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LATTICE BOLTZMANN SIMULATION OF PLUME BEHAVIOR FROM AN ECCENTRIC ANNULUS CYLINDER

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ABSTRACT

In this paper, a double-population thermal lattice Boltzmann was applied to solve two dimensional, incompressible, thermal fluid flow problems. The simplest lattice BGK D2Q4 model was applied to determine the temperature field while D2Q9 for the density and velocity fields. The simulation of natural convection from a concentrically and eccentrically placed inner heated cylinder inside cold outer cylinder with Prandtl number 0.71 and Rayleigh number 5×10^5 were carried out and discussed quantitatively. It was observed that the combination of D2Q4 and D2Q9 was able to reproduce the effect of buoyancy force in the system. We also found that the flow pattern including the boundary layers and vortices with heat transfer mechanisms is significantly influenced by the position of heated cylinder in the enclosure and excellent comparisons with previous studies.

Keywords: Double population, Lattice Boltzmann, distribution function, BGK collision, natural convection.

1. INTRODUCTION

The interaction between the fluid flow behavior and heat transfer mechanism can be seen not only in almost all industrial processes such as metal furnace, power plants, jet engine, etc, but also in everyday situation such as ventilation, air conditioning, hair dryer and so on. In some applications, such as micro-electro mechanical systems (MEMS), the detail understanding of the fluid flow and heat transfer phenomenon is unrelentingly required in order to achieve the most effective method of microchips cooling. On the other hand, lack of understanding in this problem can result in huge cost lost and inefficiency repercussions. For instance, inaccurate prediction of heat transfer and fluid flow can also leads to loss of human lives in the reentry of space shuttle due to the great heat involved in this activity.

Among the main three types of heat transfer mechanism, the convection type has a more pronounced effect on fluid flow. In fact, the convective heat transfer dominates the heat transfer mechanism in most cases when interact with surrounding fluid. This mechanism is very difficult to measure because the effect on fluid flow only appears when dealing at severe conditions such as high Rayleigh number or Grashof number. Furthermore, when the contact fluid is gas, it becomes difficult to visualize this flow configuration experimentally. For these reasons, a numerical approach is solely adopted herein. In this paper, we investigate the phenomenon of natural convection heat transfer from a heated cylinder placed concentrically and eccentrically in a cold outer cylinder. The problem of heated cylinder cylinders in an enclosure plays a significant role in many engineering applications, such as solar collector-receivers, insulation and flooding protection for buried pipes, cooling system in nuclear reactors, etc. Therefore, the purpose of this paper is to give a deeper understanding and qualitative analysis for numerically observed plume behaviour for the case in hand.

The natural convection heat transfer for concentric and eccentric cases between two circular cylinders, the basic and fundamental configuration, the flow and thermal fields have been studied by few researchers. For example, Date (1986) used a modified finite volume SIMPLE procedure to investigate the convergence rate of the numerical scheme. Young et al. (2009) applied the finite difference based lattice Boltzmann scheme to test their method for various conditions. Warrington and Powe (1985) reported some experimental results of natural convective heat transfer between concentrically mounted bodies at low Rayleigh numbers. Hasanuzzaman et al. (2007, 2009) investigated natural convection heat transfer. For eccentric cases, Shu et al. (2001) studied numerically natural convective heat transfer from a horizontal cylinder placed eccentrically inside a square enclosure. The vorticity-stream function formulations are solved using differential quadrature method. Sasaguchi et al. (1998) numerically investigated the effect of the position of cooled cylinder on the cooling process of liquid.

The rest of this paper is consisted of six sections. The physical domain of interest and the boundary conditions are described in the next section. The mathematical formulation and computational methodology are then presented, which are followed by a detailed presentation and discussion of the numerical results. Some concluding remarks are finally drawn based on the foregoing analysis.

