Comparison of Techniques for Estimating the Frequency Selectivity of Bandlimited Channels

Ahmad Zuri Sha'ameri

Digital Signal Processing Lab
Dept of Microelectronics and Computer Engineering
Faculty of Electrical Engineering
Universiti Teknologi Malaysia
81310 UTM Skudai
Johor Darul Takzim
Malaysia
Fax: 60 7 556 6272
Email : ahmadzs@safenet.utm.my

Abstracts - A transmission channel used in application such as telecommunications can be modeled as a bandpass filter. Measurement of the frequency selectivity of the channel is important to ensure that the information-bearing signal has minimal distortion and loss of information. A comparison is made for several methods used for estimating the frequency selectivity of the transmission. The methods presented in this paper are the correlation method, instantaneous energy and frequency estimation and the cross Wigner-Ville distribution. The theoretical foundations and assumptions are described for each method. In general, all the methods gave similar performance in terms of the frequency selectivity. Due to the shorter analysis duration, both the instantaneous energy and frequency estimation and cross Wigner-Ville distribution is ideal for estimating the frequency selectivity of time-varying channels.

I INTRODUCTION

In applications such as in telecommunications, it is important to measure the frequency response of the transmission channel. Since subscribers are multiplexed in frequency, some subscribers may have lower quality of service compare to others because attenuation or selectivity in the channel that is a function of frequency. In addition, some information present may be corrupted or lost if the signal transmitted through the channel has large bandwidth. Methods that are used for estimating the selectivity of the transmission channel are the correlation method [1][2], instantaneous energy and frequency estimation (IEFE), and the cross Wigner-Ville distribution CWVD [3][4]. Their performance is compared based on a set of known transmission channels.

II INPUT-OUTPUT RELATIONSHIP OF A LINEAR SYSTEM

The transmission channel is assumed a discrete-time linear time-invariant system and the input-output relationship is defined by the convolution sum

$$y(n)=\sum_{\lambda=-\infty}^{\infty}h(\lambda)x(n-\lambda) \quad (1)$$

where $y(n)$ is the output, $x(n)$ is the input, and $h(n)$ is the channel impulse response. The channel is stable if

$$\sum_{n=-\infty}^{\infty}|h(n)|<\infty \quad (2)$$

and causal if

$$\begin{aligned} h(n)&\neq 0 && n\geq 0 \\ &=0 && \text{elsewhere} \end{aligned} \quad (3)$$

If the Fourier transform exist for the input signal and the system impulse response, then the input-output relationship when expressed in the frequency domain is

$$Y(f)=H(f)X(f) \quad (4)$$

where $X(f)$ is the Fourier transform of $x(n)$ and $H(f)$ is the Fourier transform of $h(n)$.

The specifications of the filters are defined in Table 1 and an example of the frequency response for Filter 1 is shown in Figure 1.

Filter	Filter Type	lower cutoff freq, $f_{c,lo}$	upper cutoff freq, $f_{c,hi}$
1	Butterworth, 8 order	0.0625	0.125
2	Chebyshev I, 6 order	0.20	0.25
3	FIR Rectangular, 45 order	0.0625	0.125
4	FIR Hamming, 33 order	0.20	0.25

Table 1 The specifications for the bandpass filters used to model the transmission channels.

0-7803-6355-8/00/ $10.00 © 2000 IEEE

III ESTIMATION METHODS

Methods that can be used for estimating the frequency selectivity of the transmission channel are the correlation method, instantaneous energy and frequency estimation (IEFE), and cross Wigner-Ville distribution (CWVD). The theoretical and assumptions for each method will be presented in this section.

A CORRELATION METHOD

This method uses a pseudorandom sequence as input to the linear system and the input-output relationship is expressed in terms of the correlation function. From here, the frequency selectivity is estimated from the spectrum of the output of the linear system. When expressed in terms of the correlation function, the input-output relationship [1] is

$$R_{xy}(m) = \sum_{\lambda=-\infty}^{\infty} h(\lambda) R_{xx}(m-\lambda) \quad (5)$$

where $R_{xy}(m)$ is the crosscorrelation function of the input $x(n)$ and output $y(n)$, $R_{xx}(m)$ is the autocorrelation function input $x(n)$ and $h(n)$ is the system impulse response. The correlation functions are defined as

$$R_{xy}(m)=E[x(n)y^*(n-m)] \quad (6)$$
$$R_{xx}(m)= E[x(n)x^*(n-m)] \quad (7)$$

If the autocorrelation function of the input is a delta function, then the crosscorrelation function in Equation (6) becomes

$$R_{xy}(m) = \sum_{\lambda=-\infty}^{\infty} h(\lambda)\delta(m-\lambda) = h(m) \quad (8)$$

