MODELLING AND CONTROL OF A BALANCING ROBOT USING DIGITAL STATE SPACE APPROACH

HERDAWATIE BINTI ABDUL KADIR

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Faculty of Electrical Engineering
Universiti Teknologi Malaysia

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To my dearest mother, father and family for their encouragement and blessing
To my beloved classmate for their support and caring … … …

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ABSTRACT

This thesis is concerned with the problems of modelling a complete mathematical model of a balancing robot and control the system using digital state space approach to pilots the motors so as to keep the system in equilibrium. The research work is undertaken in the following development stages. In order to analyze and design the control system the dynamic of model of the system was first established in discrete-time. Then the difference equation approach is used to obtain the dynamic equations of an actual experimental test-rig. The dynamic of the DC motors as well as chassis and wheels of balancing robot are incorporated in the overall dynamic model, which is in the form of continuous state-space. Two type of controllers, namely pole placement controller and LQR controller are considered in this work. The performance and reliability of both controller will be determined by performing extensive simulation using MATLAB/SIMULINK as the platform.

ABSTRAK

Tesis ini berkenaan dengan masalah untuk memformulasikan model lengkap dinamik robot dan juga kawalan sistem menggunakan ruang digital yang boleh mengawal motor bagi mengekalkan keseimbangan system. Kajian ini telah dibahagikan kepada beberapa peringkat. Bagi menganalisa dan mereka kawalan sistem model dinamik system diambil kira dalam bentuk digital. Selepas itu kaedah persamaan perbezaan digunakan untuk memperolehi dinamik bagi platfom ujian experimen yang sebenar. Dinamik bagi motor DC juga kasis dan tayar bagi robot seimbang itu telah diaplikasikan didalam model dinamik ,dimana ia telah diubah kepada keadaan berterusan dan diubah kepada ruang digital. Dua jenis pengawal yang digunakan adalah pengawal penentuan kutub dan LQR. Prestasi dan kepercayaan bagi kedua-dua pengawal akan ditentukan melalui simulasi secara extensive menggunakan MATLAB/SIMULINK sebagai platform.

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LIST OF SYMBOLS

x	-	Displacement
$\dot{x}$	-	Displacement velocity
ϕ	-	Angle
$\dot{\phi}$	-	Angular velocity
θ	-	Parameters position
ω	-	Velocity
V_a	-	Applied torque
H_∞	-	H-infinity
k_m	-	Torque constant
τ_m	-	Motor torque
I	-	Current that flow through the armature circuit
k_e	-	Back emf constant
R	-	Lumped armature winding resistance
L	-	Self inductance of the armature winding
J_a	-	Moment inertia of the armature
k_f	-	Frictional constant
R	-	Radius of the wheels
Θ	-	Rotation angle of the wheels which is the same as the rotation angle of the armature
P_R	-	Reaction force between the wheel and the chassis of y-component of the force
H_R	-	Reaction force between the wheel and the chassis of x-component
H_{fR}	-	Friction force between ground and the wheel
M_w	-	Mass of the wheel

J_W	-	Moment inertia of the wheels
$H_{fR}r$	-	Conversion of translational force into rotational force
G	-	Gravity 9.81 m/s^2
J_P	-	Moment inertia of the robot chassis
l	-	Distance between the centre of the wheel and the robot’s centre gravity
M_P	-	Mass of the robot’s chassis

LIST OF ABBREVIATIONS

DC	Direct Current
DOF	Degree of Freedom
ZOH	Zero Order Hold
T	Sampling time
LQR	Linear Quadratic Regulator
VSC	Variable Structure Control

CHAPTER 1

INTRODUCTION

1.1 Overview

Balancing robots are characterised by the ability to balance on its two wheels and spin on the spot similar to inverted pendulum. The inverted pendulum problem is common in the field of control engineering thus the uniqueness and wide application of technology derived from this unstable system has drawn interest from many researches and robotics enthusiasts around the world. In recent years, researchers have applied the idea of a mobile inverted pendulum model to various problems like designing walking gaits for humanoid robots, robotic wheelchairs and personal transport systems

This nonlinear control problem is surprisingly difficult to solve in a methodological approach due to two degrees of freedom, i.e, the balancing robot position and chassis angle using only one control input force. A practical problem with regard to control the balancing robot is similar to the concept designing a controller to swing the inverted pendulum up from a pendant position, achieve inverted stabilization, and simultaneously position.

