The Parallel AGE Variances Method for Temperature Prediction on Laser Glass Interaction

Lizawati Mi Yusuf, Norma Alias, Mohammed Shariff Bashir Ghouse,
Siti Mariyam Shamsuddin and Mohd Shahizan Othman
Faculty of Computer Science and Information Systems,
Universiti Teknologi Malaysia, 81310 Skudai, Johor, Malaysia.
lizawati@utm.my, norma@ibnusina.utm.my, mshariff5@live.utm.my,
{mariyam, shahizan}@utm.my

***Abstract* - This paper describes the formula of three-dimensional parabolic equations for cylindrical coordinate glass that is used for mathematical simulation in simulating the temperature behavior of the laser glass cutting. There are three methods have been used for the simulation purposes which is the Alternating Group Explicit (AGE) which compromises two variances that is Brian and Douglas variant, and Gauss Seidel Red Black method. The simulation for these three methods is conducted in a parallel computing environment in order to speed up the calculation process and to achieve an accurate and convergence results. From the simulation, the results will be compared by conducting parallel performance measurement, which include execution time, speedup, efficiency, effectiveness and temporal performance.**

***Keywords* - Alternating Group Explicit Method (AGE); Gauss Seidel Red Black; Partial Differential Equation (PDE); Parallel Computing**

I. INTRODUCTION

The traditional method used on laser glass cutting use mechanical scribe and break processes. The diamond tip tool is used to create a mark or a scratching zone that will eventually break the glass [1]. This technique will produce fragmentation, micro cracks on the surface of the glass resulting an uneven glass structure. Then, came the laser technology to solve the problem arises in the traditional method. The laser technique serves a higher quality cutting and high precision compared to the mechanical scribe method [1, 2]. However, the laser technique is expensive and requires security aspects and expertise [1, 2, 3]. Hence, the mathematical simulation is needed to conduct the simulation of the laser glass technique.

To conduct mathematical simulation, a mathematical model is needed to represent the actual problem of the laser technique. In this research, the mathematical model that is used will be the partial difference equation (PDE). PDE is chosen because it can support high complexity, infinite dimensional and process that is difficult to estimate [4]. The PDE will need to undergo discretization to simplify the equation for numerical simulation. However, the discretization technique requires too many numerical analysis and iterations that will consume a lot of time particular in sequential computing platform [5]. Computer science field has advanced into a greater stage to help engineers and scientist to solve the problem by introducing a new platform using high-speed computing machine [6]. Thus, the parallel computing which is a high-speed computing machine is able to conduct mathematical simulation to simulate nanoscale temperature behavior on laser glass interaction.

The main objective of this paper is to compare the parallel performance of Alternating Group Explicit (AGE) method by using Brian and Douglass variances with Gauss Seidel Red Black (GSRB) in solving laser glass cutting problems in parallel environment.

II. PROBLEM FORMULATION

In solving large sparse problems, numerical methods are ideally used to solve partial difference equation (PDE), which is based on domain decomposition. Numerical methods can solve multidimensional problems such as the laser glass interaction problem which requires accurate results and highly convergent. Here is the three-dimensional parabolic equation for cylindrical coordinate glass that is used to formulate the laser glass interaction problem:

$$\frac{1}{a}\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial r^2} + \frac{1}{r}\frac{\partial U}{\partial r} + \frac{1}{r^2}\frac{\partial^2 U}{\partial \theta^2} + \frac{\partial^2 U}{\partial z^2} + \frac{q}{k} \quad (1)$$

with the initial condition:

$$T(r,\theta,z,t)|_{r=0} = T_0 \quad (2)$$

with T_0 is the initial temperature, where the temperature of the glass cylinder before laser process and boundary conditions:

$$T(r,\theta,z,t)|_{s_1=0} = T_1 \text{ and } \quad (3)$$
$$T(r,\theta,z,t)|_{s_2=0} = T_2$$

where,

T_1, T_2	:	Temperature (K) of inner and outer surface of glass cylinder
s_1, s_2	:	Area of inner (m^2) and outer surface (m^2) of glass cylinder
r, θ, z	:	Radius (m), angle (^{o}) and length (m) of

		glass cylinder
q	:	Rate of heat flow (W/m^2)
a	:	Density (Kg/m^3) and heat capacity (J)
K	:	Thermal conductivity (W)
t	:	Time (s)

The discretization of (1) is done by using Finite Difference Method (FDM). From the equation obtained after discretization, the AGE BRIAN method, AGE Douglas method and GSRB is used for simulation purposes.

