# STATE-DEPENDENT BOUNDARY LAYER METHOD FOR CHATTERING ELIMINATION ON SLIDING MODE CONTROL

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In (or with) the name of Allah, who is Beneficent and Merciful.

"….and say O' my Lord increase me in knowledge…"

The Subline Quran (20:114)

To my beloved mother Rohana Binti Abdul Majid, my family and my future wife.

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# ABSTRACT

The purpose of this study is to reduce/eliminate the chattering effect inside boundary layer by using Sliding Mode Control (SMC) technique while maintaining the robustness of the system by improving the existance SMC input control. In this project, state-dependent boundary layer control technique is chosen since this method have an unique characteristic in order to manipulate the state variables value where the boundary layer width can be adjusted by following state variables current value. Then, this state-dependent boundary layer also will be implimented to the inverted pendulum dynamic system where the chattering elimination inside the boundary layer, error convergence inside the boundary layer and robustness of the system will be investigated by using Matlab-simulink. Besides that, the mathemathical equations of inverted pendulum dynamic system will be constructed by using Langrange's equation which this system is forth order class system. For instance, the comparison result between basic SMC and improved SMC also will be investigated in order to observe the developed controller performance.

# ABSTRAK

Tujuan kajian pembelajaran ini adalah bertujuan untuk mengurangkan/meminimakan kesan celoteh di dalam lapisan sempadan dengan menggunakan teknik Pengawal Ragam Gelincir (SMC) sambil mengekalkan daya ketahanan sistem dengan menambahbaikkan SMC yang telah sedia ada. Di dalam projek ini, teknik keadaan-tanggungan lapisan sempadan di pilih kerana kaedah ini mempunyai ciri yang unik untuk memanipulasikan nilai keadaan berubah di mana ketebalan lapisan sempadan boleh di laraskan dengan berpandukan nilai keadaan berubah semasa. Selain itu juga, keadaan-tanggungan lapisan sempadan juga akan diaplikasikan kepada pengubah buah bandul jam sistem dinamik di mana penghapusan celoteh di dalam lapisan sempadan, ralat pemusatan di dalam lapisan sempadan dan ketahanan sistem yang dikawal akan di selidik dengan menggunakan perisian Matlab simulink. Persamaan matematik bagi pengubah buah bandul jam ini akan di bentuk dengan menggunakan persamaan Langrange dengan sistem ini adalah sistem kelas keempat. Di samping itu juga, perbandingan di antara SMC yang asas dan SMC yang telah ditambahbaikkan juga akan turut dikaji untuk mengenalpasti kemampuan pengawal yang telah dibina.

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# LIST OF ABBREVIATIONS

| | | |
|---|---|---|
| ANN | - | Artificial Neural Network |
| FL | - | Fuzzy Logic |
| MIMO | - | Multi input multi ouput |
| PE | - | Processing elements |
| PID | - | Proportional-integral-derivative |
| SISO | - | Single input single output |
| SMC | - | Sliding Mode Control |
| VSCS | - | Variable Structure Control Systems |

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# CHAPTER 1

# INTRODUCTION

In the formulation of any control problem there will typically be discrepancies between the actual plant and the mathematical model developed for control design. This mismatch may be due to unmodelled dynamics, variation in system parameters or the approximation of complex plant behaviour by a straightforward model. The controller's output must has the ability to produce the required performance levels in practice despite such plant/model mismatches. This has led to an intense interest in the development of so-called robust control methods which seek to solve this problem.

The theory of robust control began in late 1970's and early 1980's and soon developed a number techniques such as PID (proportional-integral-derivative) controller, Neural Network (ANN), Fuzzy Logic (FL), Sliding Mode Control (SMC) and so on [5]. These robust control can be catagorized into two parts; modern control theory and conventional control theory of robust control. As example, Neural Network, Fuzzy Logic and SMC are coming from modern theory of robust control family while vice versa for PID controller. These controllers have their own advantages and disadvantages characteristics. In this project, SMC method is proposed as the robust controller design.

SMC theory was found and advanced in the former Soviet Union (Russia) as a variable structure control systems (VSCS). In the middle of 1970s, a book by Itkis (1976) [7] and a survey paper by Utkin (1977) [11] were published in English. Since that moment, a lot of research have been done and numbers of papers were published based on this SMC in order to maximize the capability of this controller. VSCS concepts have subsequently been utilised in the design robust regulators, model-reference systems, adaptive schemes, tracking systems, state observers and fault detection schemes.

In SMC, VSCS are designed to drive and then constrain the system state lie within a neighbourhood switching function. SMC aims to force a system on a sliding mode surface (functional) in the state space plane. This corresponds ideally to an exponentional, stable reduction in time of the controlled variable error with a fixed time constant, irrespective of system parameters detuning with the latter within a given domain and without important overshooting. It is an intuitive robust control strategy from the family of VSCS [5]. There are two main advantages to this approach [5]. Firstly, the dynamic behaviour of the system may be tailored by the particular choice of the Switching Function. Secondly, the closed-loop response becomes totally insensitive to a particular class of uncertainty. The latter invariance property clearly makes the methodology an appropriate candidate for robust control. In addition, the ability to specify performance directly makes SMC attractive from the design perspective. Although SMC are enriched with these advantagous, chattering effect is a still major drawback from this controller.

