

## A STUDY ON FLEXURAL MODULUS AND DUCTILITY OF LAMINATED CEMENTITIOUS COMPOSITES

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**Abstract:** This paper presents a new approach for flexural modulus of laminated composites derived from the analysis of a flexural-section composed of several ductile and brittle layers of different moduli. The derivation is based on the assumption that the tension and compression zones of the laminated composite are symmetrical with respect to the neutral axis and thus, half of the section is needed to analyze for complete derivation of flexural modulus of the whole section. This composed of two ductile layers of higher moduli providing in both tension and compression zones to maintain the equilibrium conditions during flexure. The formulation shows that the flexural modulus of the entire section of laminated composites obtained by this method is equal to the modulus of ductile layers plus a factor of the difference of modulus of ductile and brittle layers. It is concluded that this factor is different from the usual one which is obtained under uniaxial loadings. Despite the fragile nature of brittle layers of the composite, it has been found that the inclusion of two ductile layers into the laminated cementitious composites makes it fully ductile material owing to the synergetic action between the layers of two components.

**Keywords:** *Laminated cementitious composites, wire mesh, flexural modulus, and ductility.*

### 1.0 Introduction

The history of man-made composite materials dates back to ancient Chinese (Rosato, 1968), Egyptians and Israelites (Rubin, 1969), all of whom embedded straw in bricks to improve their structural capabilities. With the advent of fine diameter elastoplastic mesh reinforcement in numerous cement matrices, the new era of laminated cementitious composite is born. The development of laminated cementitious composite is a significant step in this direction, wherein considerable saving in consumption of cement and steel has been reported as compared to reinforced cement concrete (Hossain and Inoue, 2001).

Laminated composite materials are composed of two or more materials joined together forming a new medium with properties superior to those of its

individual components. Often, the term laminated composite is used for mesh-reinforced cementitious composites, although different mesh layers exist. Mesh-reinforced composites (laminates) consist of several unidirectional layers arranged at the same or different angles and, therefore, present anisotropic properties. These materials can be classified as continuous and orthotropic laminated composites, the most commonly used reinforced meshes are carbon/steel and polypropylene. A typical laminated cementitious composite is illustrated in Figure 1.

The use of composite materials is constantly increasing in modern construction because of low-cost, low-weight and high performance in terms of both ductile and crack arresting properties. Laminated cementitious composite materials can offer superior performance over standard concrete materials including higher strength-to-weight ratios, stiffness and crack resistance. Practically laminated cementitious composites require little or no maintenance as compared to metal structures. Owing to the improved characteristics, the application of these materials has been growing rapidly during the past decades in a wide variety of fields like housing, agriculture, geotechnical and other structural applications.

It is important to understand the fundamental properties of laminated cementitious composites, such as, flexural modulus in order to obtain reliable design and construction for field applications. During the last couple of decades,