In this thesis the balancing robots are characterized by the ability to balance on its two wheels and spin on the spot. The robot is composed of a chassis based on a stack of 130mm x 130mm Perspex plates carrying a Faulhaber DC motor, the Mark 4 Eyebot controller running on Robios version 5.2, a HOTEC GY-130 digital rate gyroscope, a SEIKA N3 digital inclinometer as described in (Thomas, 2002). The wheels of balancing robot are directly coupled to the output of the dc motor.

The balancing robot chasis is constructed from a single sheet of aluminium, drilled with holes for the easy mounting of motors, controller, sensors and battery pack. A pair of Faulhaber DC motors drive the robot's wheels. Each motor has a gear reduction of 54.2:1 and a torque constant of 6.9203×10^{-4} kg^{-m}/A. These motors have encapsulated encoders, and can be used to measure displacement and velocity of the robot. The robot is controlled by an EyeBot. A Mark 4 Eyebot controller running on Robios version 5.2 is used as the ‘brain’ of the balancing robot system.

The controller consists of a powerful 32-Bit microcontroller running at 33MHz, there is 512k ROM and 2048k RAM on board. The gyroscope modifies a servo control signal by an amount proportional to its measure of angular velocity. Instead of using the gyro to control a servo, we read back the modified servo signal to obtain a measurement of angular velocity. An estimate of angular displacement is obtained by integrating the velocity signal over time. The Inclinometer outputs an analogue signal, proportional to the angular displacement of the sensor (Braunl, 2002).

The balancing robot with two degree of freedom (DOF), is able to move along x, y axes describe by displacement, x and displacement velocity, $\dot{x}$ and chassis angle corresponding the angle, ϕ and angular velocity, $\dot{\phi}$. These four state space variable fully describe the dynamics of the 2 DOF system.

The balancing robot balance the load with its wheels while dragging the weight around on a pivot in a regular differential drive robot. This thesis will delve into the suitability and performance analysis of Pole Placement and LQR controllers in balancing the balancing robot in discrete-time environment.

1.2 Objective

The objectives of this research are as follows:

1. To formulate the complete mathematical dynamic model of the Balancing Robot using differential equation method.

2. To establish the state space model of the Balancing Robot using Digital State Space Approach.

3. To show mathematically that the Balancing Robot system is controllable and observable in discrete-time.

4. To design digital state feedback regulators for the Balancing Robot using pole placement approach and Optimal Controller (LQR).

5. To simulate the Balancing Robot continuous system and hybrid system using MATLAB-SIMULINK.

6. To demonstrate that the digital state space approach is as accurate as the continuous state space approach.

1.3 Scope of Project

The work undertaken in this project are limited to the following aspects:

1. Balancing Robot as described by Thomas Braunl (2002).

2. Digital State Space Approach as described by Richard J. Vaccaro (1995)

3. State feedback with Pole Placement Approach and Linear Quadratic Regulator.

4. Simulation on MATLAB-SIMULINK.

1.4 Research Methodology

The research work is undertaken in the following nine developmental stages:

1. Formulate the complete mathematical dynamic model using differential equation method.

2. Establish continuous state space mathematical model.

3. Linearization: Nonlinear equations of motion are linearized around the operating point .

4. Choose Sampling Interval.

5. Discretize the linearized continuous state space model to digital state space model.

6. Check the controllability and observability of ZOH Equivalent Models.

7. Design continuous-time and discrete-time state feedback controller using the pole placement method and LQR.

8. Verify the controller design of the balancing robot simulated on MATLAB SIMULINK.

9. Evaluate results

1.5 Literature Review

The research on balancing robot has gained momentum over the last decade. This is due to the nonlinear and inherent unstable dynamics of the system. The balancing problem extensively studied by numerous researchers (Mori, 1976). An understanding of how to control such a system will allow us to easily solve the other related control problems, such as single-link flexible manipulators (Yeung et al., 1990) and stabilization of a rocket booster by its own thrust vector. Below is several research that has been done by researches.

