

# 4

## **CONTROL OF ACTIVATED SLUDGE WASTEWATER SYSTEM**

Norhaliza Abdul Wahab  
Reza Katebi  
Mohd Fuaad Rahmat  
Aznah Md Noor

### **4.1 INTRODUCTION**

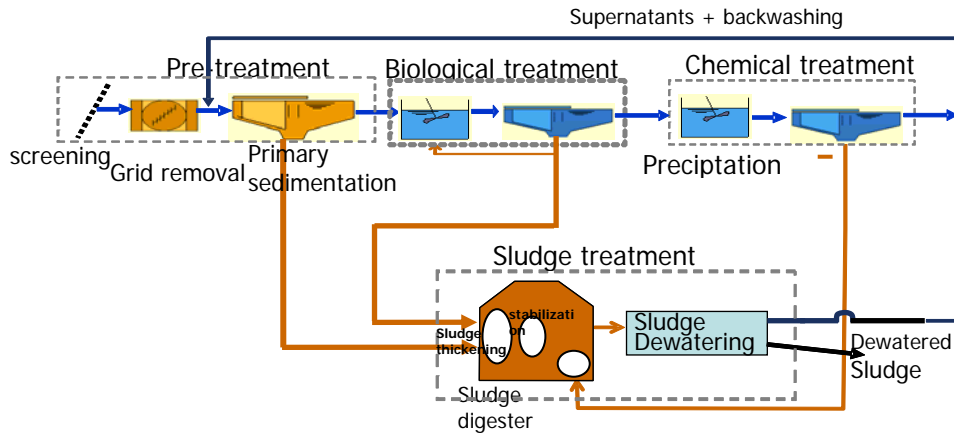
There are several stages in wastewater treatment plants (WWTP) before it can be released to a receiving water body as shown in Figure 4.1: a preliminary treatment (mechanical treatment), a primary treatment (primary sedimentation), a secondary treatment (biological treatment) and a tertiary treatment (chemical treatment).

Initially, the first treatment or so-called mechanical treatment is applied which included both preliminary and primary treatments. In preliminary treatment, large and heavy objects like solid debris, grease removal and flow equalisation are passing through grid removal by sedimentation. Therefore, lighter particles are passing through primary sedimentation or primary clarifiers. Generally, this stage is to ensure the wastewater passing to the subsequent stages is clean and could not disrupt the plant operation.

The next operation is known as biological treatment. The two main types of biological treatment plants are biofilters and activated sludge processes. The most varied animal life is found in

---

biofilter. Meanwhile, the animals in activated sludge plant are not nearly as varied in terms of species. Only activated sludge process is considered in this chapter. In activated sludge treatment plants microorganisms like bacteria and fungi contributed to the degradation of dissolved organic matter as well as suspended organic matter. All microorganisms enter the system with the influent wastewater.



**Figure 4.1:** A general layout of a wastewater treatment plant, Metcalf *et al.* (1991)

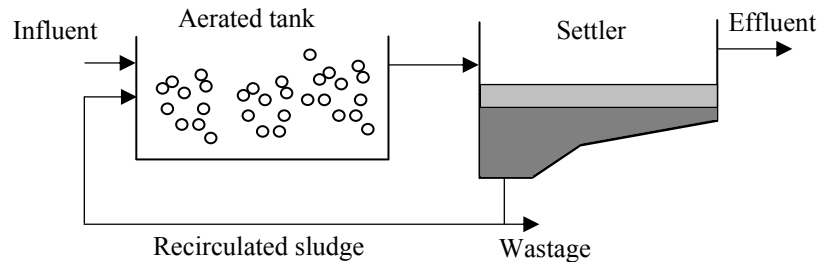
The chemical precipitation is occurred during chemical treatment. In this stage, phosphorus is removed from the wastewater. Phosphorus removal also can be applied in the activated sludge process by biological treatment. Often as a final step, chlorination is included in the water before it is released to a receiving water body. In the sludge treatment part, sludge from all different blocks is processed. Usually the sludge contain an organic matter and need to be stabilized through stabilization in the sludge digester. To minimize cost, sludge which contain about 95% water is dewatered either by mechanically or by drying before it is then transported away. The control part mainly considered in biological treatment process.

## 4.2 BIOLOGICAL PROCESSES IN WASTEWATER TREATMENT PLANTS

In biological processes, an organic matter which enters the plant in various forms is converted to other forms. *Hydrolysis* transforms the slowly degradable matter of larger organic molecules into smaller, more easily accessible molecules or readily degradable matter. This process is slower than the microorganism growth rate. The growth rate of biomass depends on several variables such as the amount of biomass, temperature, pH, the substrate and the presence of toxins. During biomass decay (bioreduction), the biologically inert matter (non-biodegradable) is produced. However, some inert matter also contain in influent wastewater. This material matter is collected and removed by the settler.