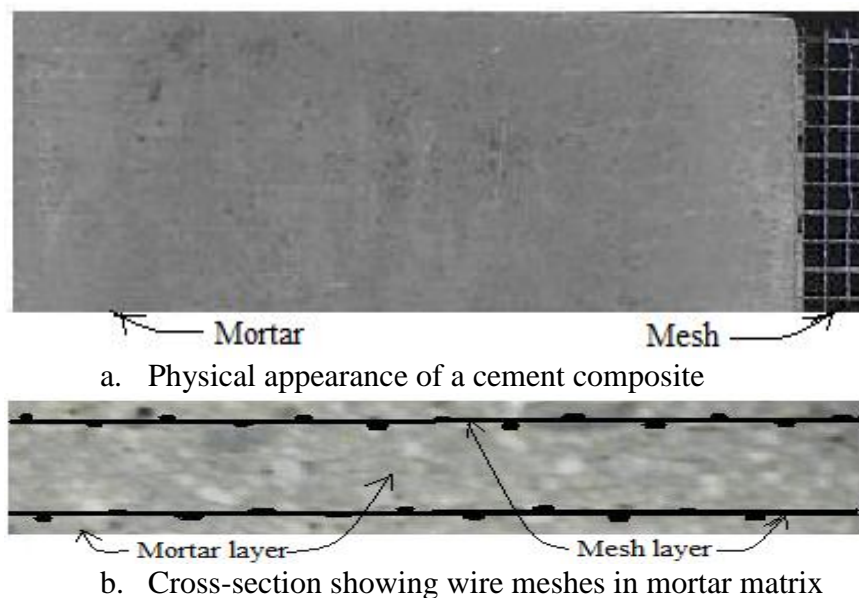


Figure 1: A laminated cementitious composite

numerous works have been carried out on mechanical properties of laminated cementitious composites (Kobayashi et al., 1992; Ghavami et al., 1999; Naaman, 2000; Hossain and Inoue, 2001). However, very little is known on flexural modulus although it presents a considerable versatility towards the reliable design of structures. The aim of this research is to conduct both analytical and experimental investigation to study flexural modulus and ductility properties of a laminated cementitious composite. Along with investigation, an analytical equation is also developed and presented to determine the relationship between the ductility and flexural modulus.

## 2.0 Materials and Methods

Various types of wire and geogrid meshes are available for the construction of laminated composite structure. The mechanical properties of mesh and mortar obtained experimentally are given in Table 1. To study the ductility characteristics of laminated composite structure, tests were carried out on specimens (two in each group) having heating and curing cycles. For better comparison, specimens with same cycles in natural drying and curing were also used without heating. For the heating group, each cycle was of 48 hours duration consisting 24 hours of heating in oven with constant temperature of 110°C and 24 hours of wetting in fresh water. In another group, here called air drying in room temperature, each cycle was of 48 hours duration consisting 24 hours air drying in room temperature at 15°C and 24 hours of wetting in fresh water.

Table 1: Mechanical properties of wire mesh, geogrid mesh and mortar

Wire mesh	Diameter (mm)	1.00
	C/c spacing (mm)	10.00
	Young's modulus (KN/mm <sup>2</sup> )	138.00
	Poisson's ratio	0.28
Geogrid	Cross-section of strands in longitudinal direction (mm)	1x5
	C/c spacing in longitudinal direction (mm)	22.00
	Diameter of strands in transverse direction (mm)	1.0 $\Phi$ x3
	C/c spacing in transverse direction (mm)	20.00
	Young's modulus (KN/mm <sup>2</sup> )	75.00
	Poisson's ratio	0.40
Mortar	Compressive strength (N/mm <sup>2</sup> )	27.84
	Young's modulus (KN/mm <sup>2</sup> )	15.47
	Poisson's ratio	0.19

The number of mesh layers selected for this investigation were two (1 top and 1 bottom) for all specimens with square steel mesh and geogrid mesh. The size of the specimen was 400×100×20mm with loading span of 360mm. Deflection was measured at the centre of the specimens tested under third point loading condition, shown in Figure 2.