2. PROBLEM FORMULATIONS

The physical domain of the problem is represented in Figure 1. The flow is induced by the buoyancy force resulting from the constant heating of the inner cylinder. In present study, the diameter ratio between the inner and outer cylinder is fixed at 4/15. The dynamical similarity depends on two dimensionless parameters: The Prandtl number and the Rayleigh number defined as follow

$$Pr = \frac{\upsilon}{\chi}$$
$$Ra = \frac{g_0 \beta \Delta T L^3}{\upsilon \chi} \qquad (1)$$

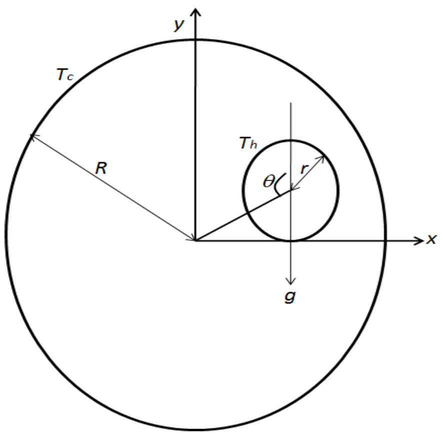


Fig. 1 Configuration of an eccentric annulus

Here, we fix the Pr at 0.71 to represent the circulation of air in the system and Ra at 5 × 10^5. At this value of Rayliegh number, the flow begins to demonstrate the complex structure in the system and often considered in real engineering applications. Through grid dependence study, the uniform square grid with the size of 300^2 is suitable for the current condition. Seven values of δ, defined as the distance of the center of inner cylinder from the center of outer cylinder over the radius of the outer cylinder, are chosen in current simulation to demonstrate the effect of eccentricity on the plume, heat transfer and fluid flow behavior. They are 0, 2/15, 4/15, 6/15, 8/15,10/15 and 12/15. In current research, we only investigate the position of inner heated cylinder located at $\theta = 90^0$. Position of inner cylinder at other values of θ will be carried out in near future.

The air in the gap between the two cylinders is treated as incompressible. Only the density is allowed to vary as a function on temperature, while all other terms are fixed. This is equivalent to Boussinesq approximation and the buoyancy force term can be expressed as follow (Shan 1997)

$$\rho \mathbf{G} = \rho \beta g_0 (T - T_m)\mathbf{j} \qquad (2)$$

3. LATTICE BOLTZMANN NUMERICAL METHOD

Recently, there are a lot of researches applying the lattice Boltzmann method (LBM) to study isothermal and thermal systems (Nor Azwadi and Syahrullail 2009, Hou et al. 1994, Nor Azwadi and Tanahashi 2006, Chen and Doolen 1998). They have demonstrated that the LBM is a powerful numerical tool in solving velocity and temperature distributions. The lattice Boltzmann method originating from the kinetic Boltzmann equation derived by Ludwig Boltzmann (1844-1906) in 1988. It considers a fluid as an ensemble of artificial particles and explores the mesoscopic features of the fluid by using the propagation and collision effects among these particles. LBM discretizes the whole flow region into a number of grids and numerically solves the simplified Boltzmann equation on the regular lattices (Peng et al. 2003). The solution to the lattice Boltzmann equation converged to the Navier-Stokes solution in continuum limit up to second order accuracy in space and time (Qian et al. 1992). This method bridges the gap between the mesoscopic world and the macroscopic phenomena. LBM has emerged as a versatile numerical method for simulating various types of fluid flow problem including turbulent (Jonas et al. 2006), multiphase (Alapati et al. 2008), magnetohydrodynamics (Breyiannis and Valongeorgis 2004), flow in porous media (Guo 2002), microchannel flow (Zhang et al. 2005), etc.