## 4.3 ACTIVATED SLUDGE PROCESSES

The activated sludge process is a biological process in which an organic matter is oxidized and mineralized by microorganisms. This section will give a brief description of physical and biological phenomena that occur in the activated sludge system. A basic configuration of activated sludge process only consists of an aerated tank and a settler as shown in Figure 4.2.



**Figure 4.2:** A basic layout of activated sludge processes

---

In aerated tank, a slowly growth of microorganisms maintain their microbiological population and allow the right concentration of microorganisms by a recirculated sludge from the settler back to the aerated tank. Oxygen is used by microorganisms to oxidize organic matter. The influent of particulate inert matter and the growth of the microorganisms is removed from the plant as excess sludge to maintain a reasonable suspended solids concentration. A component mass balance for the activated sludge process yields the following set of nonlinear differential equations, Nejari F. *et al.*(1996):

$$\dot{X}(t) = \mu(t)X(t) - D(t)(1+r)X(t) + rD(t)X_r(t) \quad (4.1)$$

$$\dot{S}(t) = -\frac{\mu(t)}{Y}X(t) - D(t)(1+r)S(t) + D(t)S_{in} \quad (4.2)$$

$$\dot{C}(t) = -\frac{K_o\mu(t)}{Y}X(t) - D(t)(1+r)C(t) + K_{La}(C_s - C(t)) + D(t)C_{in} \quad (4.3)$$

$$\dot{X}_r(t) = D(t)(1+r)X(t) - D(t)(\beta+r)X_r(t) \quad (4.4)$$

where the state variables,  $X(t)$ ,  $S(t)$ ,  $C(t)$  and  $X_r(t)$ , represents the concentrations of biomass, substrate, dissolved oxygen (DO) and recycled biomass respectively.  $D(t)$  is the dilution rate, while  $S_{in}$  and  $C_{in}$  correspond to the substrate and dissolved oxygen concentrations of influent stream. The parameters  $r$  and  $\beta$  represents the ratio of recycled and waste flow to the influent flow rate, respectively. The kinetics of the cell mass production is defined in terms of the specific growth rate  $\mu$  and the yield of cell mass  $Y$ . The term  $K_o$  is a constant.  $C_s$  and  $K_{La}$  denote the maximum dissolved oxygen concentration and the oxygen mass transfer coefficient, respectively. The Monod equation gives the growth rate related to the maximum growth rate, to the substrate concentration, and to the dissolved oxygen concentration:

$$\mu(t) = \mu_{max} \frac{S(t)}{K_s + S(t)} \frac{C(t)}{K_c + C(t)} \quad (4.5)$$

where  $\mu_{max}$  is the maximum specific growth rate ,  $K_s$  is the affinity constant and  $K_c$  is the saturation constant.

In this study, the following assumptions have been made regarding the waste water treatment system shown in Figure 4.2:

- The dissolved oxygen and substrate are the only controlled plant outputs.
- The parameters ( $r, \beta$ ) and the constants( $Y, C_s, K_o$ ) are known.

#### **4.4 THE ROLE OF SENSORS IN ACTIVATED SLUDGE WASTEWATER TREATMENT PLANT**

Sensors are of importance components to provide information to the plant operator. The basic (or local) control loop is usually composed of the controller loop, actuator, plant and sensor. Often, the controller loop is PID controller, implemented either in a form of digital or analogue. The output measurement from sensors can be compared to a desired value or setpoint. The difference between the two values may involve control laws that would yield to a desired value. Table 4.1 shows possible control loops in activated sludge wastewater systems shown in Figure 4.2.

**Table 4.1** Activated sludge control loops

<b>Manipulated Variable</b>	<b>Loop Input</b>	<b>Actuator Output</b>
Dillution rate	flow setpoint	pumping rate
	flow measurement	
Air flow rate	flow setpoint	pumping rate
	flow measurement	
Wastage removal	flow setpoint	pumping rate or removal scheduling
	flow measurement	
	or optimal wastage removal schedule	

---

Carbon dosing	dosing setpoint	pumping rate
Chemical dosing	dosing setpoint	pumping rate
Aeration	DO setpoint	aeration rate, gate
	DO measurement	positioning
	OUR	

---

In the activated sludge process shown in Figure 4.2, two control loops are considered that is DO and substrate control loops. The DO sensor measures the DO in the activated sludge tank, which is related to the uptake of oxygen by the micro-organisms present (OUR). The OUR depends on the content of the carbon in the influent. The greater the OUR in the influent which shows the higher carbon, the lower the DO will be. The profile of DO measurement received from DO sensor is logged on a PLC. The sensor will provide this information to the PLC, which will automatically adjust the volume of air supplied to the aeration tank, thus increasing the DO. The air supply will be increased or decreased depending on the indication of DO measurement received from DO sensor. In control of dynamic behavior of wastewater system, the performance of the controller plays a vital role for achieving the control goal.