A. AGE Brian Method

The method is based on linear interpolation that uses Brian variant and acceleration parameter, *r* with fractional concepts. The formula for BRIAN method for the three-dimensional cylindrical problems is as follows [7],

$$(G_1 + rI)u_{[xy]}^{\left(p+\frac{1}{7}\right)} = (rI - G_2 - G_3 - G_4 - G_5 - G_6)u_{[xy]}^{(k)} + f, \tag{4}$$

$$(G_2 + rI)u_{[xy]}^{(p+\frac{2}{7})} = ru_{[xy]}^{\left(p+\frac{1}{7}\right)} + G_2 u_{[xy]}^{p}$$

$$(G_3 + rI)u_{[xy]}^{(p+\frac{3}{7})} = ru_{[xy]}^{\left(p+\frac{2}{7}\right)} + G_3 u_{[xy]}^{p}$$

$$(G_4 + rI)u_{[xy]}^{(p+\frac{4}{7})} = ru_{[xy]}^{\left(p+\frac{3}{7}\right)} + G_4 u_{[xy]}^{p}$$

$$(G_5 + rI)u_{[xy]}^{(p+\frac{5}{7})} = ru_{[xy]}^{\left(p+\frac{4}{7}\right)} + G_5 u_{[xy]}^{p}$$

$$(G_6 + rI)u_{[xy]}^{(p+\frac{6}{7})} = ru_{[xy]}^{\left(p+\frac{5}{7}\right)} + G_6 u_{[xy]}^{p}.$$

$$u_{[xy]}^{(k+1)} = u_{[xy]}^{(p)} + 2(u_{[xy]}^{\left(p+\frac{6}{7}\right)} - u_{[xy]}^{(p)}$$

Based from the equations obtained from (4), it has been simplify further into 2×2 matrix block form and the matrix calculation using AGE Brian method for cylindrical coordinate system is as follows,

$$u_{1[xy]}^{\left(p+\frac{1}{7}\right)} = \bar{C}_{1,i}^{-1}(D_{1,i}u_{1[xy]}^{(p)} + E_{1,i}u_{2[xy]}^{(p)} + F_{1,i}u_{1[xy]}^{[N]} + H_{1,i}u_{2[xy]}^{[N]} + g_{1[xy]}) \tag{5}$$

$$u_{k[xy]}^{\left(p+\frac{1}{7}\right)} = \bar{C}_{2,i}^{-1}\left(\bar{E}_{1,i}u_{k-1[xy]}^{(p)} + E_{1,i}u_{k+1[xy]}^{(p)}\right) + D_{2,i}u_{k[xy]}^{(p)} + \bar{H}_{1,i}\left(u_{k-1[xy]}^{[N]} + H_{1,i}u_{k+1[xy]}^{[N]}\right) + F_{1,i}u_{k[xy]}^{[N]} + g_{k[xy]} \tag{6}$$

$$u_{k[xy]}^{\left(p+\frac{1}{7}\right)} = \bar{C}_{1,i}^{-1}(\bar{E}_{1,i}u_{k-1[xy]}^{(p)} + E_{1,i}u_{k+1[xy]}^{(p)} + D_{1,i}u_{k[xy]}^{(p)} + \bar{H}_{1,i}\left(u_{k-1[xy]}^{[N]} + H_{1,i}u_{k+1[xy]}^{[N]}\right) + F_{1,i}u_{k[xy]}^{[N]} + g_{k[xy]} \tag{7}$$

$$u_{m[xy]}^{\left(p+\frac{1}{7}\right)} = \bar{C}_{1,i}^{-1}(\bar{E}_{1,i}u_{m-1[xy]}^{(p)} + D_{1,i}u_{m[xy]}^{(p)} + \bar{H}_{1,i}u_{m-1[xy]}^{[N]} + F_{1,i}u_{m[xy]}^{[N]} + g_{m[xy]} \tag{8}$$

