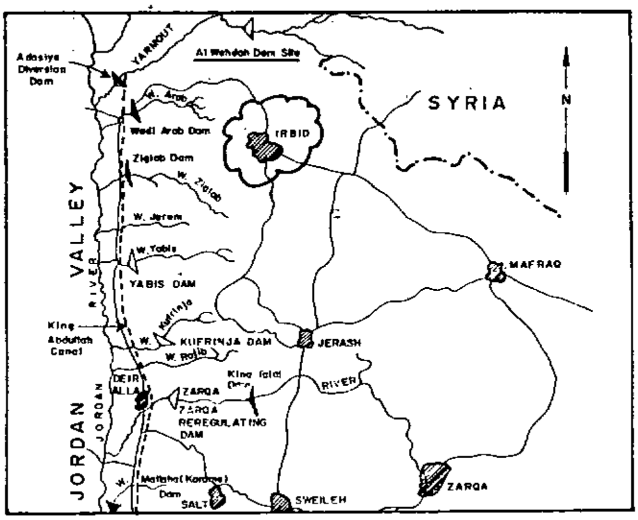

Figure 1 : Location map of study area

Mukhaiba and Wadi Arab supply surface water to the city. The supply of surface as well as ground water also comes from local resources with a limited amount. Due to high cost of exploiting ground water, it is served only for municipal and industrial demands. Surface water supplies the irrigation demand. Figure 1 shows the general location of the study area and the city of Irbid.

Problem Formulation

The mixed integer programming is formulated according to the priority arranged in descending order as shown in Table 1. In this arrangement, ground water goes last where previously explained that this resource is expensive to be exploited. In addition, the supply from Wadi Aqib was arranged at the least priority because it is the most expensive ground water resource in due to its geographic location.

	Year	1985	1990	1995	2000
	Source	MCM	MCM	MCM	MCM
Surface Water	Local	5	5	5	5
	Wadi Arab	11	20	20	20
	Mukhaiba	5	5	5	5
	Local	2	2	2	2
	Samaya	5	6.5	6.5	6.5
	Wadi Aqib	7	7	10	10

Table 1 : Total distribution of sources of water for MIP

The figure summarized in Table 1 was abstracted from the Jordan Water Resources Report 1990.

As explained earlier, the objective of this algorithm is to determine the scheduled priority of use of different types of water resources from different places as a guideline and comparison to the formulation of transportation algorithm. Then after conforming the priority of used resulted from MIP, the transportation algorithm is formulated to solve the water distribution from different sources to different water-demanding sectors.

Taking an advantage of an attractive and straight forward transportation algorithm from linear programming, the water supply for the Irbid City can be generalized us in the matrix of Table 2.

ORIGIN	DESTINATION	Domestic	Agricultural	Industrial	Category Availability
Surface Water	Source 1 Source 2 Source 3				Safe Yield (surface)
Ground Water	Source 1 Source 2 Source 3				Safe Yield (G/water)
Demand	Requirement	Domestic Requirement	Agricultural Requirement	Industrial Requirement	

Table 2 : Matrix of water resources system

The primary resources of water supply, which are surface water and ground water are the row headings in the matrix, and the demand requirements are grouped into three general sectors of domestic, agricultural, and industrial are the column headings. Even though ground water is more expensive to be used for irrigation compared to surface water, restriction is not being made for that resources as an entry in the matrix of Table 2 into that category of distination. It allows all possible combination by which the total available supply may satisfy the demand regardless the cost of the source.

The transportation programming model is capable to give an optimal solution to the objective function of this distribution operation. In a general transportation problem, a homogeneous source is available in the amounts of S_i (i = 1,2, ...,m) for each of *m* shipping origins and is required to distribute to *n* destination with the required amount of d_j (j = 1, 2, ..., h). The structure of this problem can be summarized in Table 3 shown below.

	DESTINATIONS 1	2	3	.....	n	SUPPLY
Source (Origin) 1	C_{11}	C_{12}	C_{13}	.....	C_{1n}	S_1
2	C_{21}	C_{22}	C_{23}	.....	C_{2n}	S_2
3	C_{31}	C_{32}	C_{33}	.....	C_{3n}	S_3
.	.					.
.	.					.
m	C_{m1}	C_{m2}	C_{m3}	.....	C_{mn}	S_m
DEMAND	d_1	d_2	d_3	.....	d_n	

Table 3 : Table for the transportation algorithm

The objective of this problem is to determine the amount of X_{ij} (i = 1, 2, ..., m ; j = 1, 2, ... , n) to be distributed over all possible routes so as to minimize the total cost z. The term X_{ij} denotes the amount of water delivered from city i to j_{th} water demanding sector. The cost coefficient C_{ij} represents the cost of distributing a unit of water from i_{th} origin to j_{th} destination. This coefficient should be known for all combinations as previously summarized in Table 3.