Yangsheng Xu (2004), developed a dynamic model for the single wheel robot and verified it through simulations and experiments. Using the linearization method, a linear state feedback approach to stabilize the robot at any desired lean angle was developed. This feedback provides means for controlling the steering velocity of the robot. Line following controller is developed for tracking any desired straight line while keeping balance. The controller is composed of two parts: the velocity control law and the torque control law. In the velocity control law, the velocity input (steering velocity) is designed for ensuring the continuity of the path curvature. Then, the robot can be stabilized for tracking a lean angle trajectory in which the steering velocity is identical to the desired value.

Henrik Niemann (2003) has derived a linear model of a double inverted pendulum system together with a description of the model uncertainties. For the double inverted pendulum system the trade-off between robust stability and performance is quite limited. There is not much space for reduction of the robustness to increase the performance of the system. The reason is the nonlinearities in the system together with the limitations/saturations in the system. The limitations in the system are e.g. maximal power to the motor (maximal acceleration of the cart), maximal length of the track for mention the two most important limitations.

Rich Chi Oii (2003) discusses the processes developed and considerations involved in balancing a two-wheeled autonomous robot based on the inverted pendulum model. The experimental examines the suitability and evaluates the performance of a Linear Quadratic Regulator (LQR) and a Pole-placement controller in balancing the system. The LQR controller uses several weighting matrix to obtain the appropriate control force to be applied to the system while the Pole placement requires the poles of the system to be placed to guarantee stability.

Felix Grasser (2002) has built a prototype of a revolutionary two-wheeled vehicle (JOE). The goal was to build a vehicle that could balance its driver on two coaxial wheels – a mobile, inverted pendulum. In order to reduce cost as well as danger for the test pilots it was decided on building a scaled down prototype carrying a weight instead of a driver. The control system used to guarantee stability of the system is based on two state space controllers, interfaced via a decoupling unit to the two DC motors driving the wheels. The performance of the system is shown that its have ability to reject force and angular disturbances as well as its capability of tracking a pilot's driving inputs. A control system varying the pole placement in real time depending on the states and inputs of the system has the potential to further increase JOE's performance. The implementation of these controllers can be seen in papers published by Nakajima et al. (1997), Shiroma et al. (1996), Takahashi et al. (2001) and Grasser et al (2002).

Chinichian (1990) design and analyze a controller for balancing one pendulum with two degrees of freedom, "spatial inverted pendulum". The pendulum, with two degrees of freedom, has a three dimensional motion, and it will be more analogous to the design of a controller for attitude control during launching a rocket. A full state-variable feedback controller design for a state-space linear model of a three dimensional inverted cart/pendulum system is presented. This design was based on pole-placement technique. Alternative solutions to the simple pole-placement technique were also proposed to exploit non-uniqueness of the feed-back gains for a certain closed-loop pole locations and the closed-loop system response was simulated on a digital computer.

Shiroma et al. (1996) presented the 'Cooperative Behaviour of a Wheeled Inverted Pendulum for Object Transportation' by showing the interaction of forces between objects and the robot by taking into account the stability effects due to these forces. This research highlights the possibility of cooperative transportation between two similar robots and between a robot and a human.

The rapid increase of the aged population in countries like Japan has prompted researchers to develop robotic wheelchairs to assist the infirm to move around (Takahashi et al. 2000). The control system for an inverted pendulum is applied when the wheelchair manoeuvres a small step or road curbs.

On a higher level, Sugihara et al. (2002) modelled the walking motion of a human as an inverted pendulum in designing a real time motion generation method of a humanoid robot that controls the centre of gravity by indirect manipulation of the Zero Moment Point (ZMP). The real time response of the method provides humanoid robots with high mobility.

1.6 Layout of Thesis

This section outlines the structure of the thesis.

Chapter 2 deals with the mathematical modelling of the balancing robot. The formulation of the integrated dynamic model of this robot is presented in detail. First, the state space representations of the chassis and wheel dynamics comprising of DC motors are formulated. In addition, the assumptions and limitations that been added to the model will be described.

Chapter 3 discusses control algorithm design for controlling balancing robot. Analysis regarding on performance of designed controller will be conducted.

Chapter 4 explains the discretization method that can be used in discretizing an analog plant.