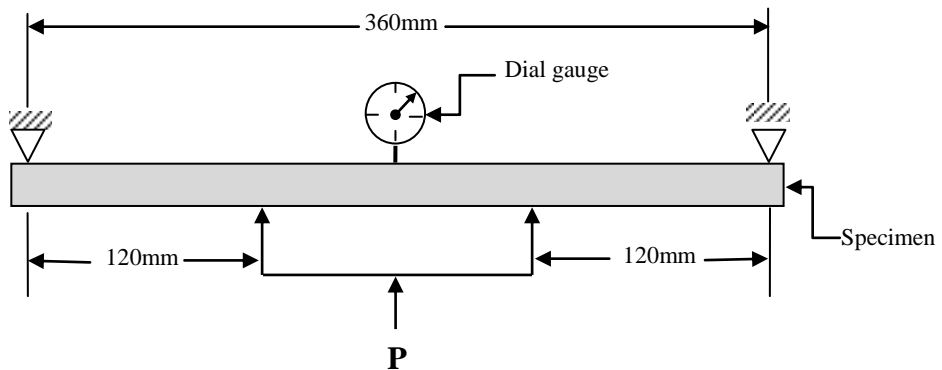


Figure 2: Specification for the test setup

### 3.0 Analytical Investigation: Flexural Modulus

Figure 3 illustrates a laminated cementitious composite of thickness  $t$  subjected to flexural loadings where  $y_0$  and  $\Delta y$  are the brittle (mortar) and ductile (mesh) portions, respectively. Here,  $\sigma_b$  and  $\sigma_d$  are stresses developed in brittle and ductile layers, respectively, and  $y$  is the distance from the neutral axis to the area  $dA$ . The  $dA$  can be expressed as  $w dy$  where  $w$  is the width of the flexural section and  $dy$  is the height of strip taken in the brittle and ductile portion. It can be given that the deformation ( $\Delta dx$ ) of any layer having original length  $dx$  at any distance  $y$  from the neutral axis to be varied proportionally with the variation of the distance from the neutral axis ( $y$ ). Therefore, the equation of strain ( $\varepsilon$ ) in any layer can be derived as follows:

$$\varepsilon = \frac{\Delta dx}{dx} = c.y \quad (1)$$

Where,  $c$  is the proportional constant. By using the Hook's law for any layer, equation 1 can also be written as follows:

$$\varepsilon = \frac{\sigma_b}{E_b} = \frac{\sigma_d}{E_d} \quad (2)$$

This is to note that the strains occurred in the brittle ( $\varepsilon_b$ ) and ductile ( $\varepsilon_d$ ) layers due to the deflection of beam are equal because of their relative changes with respect to neutral axis, i.e.  $\varepsilon_b = \varepsilon_d$ .

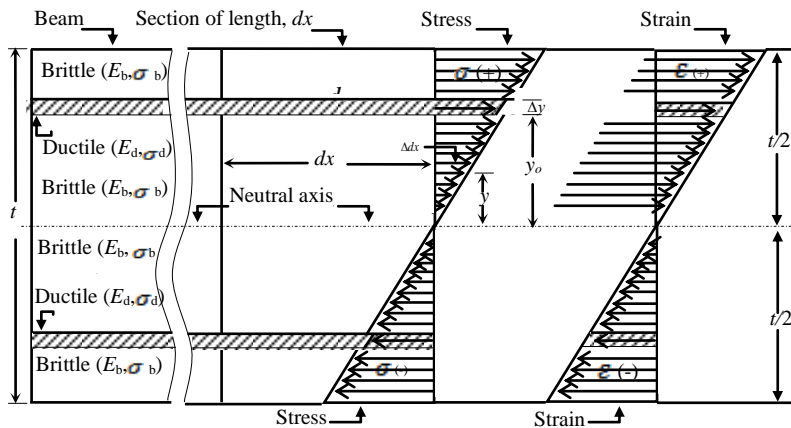


Figure 3: A laminated composite section with stress-strain diagram

However, the stress developed in brittle ( $\sigma_b$ ) and ductile ( $\sigma_d$ ) layers is varied depending on the rigidity of the layers, i.e.  $\sigma_b \neq \sigma_d$ .

By applying the condition of equilibrium in a section of a laminated cementitious composite consisting of several ductile (mesh) and brittle (mortar) layers, the following equation of balance may be formulated.

$$M = 2 \int_0^{y_0} y \sigma_b dA + \int_{y_0}^{y_0+\Delta y} y \sigma_d dA + \int_{y_0+\Delta y}^{t/2} y \sigma_b dA \quad (3)$$

By substituting the values of  $\varepsilon$  and  $\sigma$  from equations 1 and 2 into equation 3, the following equation can be obtained.

$$M = 2w \left[ \int_0^{y_0} E_b \cdot c \cdot y^2 dy + \int_{y_0}^{y_0+\Delta y} E_d \cdot c \cdot y^2 dy + \int_{y_0+\Delta y}^{t/2} E_b \cdot c \cdot y^2 dy \right] \quad (4)$$