Unlike the asignment problem in linear programming, the TP has a restriction to have a feasible solution is that the row sums equals to the column sum, i.e supply equal demand, that

$$\sum_{i=1}^{m} S_i = \sum_{j=1}^{n} d_j$$

where d_j and s_i are demand requirement and supply respectively. With this restriction, a dummy constraint should be introduced in formulating the problem.

The cost element in the matrix, $C_{ij,}$ which can be translated as the cost of delivering one unit of water from i_{th} source to j_{th} destination involve three components namely :

1. Raw water cost : This includes the cost of getting (buying) the Water Rights and transmission costs.
2. Treatment cost : To provide a safe and clean water, the raw water should be treated to meet the standards set by local environmental agency. It is the cost of treating one unit of water to meet the standard required by different water demanding sectors.
3. Distribution cost : This cost includes the cost of storage, pipe laying and pumping from a source to a destination.

As reported in the Jordan Water Resources Report (1990), a lump sum approach was adopted in the prediction of the cost incorred to deliver water supply. Due to this approach the same cost appears each time and are not necessary to be discounted to a present value.

Having all the basic required data, the problem can be solved by linear programming technique so called the transportation problem algorithm after it is first solved by MIP to evaluate and assess the supply of water according to its priority of use.

			DOMESTIC	AGRIC	INDUSTRIAL	AVAILABLE SUPPLY (MCM)
SURFACE WATER	Mukhaiba	1985 1990 1995 2000	783*	460	783	5.0 5.0 5.0 5.0
	Wadi Arab	1985 1990 1995 2000	783	460	783	11.0 20.0 20.0 20.0
	Local	1985 1990 1995 2000	500	350	500	5.0 5.0 5.0 5.0
GROUND WATER	Samaya	1995 1990 1995 2000	783	800	783	5.0 6.5 6.5 6.5
	Wadi Aqib	1985 1990 1995 2000	783	800	783	7.0 7.0 10.0 10.0
	Local	1995 1990 1995 2000	500	350	500	2.0 2.0 2.0 2.0
	Demand	1985 1990 1995 2000	11.0 12.6 14.5 16.6	14.0 16.4 19.2 22.4	7.0 7.0 8.0 8.5	

* Thousand Dollars per MCM

Table 4 : Transportation problem table for Irbid city

Result and Discussion

The application of MIP and transportation problem algorithms is illustrated by using the available data of water supply and demand for the city of Irbid, Jordan. The data used on these formulations are based on the 1985 data with the projection to the year 2020. It is estimated that the annual population increase is 3.5% with the 1985 population of 143,042. The water demand requirement for each sectors appears in the bottom row of the matrix and the availabilities are recorded in the right hand column of the matrix as summarized in Table 4. The cost incurred to convey the water supply from each source to different sectors appear in the matrix element in the table. In this study, the cost function was assumed constant. The cost function could have been non-linear which has a function of distance from the supply delivers to water-demanding ssectors which could look like the illustration in Figure 2.

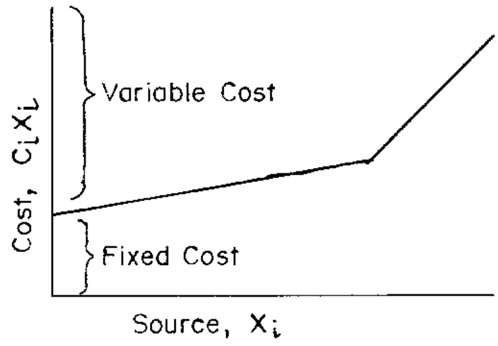

Figure 2 : Illustration of cost variation

The mixed integer programming is formulated as :

$$Max\ Z = \sum_{i=1}^{m} Ci\ Xi$$

subject to;

$$X_i \leq U_i$$

$$X_j \leq 1$$

$$X_j \geq 0$$

$$X_i - U_i\ X_j \geq 0$$

$$-X_{i+1} + U_{i+1}\ X_j \geq 0$$

$$\sum_{i=1}^{m} X_i = demand$$

for i = 1, 2, ..., 5
j = 7, 8, ..., 11, and
m = 6 [number of sources]

Non-negativity

All $X_i \geq 0$ for all i's

where

C_i denotes cost of delivering water from source i
X_i for i = 1, 2, ..., 6 denotes the source from city i
X_j = 1 if $Xi > 0$
= 0 otherwise for j = 7, 8, ... 12
U_i denotes total amount of available supply from source (city) i.

The results of this formulation for 1985, 1990, 1995 and 2000 is summarized in Table 5. The figures in table show the supply from each available source to meet the required total demand and figures in bracket are the available supply.