#### 4.5 MODEL IDENTIFICATION OF ACTIVATED SLUDGE PROCESSES

The nonlinear model is linearised about an operating point,  $x_s$ . At this operating point, it is assumed that the plant can be described by the following state space model:

$$\begin{aligned}\dot{\underline{x}} &= A\underline{x} + B\underline{u} \\ \underline{y} &= C\underline{x} + D\underline{u}\end{aligned}\tag{4.6}$$


---

where A, B, C and D represents matrices with constant parameters. To simplify the notation the reactor dynamics in equations (4.1)-(4.5) is expressed as:

$$\dot{\underline{z}} = f(\underline{z}, \underline{u}) \quad ; \delta \underline{z} = A\delta \underline{z} + B\delta \underline{u} \quad (4.7)$$

where,

$$\dot{\underline{z}} = \begin{bmatrix} \dot{X} \\ \dot{S} \\ \dot{C} \\ \dot{X}_r \end{bmatrix}, \quad \underline{z} = \begin{bmatrix} X \\ S \\ C \\ X_r \end{bmatrix}, \quad \underline{u} = \begin{bmatrix} D \\ W \end{bmatrix} \quad (4.8)$$

and

$$\delta \dot{\underline{z}} = \begin{bmatrix} \frac{\delta f\dot{X}(t)}{\delta X(t)} & \frac{f\dot{X}(t)}{\delta S(t)} & \frac{f\dot{X}(t)}{\delta C(t)} & \frac{f\dot{X}(t)}{\delta X_r(t)} \\ \frac{\delta f\dot{S}(t)}{\delta X(t)} & \frac{\delta f\dot{S}(t)}{\delta S(t)} & \frac{\delta f\dot{S}(t)}{\delta C(t)} & \frac{\delta f\dot{S}(t)}{\delta X_r(t)} \\ \frac{\delta f\dot{C}(t)}{\delta X(t)} & \frac{\delta f\dot{C}(t)}{\delta S(t)} & \frac{\delta f\dot{C}(t)}{\delta C(t)} & \frac{\delta f\dot{C}(t)}{\delta X_r(t)} \\ \frac{\delta f\dot{X}_r(t)}{\delta X(t)} & \frac{\delta f\dot{X}_r(t)}{\delta S(t)} & \frac{\delta f\dot{X}_r(t)}{\delta C(t)} & \frac{\delta f\dot{X}_r(t)}{\delta X_r(t)} \end{bmatrix}_{t=z_0} \delta \underline{z} + \begin{bmatrix} \frac{\delta f\dot{X}(t)}{\delta D(t)} & \frac{\delta f\dot{X}(t)}{\delta W(t)} \\ \frac{\delta f\dot{S}(t)}{\delta D(t)} & \frac{\delta f\dot{S}(t)}{\delta W(t)} \\ \frac{\delta f\dot{C}(t)}{\delta D(t)} & \frac{\delta f\dot{C}(t)}{\delta W(t)} \\ \frac{\delta f\dot{X}_r(t)}{\delta D(t)} & \frac{\delta f\dot{X}_r(t)}{\delta W(t)} \end{bmatrix} \underline{u}(t) \quad (4.9)$$

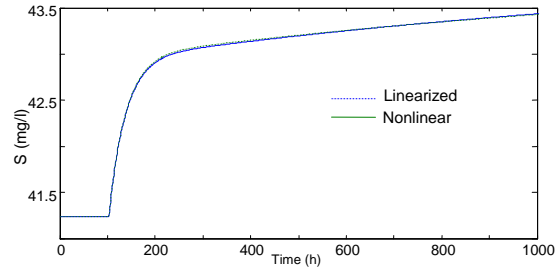
The nonlinear dynamics, described by (4.7), was linearized around the operating point described by the data presented in Table 4.2. This yielded the linearized matrices A, B, C and D. These were then manipulated into a transfer function form using:

$$\begin{aligned} \underline{x}(s) &= (sI - A)^{-1} B\underline{u}(s) = G(s)\underline{u}(s) \\ \underline{y}(s) &= G(s)\underline{u}(s) \end{aligned} \quad (4.10)$$