The spectrum of the system is related to the cross power spectrum by

$$\sum_{m=-\infty}^{\infty} R_{xy}(m)\exp(-j2\pi fm) = \sum_{m=-\infty}^{\infty} h(m)\exp(-j2\pi fm)$$
$$S_{xy}(f)=H(f) \quad (9)$$

Besides additive white Gaussian noise, a random signal that has an autocorrelation function that is approximately a delta function is a pseudorandom binary sequence [1]. Since a bandpass channel is assumed, the binary data is assumed as

$$x(n)=\cos(2\pi f_1 n). \quad \ldots -N_0/2<n<N_0/2 \quad \text{for binary bit '1'}$$
$$=\cos(2\pi f_1 n+\pi) \quad -N_0/2<n<N_0/2 \quad \text{for binary bit '0'} \quad (10)$$

where N_0 is the bit-duration and f_1 is the frequency. The sequence can be generated using a linear feedback shift register [5] of length L. If the polynomial that represents the position of the taps in the linear feedback shift register is primitive, then the resulting sequence will have a length of 2^L-1 bits that is valid for all possible initial conditions. If the chosen sequence length is $N_{seq} \leq 2^L-1$ bits, the autocorrelation function for the input binary sequence is

$$R_{xx}(m) = \left[1-\frac{2|m|}{N_0}\right]\cos(2\pi f_0 m) \quad m \geq 0$$
$$= R_{xx}^*(m) \quad m<0 \quad (11)$$

where N_0 is the bit-duration in samples. If $R_{xx}(0)$ is the total energy of the N_{seq} length sequence, then the average power is $R_{xx}(0)/N_{seq}$.

C INSTANTANTANEOUS ENERGY AND FREQUENCY ESTIMATION

In this method, a time-varying signal is used as an input to the transmission channel and the frequency selectivity is measured by comparing the instantaneous energy and frequency of both the input and output signals. This is referred as the instantaneous energy and frequency estimation (IEFE) technique, requires the signal to be in analytical form and is achieved by using the Hilbert transform [3].

In the complex form, a time-varying signal can be expressed as

$$x(n) = c(n)\exp\big(j\phi(n)\big)+w(n)$$
$$= c(n)\exp\left(j2\pi\sum_{\lambda=-\infty}^{n} f_i(\lambda)\right)+w(n) \quad 0<n<N\text{-}1 \quad (12)$$

where $f_i(n)$ is the instantaneous frequency and $w(n)$ is the interference due to additive white noise.

Assuming high signal-to-noise ratio conditions, then the instantaneous frequency obtained by

$$f_i(n) = \frac{1}{2\pi}\frac{d}{dn}[\arg[x(n)]] = \frac{1}{2\pi}\frac{d}{dn}[\phi(n)] \quad (13)$$

Based on the definition of the derivative, the instantaneous frequency in Equation (13) is expressed as

$$f_i(n) = \lim_{\Delta n T_s \to 0}\frac{1}{2\pi T_s}\left[\frac{\phi(n)-\phi(n-\Delta n)}{\Delta n T_s}\right] \quad (14)$$

where T_s is the sampling interval and Δn is an infinitesimal difference in time sample. If the sampling interval T_s and difference Δn are assumed as one, the instantaneous frequency is

$$f_i(n) = \frac{1}{2\pi}[\phi(n)-\phi(n-1)] \quad (15)$$

The result in Equation (15) is known as the backward finite difference (BFD). It is based on comparing the present instantaneous phase with its previous. Depending on the choice of sample instants, the forward finite difference (FFD) and central finite difference (CFD) are defined as

$$f_i(n) = \frac{1}{2\pi}\left[\phi(n+1) - \phi(n)\right] \tag{16}$$

$$f_i(n) = \frac{1}{4\pi}\left[\phi(n+1) - \phi(n-1)\right] \tag{17}$$

For nonstationary signals, it has been proven [4] that the central finite difference (CFD) gives an unbiased estimate of the signal frequency compared to the FFD and BFD. Thus, the CFD is chosen as the method for estimating the instantaneous frequency.

Another characteristic of the analytical signal is the relationship between the instantaneous energy and the signal amplitude. Using the signal definition in Equation (12), the instantaneous energy is

$$\begin{aligned} E_z(n) &= z(n)z^*(n) \\ &= c(n)c^*(n) \end{aligned} \tag{18}$$

Thus, the instantaneous energy is related directly to the amplitude of the signal. Similar to the instantaneous frequency, this relationship is only true for the analytical signal.