4. THERMAL LATTICE BOLTZMANN MODEL

In present study, the thermal lattice Boltzmann model is based on the work of He et al. (1998), which involves two evolution equations of density distribution function *f* and temperature distribution function *g* and can be written as

$$f_i(\mathbf{x} + \mathbf{c}_i \Delta t, t + \Delta t) - f_i(\mathbf{x}, t) = -\frac{f_i - f_i^{eq}}{\tau_f} + F \qquad (3)$$

$$g_i(\mathbf{x} + \mathbf{c}_i \Delta t, t + \Delta t) - g_i(\mathbf{x}, t) = -\frac{g_i - g_i^{eq}}{\tau_g} \qquad (4)$$

where f_i^{eq} and g_i^{eq} are the density and temperature equilibrium distribution functions, respectively. c_i is the lattice velocity and *i* is the lattice direction, Δt is the time interval, τ_f and τ_g are the relaxation times of the density and temperature distribution functions, respectively. In LBM, the magnitude of c_i is set up so that in each time step Δt, the distribution function propagates in a distance of lattice nodes spacing Δx. This will ensure that the distribution function arrives exactly at the lattice nodes after Δt and collides simultaneously. The macroscopic variables such as the densityρ, fluid velocity u and temperature T can be computed in terms of the particle distribution functions as

$$\rho = \int f d\mathbf{c}$$
$$\rho\mathbf{u} = \int \mathbf{c} f d\mathbf{c}$$
$$T = \int g d\mathbf{c} \qquad (5)$$

To simulate the flow and thermal processes of the fluid in a system, one uses the D2Q9 model (He and Luo 1997) with nine velocities assigned on a two-dimensional square lattice. These velocities include eight moving velocities along the links connecting the lattice nodes of the square lattice and a zero velocity for the rest particle. The rest of the particles is defined by the distribution functions f_0, the particles moving in the orthogonal direction by the function f_i (i = 1,2,3,4) and the particles moving in the diagonal directions by the function f_i (i = 5,6,7,8). The equilibrium distribution functions f_i^{eq} and g_i^{eq} are given as

$$f_i^{eq} = \rho\omega_i \left[1 + 3\mathbf{c}_i \cdot \mathbf{u} + 4.5(\mathbf{c}_i \cdot \mathbf{u})^2 - 1.5\mathbf{u}^2\right] \qquad (6)$$

$$g_i^{eq} = \frac{T}{4}\left[1 + 3\mathbf{c}_i \cdot \mathbf{u}\right] \qquad (7)$$

where ω is the weight function and depends on the direction of the lattice velocity.

Through the multiscaling expansion, the mass and momentum equations can be derived for the D2Q9 model of the evolution equation of the density distribution function. Details derivation can be found in (Cercignani 1988).

It is well known that for the prediction at low and moderate Rayleigh numbers, the viscous heat dissipation and compression work carried out by the pressure are negligible. The temperature field is then passively advected by the fluid flow and obeys a simpler passive-scalar equation

$$\frac{\partial T}{\partial t} + \nabla \cdot (\mathbf{u}T) = \chi \nabla^2 T \qquad (8)$$

The detailed derivation of (8) from the D2Q9 model of the temperature distribution function can be seen in Azwadi and Tanahashi (2008). The time relaxations τf and τg in mesoscopic world can be related to the viscosity and diffusivity in the macroscopic world as follow

$$\tau_f = 3\upsilon + 0.5$$
$$\tau_g = \chi + 0.5 \qquad (9)$$

5. RESULTS AND DISCUSSION

In the previous section, we have discussed a numerical approach to predict the phenomenon of the natural convection between two differentially heated cylinders. In this section, we will demonstrate the predicted results in terms of streamlines, isotherms and contour plots. The obtained results are presented in dimensionless form from the heated to the cooled walls.

For the sake of code validation, we firstly carried out numerical investigation for the natural convection phenomenon between two concentric cylinders at aspect ratio of 2.6 for the conditions shown in Tab. 1. These conditions are set according to the published results by Shi et al. (2006).