Equations (5) to (8) describe the point calculation for its fraction of the grid from $\frac{1}{7}$, $\frac{2}{7}$, $\frac{3}{7}$ until $\frac{7}{7}$. Thus, on calculation for every$(p + 1)$,

$$u_{[x,y]}^{(p+1)} = u_{[xy]}^{(p)} + 2(u_{[xy]}^{\left(p+\frac{6}{7}\right)} - u_{[xy]}^{(p)}) \tag{9}$$

B. AGE Douglas Method

This method is based on the Douglas-Rachford formula which uses fractional scheme involves splitting a matrix system of linear equations [8]. Matrix A is been split into consistent symmetric and positive definite matrices $G_1, G_2, G_3, G_4, G_5, G_6$ and the calculation of these matrices will be simplified into four equations as follows,

$$u_{1[xy]}^{\left(p+\frac{1}{6}\right)} = \bar{C}_1^{-1}(D_1 u_{1[xy]}^{(p)} + E_1 u_{2[xy]}^{(p)} + F_1 u_{1[xy]}^{[N]} + F_2 u_{2[xy]}^{[N]} + 2g_{1[xy]}) \tag{10}$$

$$u_{k[xy]}^{\left(p+\frac{1}{6}\right)} = \bar{C}_2^{-1}(E_1(u_{k-1[xy]}^{(p)} + u_{k+1[xy]}^{(p)} + D_2 u_{k[xy]}^{(p)} + F_2\left(u_{k-1[xy]}^{[N]} + u_{k+1[xy]}^{[N]}\right) + F_1 u_{k[xy]}^{[N]} + 2g_{k[xy]} \tag{11}$$

$$u_{k[xy]}^{\left(p+\frac{1}{6}\right)} = \bar{C}_1^{-1}\left(E_1\left(u_{k-1[xy]}^{(p)} + u_{k+1[xy]}^{(p)}\right) + D_1 u_{k[xy]}^{(p)} + F_2\left(u_{k-1[xy]}^{[N]} + u_{k+1[xy]}^{[N]}\right) + F_1 u_{k[xy]}^{[N]} + 2g_{k[xy]}\right) \tag{12}$$

$$u_{m[xy]}^{\left(p+\frac{1}{6}\right)} = \bar{C}_1^{-1}\left(E_1\left(u_{m-1[xy]}^{(p)} + D_1 u_{m[xy]}^{(p)}\right) + F_1 u_{m[xy]}^{[N]} + F_2 u_{m-1[xy]}^{[N]} + 2g_{m[xy]}\right) \tag{13}$$

Thus, based from (10) to (13), we can derive the calculation using AGE Douglas as (14).

$$u_{[x,z]}^{(p+1)} = (G_1 + rI)^{-1}[G_1 u_{[xz]}^{(p)} + ru_{[xz]}^{(p+\frac{5}{6})} \tag{14}$$

C. Gauss-Seidel Red Black (GSRB) Method

The GSRB method is been used as the control scheme to solve three dimensional cylindrical glass interactions. GSRB method is based on domain decomposition for each odd sub domain, Ω^{R} and even sub domain, Ω^{H}[9]. GSRB calculation is as follows,

$$u_{i,j,k}^{(p+1)} = \frac{\begin{bmatrix} -a_i u_{i-1,j,k}^{(p)} - h_i u_{i,j-1,k}^{(p)} - du_{i,j,k-1}^{(p)} - \\ b_i u_{i+1,j,k}^{(p)} - gu_{i,j,k+1}^{(p)} + \bar{b}u_{i-1,j,k}^{[N]} + \\ \bar{d}_i u_{i,j,k}^{[N]} + \bar{h}\, u_{i,j-1,k}^{[N]} + \bar{d}u_{i,j,k-1}^{[N]} + \\ \bar{e}u_{i+1,j,k}^{[N]} + \bar{f}u_{i,j+1,k}^{[N]} + \bar{g}u_{i,j,k+1}^{[N]} + \frac{p}{k} \end{bmatrix}}{c_i} \tag{15}$$

where i, j and k are odd or even number depending on the type of subdomain.