Where,  $E_b$  and  $E_d$  are the moduli of elasticity of brittle (mortar) and ductile (mesh) layers, respectively. Integration of equation 4, gives:

$$M = 2w \left[ E_b c \frac{y_0^3}{3} + E_b c \frac{1}{3} \{ (y_0 + \Delta y)^3 - y_0^3 \} + E_b c \frac{1}{3} \left\{ \frac{t^3}{8} - (y_0 + \Delta y)^3 \right\} \right] \quad (5)$$

The ductile layer  $\Delta y$  can be written as the part of the composite indicating by  $\alpha$  which is expressed as  $\Delta y = \alpha \frac{t}{2}$  for half of the thickness of the composite due to symmetry. Similarly, the brittle layer  $y_0$  can be written as the part of the composite indicating by  $\beta$  which is expressed as  $y_0 = (1 - \alpha) \frac{t}{2} = \beta \frac{t}{2}$  for half of the thickness of the composite due to symmetry. Substituting the values of  $\Delta y$  and  $y_0$  into equation 5, the following equation can be obtained.

$$M = c \left[ E_b \frac{wt^3}{12} + 3(E_d - E_b) \alpha \beta^2 \frac{wt^3}{12} + 3(E_d - E_b) \alpha^2 \beta \frac{wt^3}{12} + (E_d - E_b) \alpha^3 \frac{wt^3}{12} \right] \quad (6)$$

It is known that the term  $\frac{wt^3}{12}$  of equation 6 is the moment of inertia of the composite flexural section with respect to neutral axis which can be indicated as  $I_c$  i.e.  $\frac{wt^3}{12} = I_c$ . Therefore, equation 6 can be written in terms of moment of inertia as follows:

$$M = c \left\{ E_b + (3\alpha\beta^2 + 3\alpha^2\beta + \alpha^3)(E_d - E_b) \right\} I_c \quad (7)$$

Combining equations 2 and 7, it takes the following form:

$$\varepsilon = \frac{M y}{\left\{ E_b + (3\alpha\beta^2 + 3\alpha^2\beta + \alpha^3)(E_d - E_b) \right\} I_c} \quad (8)$$

Equation 8 is the general equation for expressing the strain in any point of a laminated composite section due to the bending moment  $M$ , where the term  $E_b + (3\alpha\beta^2 + 3\alpha^2\beta + \alpha^3)(E_d - E_b)$  indicates the flexural modulus of the whole section composed of brittle and ductile layers, indicated as  $E_c$ . Hence, equation 8 can be written as:

$$\varepsilon = \frac{M y}{E_c I_c} \quad (9)$$

Where,

$$E_c = E_b + (3\alpha\beta^2 + 3\alpha^2\beta + \alpha^3)(E_d - E_b) \quad (10)$$

The synergy between the brittle and ductile material is evident from equation 10 indicating that the flexural modulus of laminated matrix depends on the third order of ductile material. This means that the ductile material may play a significant role on the brittle matrix which needs to be substantiated through laboratory experiment.

#### 4.0 Experimental Investigations: Ductility under Flexural Loadings

The ductility behavior of a laminated cementitious composite is illustrated in Figure 4. It can be seen that the laminated cementitious composite behaves like ductile material despite the 90% of the constituents is of brittle nature. This indicates the effect of the factor depicted in equation 10 which is the third order of the ductile layer. This means laminated cementitious composites under flexural loadings behaves like a plastic flexible material which can withstand any load without sudden failure. Indeed, this will be of great benefit for safe and reliable design of structures.