Table 1. Values of Pr and Ra for code validation test

Case	Prandtl Nu.	Rayleigh Nu.
1	0.716	2.38×10^3
2	0.717	9.50×10^3
3	0.717	3.20×10^4
4	0.718	6.19×10^4
5	0.718	1.02×10^5

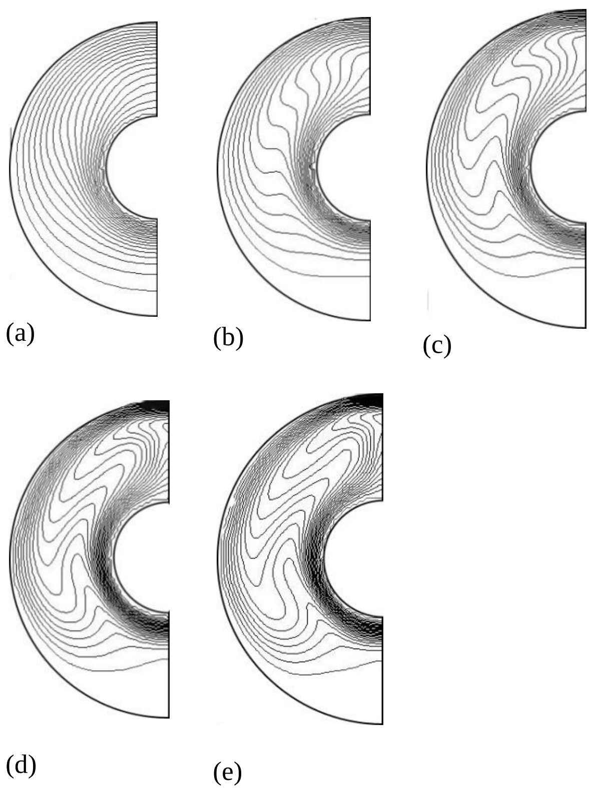
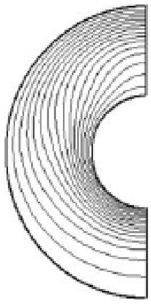
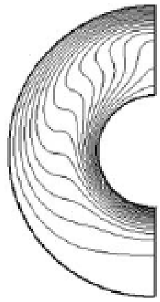


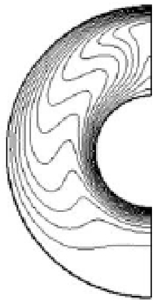
Fig. 2 Streamline plots for (a) Case 1 (b) Case 2 (c) Case 3 (d) Case 4 and (e) Case 5 using LBM



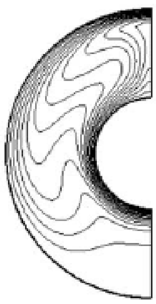
(a)



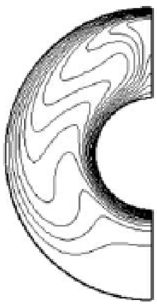
(b)



(c)



(d)



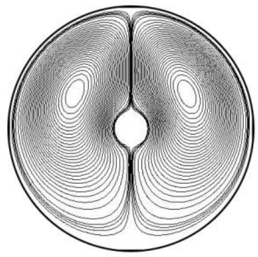
(e)

Fig. 3 Streamline plots for (a) Case 1 (b) Case 2 (c) Case 3 (d) Case 4 and (e) Case 5 from Shi et al. (2006).

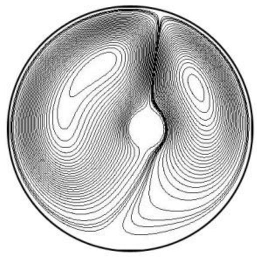
Figure 2 shows the plots of isotherms for every case obtained from LBM simulation while Figure 3 represents the same numerical results obtained Shi et al. (2006). As can be seen from the figures, the results are almost identical and qualitatively good agreement with each other.

Figure 4 shows the plots of streamline for every simulated case in present study. For the case of the heated cylinder placed on the vertical symmetrical axis (δ=0), the vortex splits into two with the same size indicates the equal strength of recirculation on the left and right area of the system. They show that the hot fluid rises near to the hot heated inner cylinder wall until it reaches the most top wall, then moves downwards along the outer cylinder wall under the effect of cooling.

When the heated inner cylinder is shifted to the right (δ>0), the flow dominates at the region of larger gap spacing between the inner and outer cylinders for every case. This phenomenon can be clearly seen when the heated inner cylinder is located δ=4/15. The vortex in the larger gap spacing shows a bigger size than in the smaller area and elongates to the lower area of the systems. This indicates that the flow drags the fresh cold air upwards and the replaced by the hot air heated by the inner cylinder. The center of the vortices mainly located at the upper half of the system demonstrating the most active air flow activity in this area.