III. Methodology

The flowchart in Fig. 1 is used to solve the laser glass interaction problem by using AGE method with Brian and Douglas variances in a parallel environment. This flowchart contains two levels of computers where the top level of the hierarchy is the master and the lower level contains numbers of worker. The master conducts the first task, t_1 that will initiate workers t_i where $i = 1,2,3,\ldots,N$. It is also responsible to process the domain decomposition, data distribution and output from workers. Whilst the workers will conduct data processing, establish communication between neighboring processors and sending outputs back to the masters. This model is more practical and suitable to be implemented in the parallel program.

The initial boundary, domain size and task load are set to the master. The master will receive these information and it will initiate workers to activate them. As the main responsible for the master is domain decomposition, hence the master will start by decomposing data into data blocks. The data decomposition technique used is static data decomposition. Static data decomposition is a technique that sends data following the size of the domain and the task load. To ensure that all task loads are equally distributed to each processor, all processors must receive and process these data simultaneously. To allow this, iteration method is used by every processor will execute the same amount of arithmetic operation with the same quantity of data block distribution. This paper focuses on this decomposition technique to ensure that there are no processors idle during the simulation.

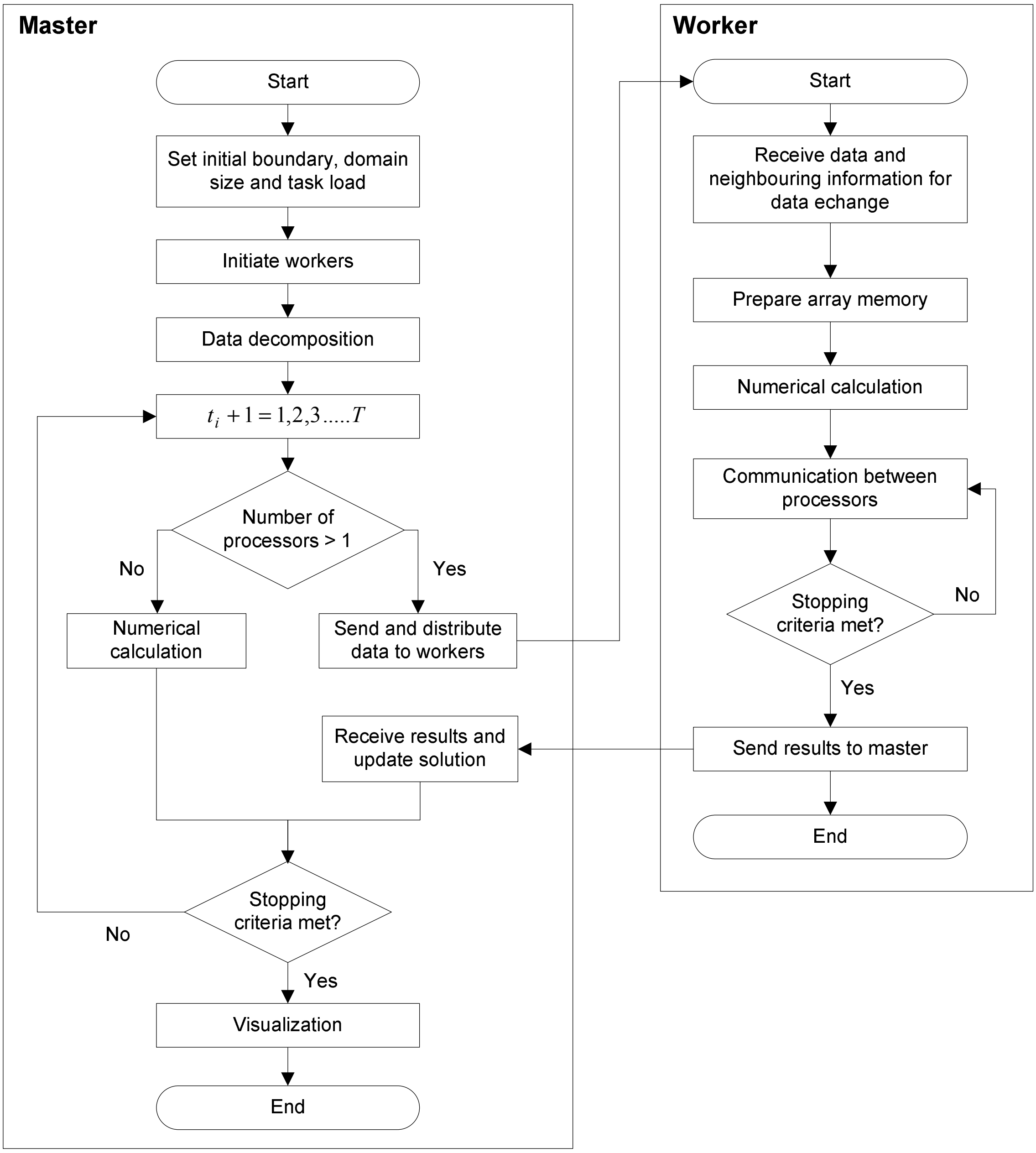

Figure 1. Parallelization flowchart

Then, the master will establish a global type of communication that is distributed to workers and received the data processed by the workers back to the master. Global communication occurs during the sending and receiving an absolute error, Ω for each iteration until the convergence needed is met then the communication will be terminated. At the workers level, each processor establishes a communication between them. The communication that is established is unstructured type of communication where it uses network of communication