## 1.1 Project Background

Chattering is the major effect when SMC is applied as controller in a system. Incorporating infinite switching such that the system can be driven from trajectory point towards to the stable sliding surface is the factor which contribute to this really unwanted phenomena. Chattering effect produces loud noise which can contribute to the noise pollution and high wear of moving mechanical parts that can increase the financial cost to replace the wear parts especially in indutrial sector [3].

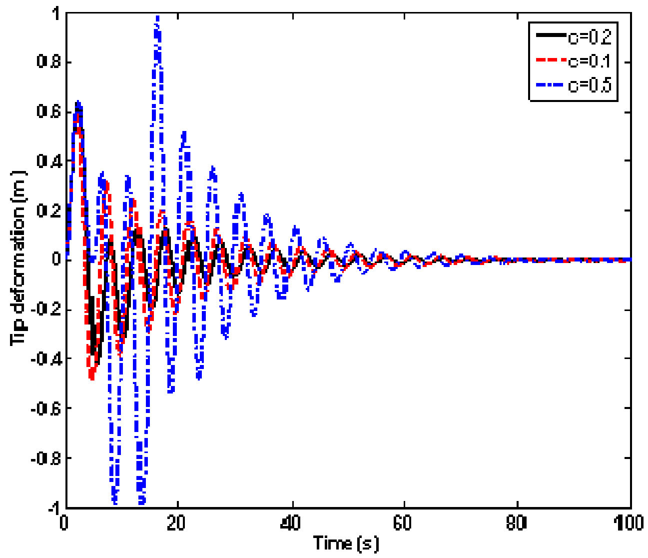


**Figure 1:** SMC with chattering effect [12]

A lot of modifications had been made to overcome this chattering effect such as using boundary layer around the sliding mode surface. This one of the best chattering elimination techniques which will convert all the chattering value within the boundary layer to sliding surface line. The chattering effect totally can be reduced or eliminated but it also reduces the robustness performance of the system [2][3]. Finally, to overcome both effects, the idea to use the boundary layer width in multiple functions as one of the solutions is introduced [1][2][3].

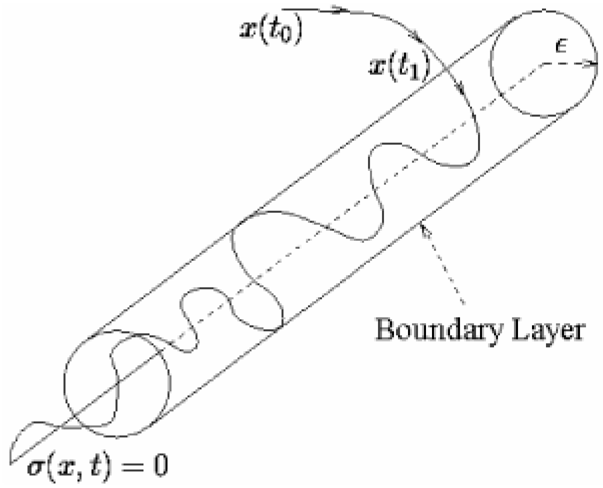


**Figure 2**: Boundary layer enclosing sliding surface [13]

Finally, based on this idea, state-dependent boundary layer method is proposed in this project since state variables can manipulate the boundary layer width in order to eliminate the chattering effect while maintaining the robustness of the system. This concept is proposed since the state variables moving from trajectory point to the origin. It means that the value of the state variable will also be reduced. This state variables value can be used as the guide to determine the width of the boundary layer such as using multiple functions of boundary layer's width when the state variables located either inside or outside the boundary layer region.

## 1.2 Objectives

The purpose of this project is to reduce or eliminate the chattering effect inside the boundary layer in SMC while maintaining system robustness. An objective has been set to realize this purpose below:

(i) To improve the existing state-dependent boundary layer method to reduce the chattering effect inside the SMC up to forth order system with multiple types of disturbances while maintaning the robustness of the system.

## 1.3 Project Scopes

This project will focus on several scopes to achieve the objective as highlighted above. The scopes of the project are:

(i) Develop a new SMC control algorithm to let the control system free from chattering effect while robustness and stability of the system are maintained.

(ii) Research and investigation of the previous methods are implemented in order to gain the main ideas and concepts to eliminate/minimize the chattering effect and stability and robustness of the system are guaranteed.

(iii) Simulation will be used to verify the proposed control method performance.

Finally, based on these project background, objective and project scopes, a new SMC control algorithn is investigated in order to fullfill all the requirements highlighted before.