δ=0



δ=2/15

δ=4/15

δ=6/15

δ=8/15

δ=10/15

δ=12/15

Fig. 4 Streamlines plots for concentrically and eccentrically heated cylinders.

Figure 5 shows the isotherms plot for every simulation condition. The isotherms line in all figures shows that the steep temperature gradient occurs around the heated cylinder. This can be seen from the density of the lines where it is denser near the inner cylinder.

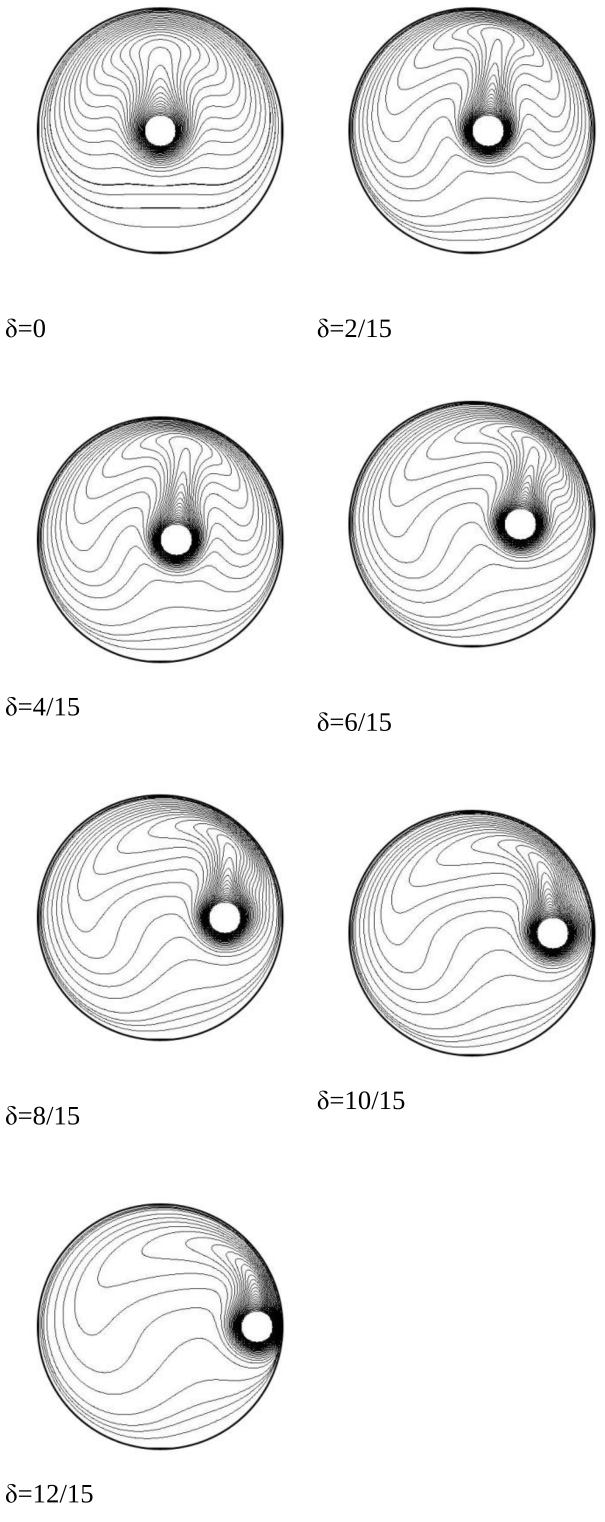


Fig. 5 Isotherms plots for concentrically and eccentrically heated cylinders.

Very thin temperature boundary layer also can be seen from all the figures. The isotherms lines show distortion almost everywhere in the system indicates the main heat transfer mechanism at this condition is by convection.