for any complex graph that requires domain decomposition. Moreover, this communication needs mapping s and algometric process. These processes will ensure the data that are passed do not depend on a particular task to complete and prevent processors to be idle. Again, the global communication is used to determine the convergence criteria. If the criteria is met, then the data that been processed will be send back to the master for data collection to update the solution. The updated solution then will be visualized in a graph as shown in the next section.

IV. Experiment And Results

To measure the parallel performance on the simulation of the laser glass interaction, the following definitions are used,

Speed up: $S_p = T_1/T_p$ (16)

Efficiency: $C_p = S_p/p$ (17)

Effectiveness: $F_p = S_p/T_p$ (18)

Temporal Performance: $L_p = T_p^{-1}$ (19)

where, T_1 and T_p are execution time on one and p processors.

In determining the parallel performance, the parallelization is based on the a number of homogenous PC cluster systems which contains 20 units Intel Pentium processors that are supported by PVM software and C programming. The following graphs visualize the parallel performance in simulating the calculations on laser glass interaction.

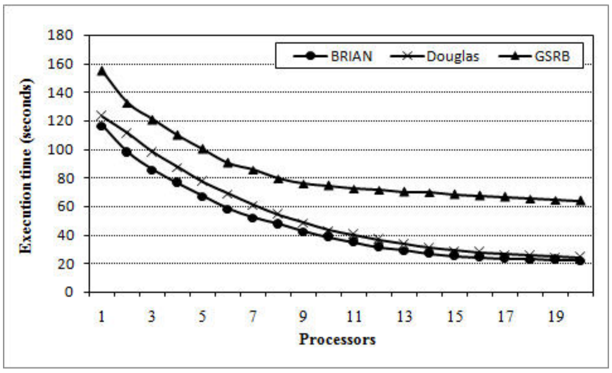

Figure 2. Execution time

Fig. 2 shows the results of parallel performance based on the execution time obtained from Brian, Douglas and GSRB methods. The results clearly show that as the number of processors p increases, the execution time decreases. It proves that when more processors are working together, the execution time will decrease gradually. However, by using Brian method the execution time has the lowest value compared with Douglas and GSRB methods.

Fig. 3 illustrates the performance of speed up using Brian, Douglas and GSRB methods. Based from the visualization obtained, BRIAN method records the highest value of speed up followed by Douglas and GSRB method. Speed up is a measurement on the speed of the processors to simulate the simulation in producing the results that intended. Thus, by using AGE Brian method, a large sparse point system is been used to decrease the execution time by increasing the speed during the calculation.