The chirp or linear FM (Frequency Modulation) signal is a nonstationary signal that is applied as input to the linear system is

$$x(n) = \exp\left(j2\left(\pi f_1 + \frac{\alpha}{2}\right)n\right) \qquad 0<n<N\text{-}1 \tag{19}$$

where f_1 is the starting frequency, N is the signal duration and α is the frequency sweep rate. The factor α is defined as

$$\alpha = \frac{f_{BW}}{N} \tag{20}$$

where f_{BW} is the bandwidth and N is the duration of the chirp signal. For large α, the energy spectrum of $x(n)$ is

$$\begin{aligned} E_x(f) &= 1 \qquad \text{for } f_1<f<f_1+f_{BW} \\ &= 0 \qquad \text{elsewhere} \end{aligned} \tag{21}$$

Based on the definition in Equation (13), the instantaneous frequency of the chirp signal is

$$f_i(n) = f_1 + \alpha n \tag{22}$$

Since the chirp signal is assumed constant magnitude, the instantaneous energy based on Equation (18) is at unity for the duration of the signal.

If the true lower and upper cutoff frequency of the channel is defined as 0.5 of the maximum value, the estimated values from the instantaneous energy and frequency are

$$\begin{aligned} &f_{c,lo}=f_i(n_{lo}),\ E_z(n_{lo})=0.5E_{z,max} \\ &f_{c,hi}=f_i(n_{hi}),\ E_z(n_{hi})=0.5E_{z,max} \\ &n_{hi}>n_{lo},\ n_{lo}<N,\ n_{hi}<N \end{aligned} \tag{23}$$

where $E_{z,max}$ is the peak instantaneous energy.

D CROSS WIGNER-VILLE DISTRIBUTION

Time-frequency distributions were developed to analyze time-varying nonstationary signals. A generalized formulation for bilinear time-frequency distribution [6] was proposed that link all the different types of time-frequency distributions. The Wigner-Ville distribution (WVD) is one of the earliest classes of time-frequency distribution and is a member of the bilinear class of time frequency distributions. For discrete-time signals, the distribution can be expressed as

$$W_x(n,f) = \sum_{m=-\infty}^{\infty} x(n+m)x^*(n-m)\exp(-j4\pi fm) \tag{24}$$

where $x(n)$ is the signal of interest. The cross Wigner-Ville distribution (CWVD) [4] is derived from the WVD where comparison is made between two different signals. For the problem of interest, the CWVD is defined as

$$W_{y,x}(n,f) = \sum_{m=-\infty}^{\infty} y(n+m)x^*(n-m)\exp(-j4\pi fm) \tag{25}$$

where $y(n)$ is the output of the unknown linear system and $x(n)$ is the input signal. By evaluating the frequency marginal [6], the cross energy spectrum is

$$E_{y,x}(f) = \sum_{n=-\infty}^{\infty} W_{y,x}(n,f) \tag{26}$$

With the appropriate choice of signals, the cross energy spectrum can be made equal to the spectrum of the system.

If the chirp signal defined in Equation (19) is applied is applied as input, the CWVD is

$$\begin{aligned} W_{y,x}(n,f) &= \sum_{m=-\infty}^{\infty}\left[h(n+m) \underset{(n)}{*} x(n+m)\right]x^*(n-m) \\ &\quad \times\exp(-j4\pi fm) \\ &= H(f_1+\alpha n)\delta(f-(f_i+\alpha n)) \end{aligned} \tag{27}$$

The cross energy spectrum derived by substituting the CWVD in Equation (27) into the frequency marginal in Equation (26) is

$$E_{y,x}(f) = \sum_{n=-\infty}^{\infty} W_{y,x}(n,f) = H(f)E_x(f)$$
$$=H(f) \qquad (28)$$

This is only true if the energy spectrum of $E_x(f)$ is approximate defined in Equation (21). The channel spectrum $H(f)$ is the cross energy spectrum of $E_{z,x}(f)$. Thus, the frequency selectivity can be estimated directly from the cross energy spectrum.

IV RESULTS

The measured frequency responses are shown in Figure 2 to 4 using the correlation, IEFE and CWVD. All the methods present gave similar results in the estimated frequency selectivity. The similarity is reflected in the measurement of the lower and upper cutoff frequency of the selected bandpass channel. This is can only be true if autocorrelation of the pseudorandom sequence used in the correlation method approaches an impulse function. For the IEFE method and CWVD, a high sweep rate factor will ensure that the input signal has bigger bandwidth than the transmission channel. Thus, an accurate measurement of the frequency selectivity of the channel is achieved.