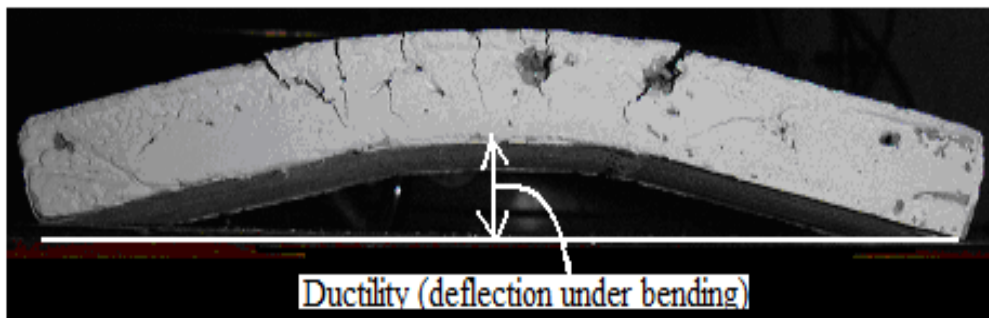


Figure 4: Ductility of the laminated cement composite

The ductility properties of some cement composites containing steel wire mesh and geogrid mesh treated under normal temperature of 25°C and elevated temperature of 110°C are given in Table 2. The symbol W0 indicates cement

composite reinforced with wire mesh having cured for 28 days (control specimens), G0 indicates cement composite reinforced with geogrid mesh with 28 days curing (control specimens), W30 indicates cement composite reinforced with wire mesh with 28 days curing and then 30 curing cycle in water, WH30 indicates cement composite reinforced with wire mesh with 28 days curing and then 30 heating cycle, G30 indicates cement composite reinforced with wire mesh with 28 days curing and then 30 curing cycle in water, GH30 indicates cement composite reinforced with geogrid mesh with 28 days curing and then 30 heating cycle and so on for other numbering of specimens. The ductility of the cement composites due to effect of reinforcement has been studied at different stages of loadings such as, at first crack load, at ultimate load and at failure load. The amount of ductility (deflection) at first crack varies depending on the types of reinforcements and variation of temperature. However, the ductility at ultimate and failure loads is almost same for all the cases. An example of the load-deflection relationships of zero heating cycle with 28 days curing is shown in Figure 5 indicating that ultimate load is obtained at the ductility range of 3 to 12 mm and the failure load is obtained at the ductility value of 15 mm.

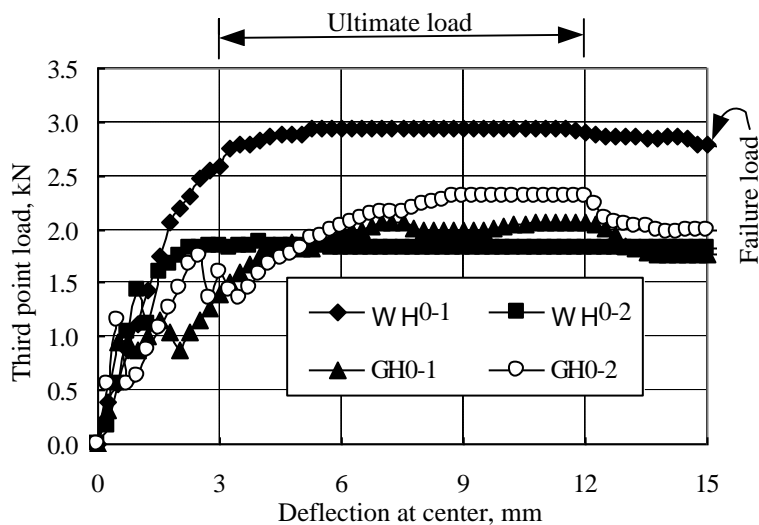


Figure 5: Load vs. deflection of laminated cementitious composite showing the range of ultimate load and failure load



Table 2: Ductility properties of composites under normal and elevated temperatures