Chapter 5 discusses the simulation results. The performance of the continous-time and discrete-time of the state feedback and LQR controller is evaluated by simulation study using Matlab/Simulink.

Chapter 6 concludes the topics and suggests recommendation for future works.

REFERENCES

1. Mori,S., Nishihara,H., and Furuta,K. (1976).Control of unstable mechanical system control of pendulum. *Int. J. Contr.*, vol. 23: pp. 673–692.

2. Chen,C.S., and Chen,W.L.(1998).Robust Adaptive Sliding-Mode Control Using Fuzzy Modeling for an Inverted-Pendulum System. *IEEE Transactions On Industrial Electronics*. vol. 45. no. 2.

3. Shiroma,N, Matsumoto, O., Kajita, S. & Tani, K, (1996). Cooperative Behaviour of a Wheeled Inverted Pendulum for Object Transportation. *Proceedings of the 1996 IEEE/RSJ International Conference on Intelligent Robots and Systems '96, IROS 96*. volume: 2:pp 396 -401.

4. Yeung,K. S. and Chen,Y. P(1990).Sliding mode controller design of a single link flexible manipulator under gravity. *Int. J. Contr.*, vol. 52, no. 1:pp. 101–117.

5. Braunl,T.,Sutherland, and Alistair (2002). *An Experimental Platform for Researching Robot Balance*. Department of Electrical & Electronic Engineering. University of Western Australia.

6. Zdzislaw Kopwalczuk (1991).On Discretization of Continuous-Time State Space Models:A Stable –Normal Approach.*IEEE Transaction on circuit and system,vol 38,no12.*

7. Henrik Niemann(2003). Analysis and Design of Controllers for a Double Inverted Pendulum. *IEEE Proceedings of the American Control Conference Denver, Colorado:* pp 2403-2408

8. Chinichian, M. (1990). State space controller design for a spatial inverted Cart/pendulum system. *IEEE proceedings:* pp633-636

9. Grasser, Felix, Alonso D'Arrigo, Silvio Colombi & Alfred C. Rufer.(2002) JOE: A Mobile, Inverted Pendulum. *IEEE Transactions on Industrial Electronics*, Vol 49.

10. Shiroma,N, Matsumoto, O., Kajita, S. & Tani, K, (1996). Cooperative Behaviour of aWheeled Inverted Pendulum for Object Transportation. *Proceedings of the 1996 ,IEEE/RSJ International Conference on Intelligent Robots and Systems '96, IROS 96*, volume: 2 , 4-8 nov. 1996 pp: 396 -401 vol.

11. Takahashi, Y., Ishikawa, N. & Hagiwara, T., (2001).Inverse pendulum controlled two wheel drive system. *Proceedings of the 40th SICE Annual Conference, International ,Session Papers SICE 2001*, pp. 112 -115.

12. Robert L. Woods (1999).*Modeling and Simulation of Dynamic Systems*.New Jersey:Prentice Hall.

13. Ernest O. Doebelin.(1972). *System Dynamics: Modeling and Response*. USA: Charles E. Merrill Publishing Company.

14. B. Gold and C. M. Rader (1969). *Digital Processing of Signals.* New York: McGraw-Hill.

15. Beauchamp, K. G. (1973). *Signal Processing Using Analog and Digital Techniques.* London: G. Allen and Unwin.

16. Temes,G. C. and Mitra,S. K. (1973). *Modern Filter Theory and Design.* New York: Wiley.

17. Glover,K. (1984). All optimal Hankel-norm approximations of linear multivariable systems and their L,-error bounds. *Int.* J. *Contr.,* vol. 9:pp. 1115-1193, 1984.

18. McClamroch N. H.(1986). Singular systems for differential equations as dynamic models for constrained robot systems. *Proc. IEEE Int.Conf. Robotics and Automation,* pp. 21-28, Apr.

19. Astrom,K. J., Hagander P., and Sternby ,J.(1984).Zeros of sampled systems. *Autornatica,* vol. 20:pp. 31-38.

20. Rich Chi Oii.(2003).*Balancing a two wheeled Autonomous Robot.*The University of Western Australia:Degree Thesis

21. Richard J.Vaccaroo (1995). *Digital Control:A State Space Approach,*McGraw-Hill,Inc,Singapore.