A more significant factor would be the analysis duration. The crosscorrelation method requires a pseudorandom sequence that is significantly long to ensure that the autocorrelation function of the pseudorandom sequence is approximately a delta function. This is not desirable if the channel is time-varying and its characteristics changes within the analysis duration. For the IEFE and CWVD, the duration of the chirp signal can be made smaller based on the selection of the sweep rate. The frequency selectivity can be estimated within shorter analysis duration. Thus, both methods would be more appropriate for time-varying channels.

Filter	Actual cutoff freq	Measured lower & upper cutoff freq		
		Correlation	IEFE	CWVD
1	0.0625-0.125	0.06 – 0.12	0.065 – 0.13	0.06 – 0.13
2	0.20-0.25	0.2 – 0.25	0.21 – 0.25	0.2 – 0.25
3	0.0625-0.125	0.07 – 1.2	0.07 – 0.125	0.06 – 0.14
4	0.20-0.25	0.21 – 0.24	0.21 – 0.25	0.19-0.26

Table 2 Comparison between the actual and measured lower and upper cutoff frequency based on the correlation, IEFE and CWVD.

V CONCLUSIONS

The paper presents several methods that can be used for estimating the frequency selectivity of bandpass transmission channels. A comparison in terms of their accuracy is made using the correlation method, IEFE and the CWVD. Similar results are obtained for all the techniques presented. For both the IEFE method and CWVD, the estimation of the frequency selectivity is achieved using a shorter duration signal compared to the correlation method and will be useful in the analysis of time-varying channels.

VI REFERENCES

[1] Proakis, J.G., Rader, C.M., *Advanced Digital Signal Processing*, MacMillan, New York, 1992.

[2] Haykin,S., *Adaptive Signal Processing*, Prentice-Hall, New Jersey, 1991

[3] Boashash, B., "Estimating & Interpreting the Instantaneous Frequency of a Signal-Part I: Fundamentals", *Proc. of the IEEE*, Vol.80, No.4, April 1992, pp. 519-538.

[4] Boashash, B., "Estimating & Interpreting the Instantaneous Frequency of a Signal-Part 2: Algorithms and Applications", *Proc. of the IEEE*, Vol 80, No 4, April 92, pp540-568.

[5] Golomb, S.W., *Shift Register Sequences*, Holden Day, 1967.

[6] Boashash, B. "Time-Frequency Signal Analysis", *Advances in Spectrum Estimation and Array Processing*, Vol.1, Ed. S. Haykin, Prentice-Hall, 1990, pp. 418-517.

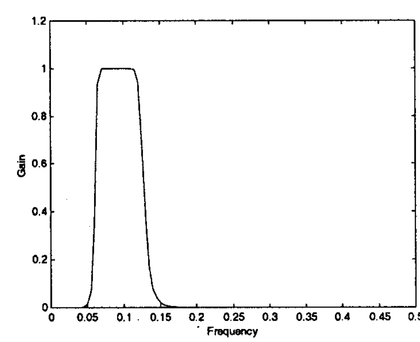

Fig 1 Actual frequency response of the 8 th order Butterworth filter (Filter 1).

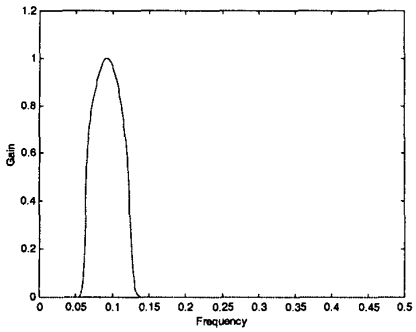

Fig 2 Measured frequency response using the correlation method.

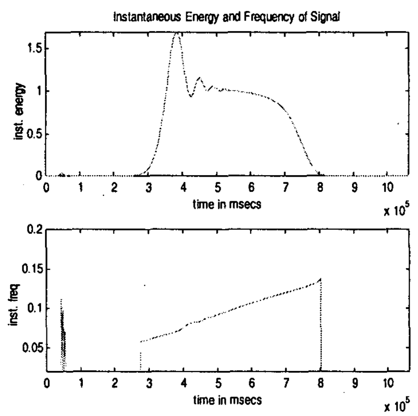

Fig 3 Instantaneous energy and frequency estimate for Filter 1.

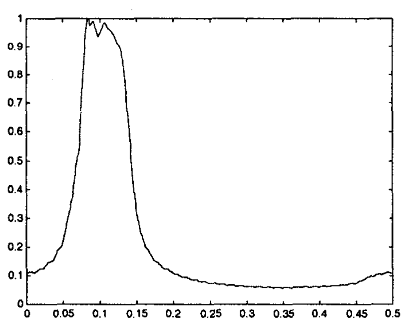

Fig 4 Measured frequency response using the marginal of the C.WVD