	YEAR	1985	1990	1995	2000
	SOURCE	MCM	MCM	MCM	MCM
SURFACE WATER	Mukhaiba	5(5)*	5(5)	5(5)	5(5)
	Wati Arab	11(11)	20(20)	20(20)	20(20)
	Local	5(5)	5(5)	5(5)	5(5)
GROUND WATER	Samaya	5(5)	4(6.5)	6.5(6.5)	6.5(6.5)
	Wadi Aqib	4(7)	0(7)	3.2(10)	9.0(10)
	Local	2(2)	2(2)	2.0(2)	2.0(2)
Total Demand		32	36	41.7	47.5

* Total supply (available supply)

Table 5 : Supply of water from different sources as resulted from MIP

The transportation problem algorithm was then formulated to determine the amount of water taken from sources to supply the demand to different water demanding sectors in Irbid City. This problem is formulated as follow:

$$Min\ Z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} X_{ij}$$

Subject to;

Demand Constraints

$$\sum_{i=1}^{m} X_{ij} = d_j$$

Supply Constraints

$$\sum_{i=1}^{n} X_{ij} = S_i$$

Non-negativity

$$X_{ij} > 0 \text{ for all } i \text{ and } j$$

for

i = 1, 2, ..., 6
j = 1, 2, 3, 4
m = 6 [number of sources]
n = 4 [number of water-demanding sectors]

where

C_{ij} denotes the cost to deliver water from source (city) i to j*th* water-demanding sectors,

X_{ij} denotes the amount of water delivered from source (city) i to j*th* water-demanding sectors

In this formulation city (source) i = 1, 2, and 3 are for surface water source from city of Mukhaiba, Wadi Arab and local supply respectively; and i = 4,5, and 6 are for ground water source from city of Samaya, Wadi Aqib and local supply respectively. The transportation algorithm as well as MIP were analyzed using LINDO. The results obtained from the transportation problem formulation are summarized in Table 6. The matrix elements in Table 6 show that the amount of water required from each sources to be supplied to the water demanding sectors. From this results it shows that all surface water from Mukhaiba should be supplied to agricultural use. It also shows a reasonable argument that all surface water from all source is used as well as ground water from local supply. Due to a higher cost of ground water, no amount is diverted for agricultural purposes as shown in the result. From MIP result, it also shows that surface water should be used up first-before utilizing ground water supply. Looking at ground water supply, it can be seen that the cheapest-sources 5 first-utilized as explained in Table 5.

			Domestic	Agric	Industrial	'Dummy'	Available supply
SURFACE WATER	Mukhaiba	1985	0	5	0	0	5.0
		1990	0	5	0	0	5.0
		1995	0	5	0	0	5.0
		2000	0	5	0	0	5.0
	Wadi Arab	1985	0	9.0	2.0	0	11.0
		1990	3.6	11.4	5.0	0	20.0
		1995	2.8	14.2	3.0	0	20.0
		2000	1.1	17.4	1.5	0	20.0
	Local	1985	0	0	5.0	0	5.0
		1990	5.0	0	0	0	5.0
		1995	0	0	5.0	0	5.0
		2000	0	0	5.0	0	5.0
GROUND WATER	Samaya	1995	5	0	0	0	5.0
		1990	0	0	0	6.5	6.5
		1995	6.5	0	0	0	6.5
		2000	6.5	0	0	0	6.5
	Wadi Aqib	1985	4.0	0	0	3.0	7.0
		1990	4.0	0	0	3.0	7.0
		1995	3.2	0	0	6.8	10.0
		2000	9.0	0	0	1.0	10.0
	Local	1995	2.0	0	0	0	2.0
		1990	0	0	2.0	0	2.0
		1995	2.0	0	0	0	2.0
		2000	0	0	2.0	0	2.0
	Demand	1985	11.0	14.0	7.0	3.0	
		1990	12.6	16.4	7.0	9.5	
		1995	14.5	19.2	8.0	6.8	
		2000	16.6	22.4	8.5	1.0	

Table 6 : The results of linear programming formulation using transportation algorithm for Irbid City (MCM)

From the projected figures for water use in future years, it can be explained that the city of Irbid should have found other alternative means of water supply to cater the future needs as well as planning the development of the city with respect to water availability. From the estimated figures, the city will only have one MCM extra of water supply by the year of 2000.

In conclusion, the formulation of the problem discussed in this paper gives the results or alternative on how to meet the water demands from the available sources at the least cost. The linear programming method seems to be a powerful tool in handling water supply problems and at the same time guiding the decision makers in planning a short and long terms water demand. These two algarithms from linear programming discussed here can be a useful tool to examine the water supply in a total context.

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