Specimen	Group	Central deflection(mm) at first-crack load	Central deflection(mm) at ultimate load	Central deflection(mm) at failure load
W0	1	1.75	3 to 12	15.75
	2	1	3 to 12	15.75
WH0	1	0.5	3 to 12	15.75
	2	0.3	3 to 12	15.75
G0	1	1.15	3 to 12	15.75
	2	1.2	3 to 12	15.75
GH0	1	0.6	3 to 12	15.75
	2	0.35	3 to 12	15.75
W30	1	0.75	3 to 12	15.25
	2	0.5	3 to 12	15.75
WH30	1	0.5	3 to 12	15.25
	2	0.3	3 to 12	15.25
G30	1	1.75	3 to 12	15.25
	2	1.5	3 to 12	15.25
GH30	1	0.5	3 to 12	15.25
	2	0.5	3 to 12	15.75
W60	1	1.25	3 to 12	15.25
	2	1.5	3 to 12	15.25
WH60	1	0.5	3 to 12	15.25
	2	0.5	3 to 12	15.25
G60	1	0.75	3 to 12	15.25
	2	0.5	3 to 12	15.25
GH60	1	0.5	3 to 12	15.25
	2	0.5	3 to 12	15.25
W90	1	1.25	3 to 12	15.25
	2	1.5	3 to 12	15.75
WH90	1	0.5	3 to 12	15.25
	2	0.5	3 to 12	15.25
G90	1	1	3 to 12	15.25
	2	1.25	3 to 12	15.25
GH90	1	0.5	3 to 12	15.25
	2	0.3	3 to 12	15.25
W120	1	0.5	3 to 12	15.25
	2	0.5	3 to 12	15.75
WH120	1	1.75	3 to 12	15.25
	2	0.75	3 to 12	15.25

Table 2 Contd: (Ductility properties of composites under normal and elevated temperatures)

G120	1	1	3 to 12	15.25
	2	1	3 to 12	15.25
GH120	1	1.5	3 to 12	15.25
	2	0.75	3 to 12	15.25

(W= Wire, G=Geogrid, H= Heating and 30, 60, 90 & 120 are no. of cycles)

## 5.0 Discussion

This is to note that the usual equation of modulus of elasticity ( $E_c$ ) of a composite member consisting of different materials under uniaxial loading (tension or compression) has been given as:

$$E_c = \alpha E_d + \beta E_b \quad (11)$$

Where  $\alpha$  and  $\beta$  are the volume fractions of ductile and brittle layers respectively, as defined in earlier section. Equation 11 can also be written in the following form:

$$E_c = E_b + \alpha(E_d - E_b) \quad (12)$$

This equation indicates that the modulus of elasticity of a composite composed of ductile and brittle layers under uniaxial loadings depends on the moduli of brittle and ductile layers. The terms  $E_b$  and  $(E_d - E_b)$  are common for both the cases of uniaxial and flexural conditions but the factor is different depending on the loading conditions. As can be seen from the rule of mixture (equation 12), the factor is first order of the ductile layers. In contrast to this, the factor (equation 10) obtained through analytical investigation of a composite flexural section is third order of the ductile layer. Due to this third order of ductile portion, the laminated cementitious composite having brittle material i.e. mortar as the matrix and ductile material as the reinforcement possesses ductility properties which has been verified by experimental observation.

## 6.0 Conclusion

The ductility properties in terms of flexural modulus and deflection of a brittle laminated cementitious composite have been studied analytically and experimentally. The synergy between the brittle and ductile layers of a laminated composite is clarified through an analytical equation. The analytical observation shows that the flexural modulus of a brittle matrix is affected significantly by the

ductile layers, which is of third order of the ductile portion. The effect of the ductile layer found in analytical equation has also been verified through experimental work. Laboratory test results reveal that the brittle matrix with only two layers of ductile material possesses a good ductility under flexural loadings. The design ductility value of laminated cementitious composites, for example, in case of water storage structures may be taken as 0.5 to 1.0% within the first crack loading range. However, for other structures such as partition wall where macro-cracks may not be a great problem, the design ductility value may be taken as 1.5 to 6% at the ultimate loading range.

### Acknowledgement

The present study is partly supported by the Research Grant No. 22580271 with funds from Grants-in-Aid for Scientific Research, Japan. The authors gratefully acknowledge the support. The findings, conclusions and any opinion expressed in this paper are those of the authors and do not necessarily reflect the views of the sponsor.

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