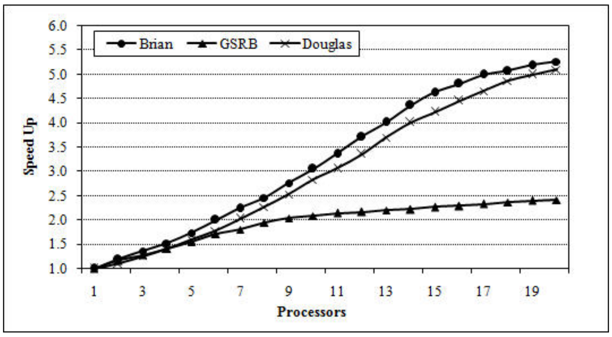

Figure 3. Speed up

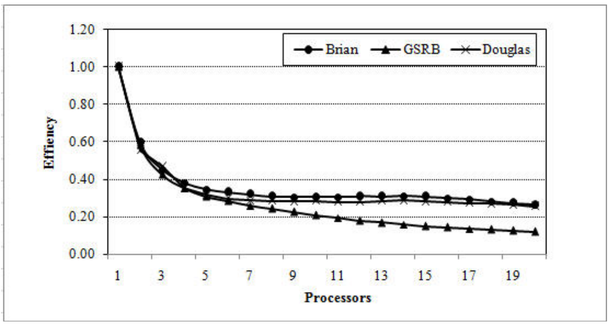

Figure 4. Efficiency

Based from Fig. 4, the efficiency slowly decreases as the number of processors p increases. However, by using Brian method, it produces a higher value of efficiency compared to Douglas and GSRB method. Hence, Brian method is much more efficient followed by Douglas and GSRB.

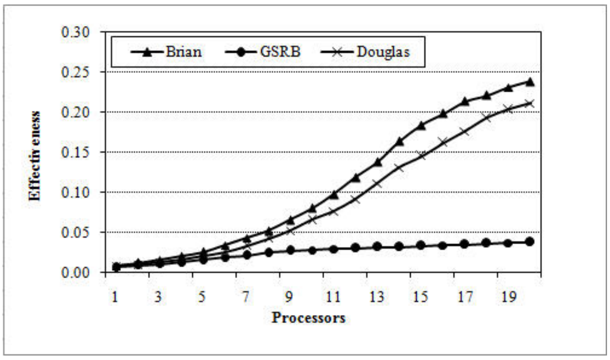

Figure 5. Effectiveness

Fig. 5 shows the effectiveness of the three methods used for the simulation on laser glass interaction. Effectiveness is the question on how the impact of these methods by cost and the accuracy provided by each method. AGE Brian method provide the highest effectiveness value compare to Douglas and GSRB methods. Therefore, AGE Brian method provides more accuracy and reduces the cost of calculation in the simulation process.

Fig. 6 shows the results of the temporal performance. Again, Brian method proves to be the better method. Brian method produces higher temporal performance followed by Douglas and GSRB methods.

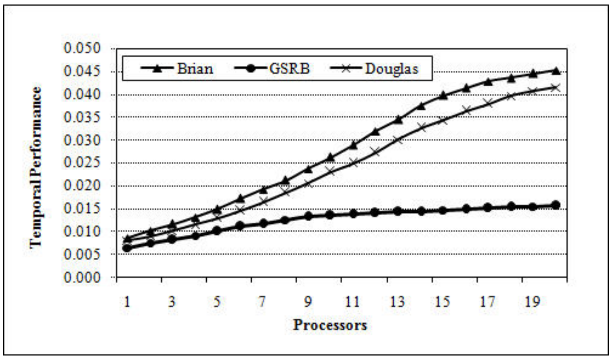

Figure 6. Temporal performance

V. CONCLUSION

To conclude, Brian method proves to be the better method to be used to simulate the laser glass interaction. From the visualization obtained, it supports Brian method to have better parallel performance compared to Douglas and GSRB methods. Douglas method shows that it is better than GSRB method. Thus to conclude, the Brian method is the best method that is more stable and more effective to simulate the temperature behavior on laser glass cutting followed by Douglas and GSRB method.

ACKNOWLEDGMENT

This material is based upon work supported by Fundamental Research Grant Scheme (FRGS), Malaysian Ministry of Higher Education under Vote No. 78598, 77264 and 78372. Any opinions, findings, and conclusions or recommendations expressed in this material are those from the authors and do not necessarily reflect the views of the Malaysian Ministry of Higher Education.

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