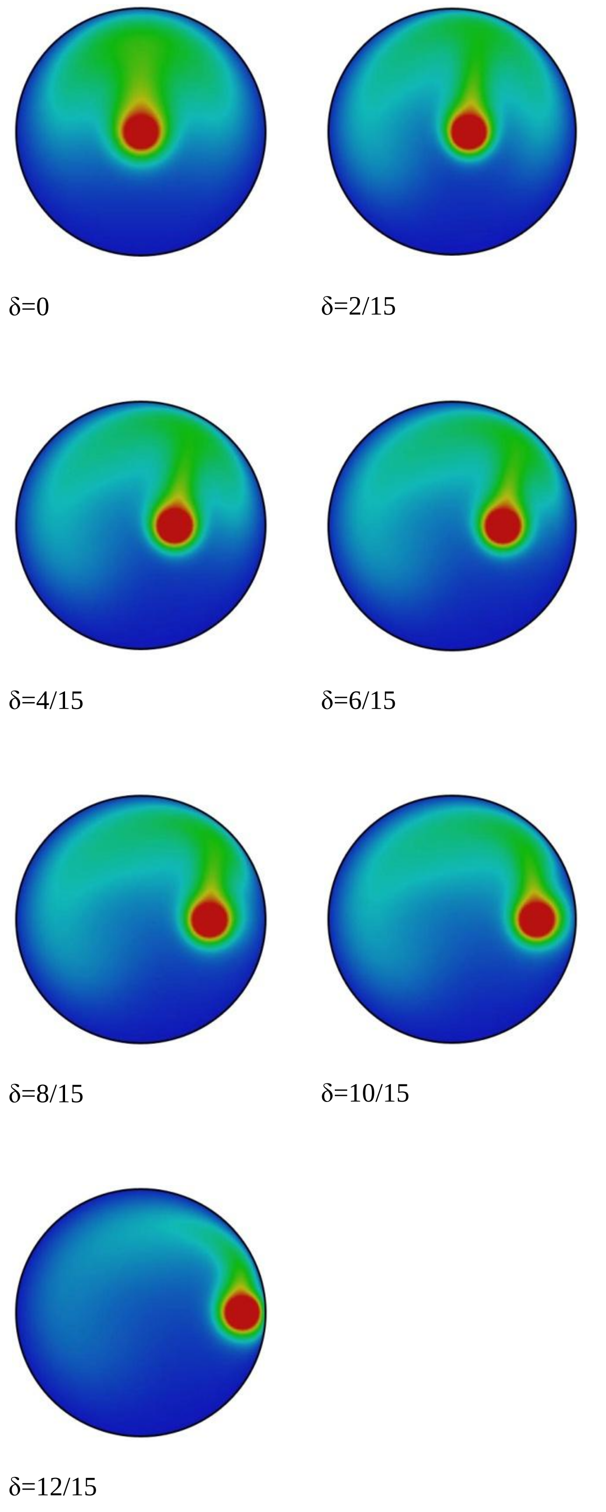


Fig. 6 Isotherms plots for concentrically and eccentrically heated cylinders.

The temperature distribution in the system can be clearly seen from the plots of temperature contour shown in Fig. 6. The heated fluid concentrated at the upper part of the system is understood. This is due to the strong buoyancy force is applied to the system at this value of Rayleigh numbers. All of the predicted phenomena are in good agreement with results presented in previous researches (Nor Azwadi and Osman 2009, Shu and Wu 2002, Kim et al. 2008)

6. CONCLUSION

Two-dimensional thermal lattice Boltzmann formulation were applied and tested on the prediction of natural convection from a heated inner cylinder placed concentrically and eccentrically in a cold outer cylinder. We found that the combination of D2Q4 with D2Q9 correctly predicted the flow features for different eccentricity and gives excellent agreement with the results of previous studies.

From Fig. 4 to Fig. 5, the boundary layers for the velocities and temperature can be observed clearly. As expected, the flow pattern including the boundary layers and vortices with heat transfer mechanisms are significantly influenced by the position of heated cylinder in the enclosure. These demonstrate the lattice Boltzmann numerical scheme the passive-scalar thermal lattice Boltzmann model is a very efficient numerical method to study flow and heat transfer in a differentially heated cubic enclosure.

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