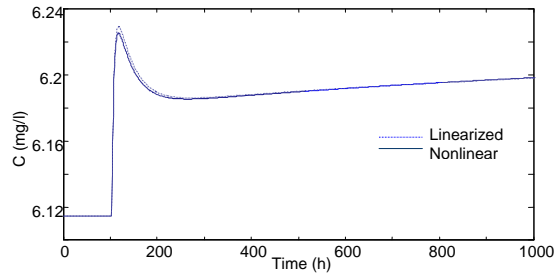
yielding the following open- loop transfer function matrix,  $G(s)$ :

$$\begin{aligned} g_{11} &= \frac{134(s+2)(s+0.1864)(s+0.01169)}{(s+1.996)(s+0.2573)(s+0.2)(s+0.008)} \\ g_{12} &= \frac{-0.03(s+0.1863)(s+0.01169)}{(s+1.996)(s+0.2573)(s+0.2)(s+0.008)} \\ g_{21} &= \frac{-9(s+1.445)(s+0.1942)(s+0.004783)}{(s+1.996)(s+0.2573)(s+0.2)(s+0.008)} \\ g_{22} &= \frac{0.07(s+0.2802)(s+0.1993)(s+0.007407)}{(s+1.996)(s+0.2573)(s+0.2)(s+0.008)} \end{aligned} \quad (4.11)$$

Clearly, there is an interaction in the system, since the off-diagonal elements are nonzero. The approximate linearized response for a small input change is plotted in Figure 4.3(a-b).



(a)



(b)

**Figure 4.3** Dynamic responses for  $\pm 5\%$  input change (a) Substrate; (b) Dissolved oxygen



### 4.6 CONTROL TUNING

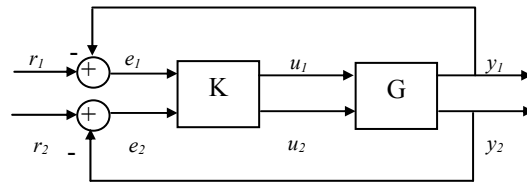
The basic idea is to adjust certain scalar tuning parameters of the controller via the different tuning methods mentioned in section 1. Figure 1.4 shows the control system under consideration in this study, which incorporates two control loops. The two control loops aims to maintain the substrate and dissolved oxygen concentrations at desired levels.

#### 4.6.1 Davison Method

In the Davison method the plant is diagonalised at low frequencies Davison, E., (1976). The expression for the controller used in the Davison method is:

$$\underline{u}(s) = K_i \frac{1}{s} \underline{e}(s), \quad K_i = \varepsilon G^{-1}(0) \tag{4.12}$$

where  $K_i$  is the integral feedback gain,  $G(s)$  is the open loop transfer function matrix and the scalar,  $\varepsilon$ , is the tuning parameter. The multiplier  $\varepsilon$  can be tuned and adjusted simultaneously so the closed loop plant has a maximum speed of response for a step input. In this case, we can find  $G(0)$  matrix from the different value between the operating point and controlled output.



**Figure 4.4** Multiple single loop controllers, F.G.Shinsky (1981)

### 4.6.2 Penttinen -Koivo Method

In this method, the plant is diagonalised at high frequencies, Penttinen, J. and Koivo, N.H.(1980). The expression for the controller is slightly altered from previous method to form:

$$\underline{u}(s) = (K_p + K_i \frac{1}{s}) \underline{e}(s) \quad (4.13)$$

where,

$$K_p = (CB)^{-1} \rho, \quad K_i = \varepsilon G^{-1}(0) \quad (4.14)$$

From equation (4.6), it can be found that  $\underline{y} = C\underline{\dot{x}}$ . In the case that plant is at the operating point or  $\underline{x} = 0$ ,  $\underline{\dot{y}} = CB\underline{u}$  or  $\underline{\dot{z}} = f(CB, \underline{u})$ .

Therefore, by applying each outputs with a unit step and measuring the degree of slope of each output immediately after:

$$CB = [\underline{\dot{z}}_1, \underline{\dot{z}}_2, \dots, \underline{\dot{z}}_m] \quad (4.15)$$

where  $m$  is the number of plant inputs and  $\underline{\dot{z}}_k$  is the output degree of slope in response to the  $k^{th}$  input step. Given a plant in state space form:

$$G(s) = C(sI - A)^{-1} B \quad (4.16)$$

By applying Laurent series expansion of the transfer function as follows:

$$G(s) = \frac{CB}{s} + \frac{CAB}{s^2} + \frac{CA^2B}{s^3} + \dots \quad (4.17)$$

Therefore at high frequencies,  $G(s) = CB/s$  and  $G(s)K_p = I/s$ , thus give a closed-loop transfer function in Figure 4.3 to form :

$$(I + GK)^{-1} GK = \begin{bmatrix} H_1(s) \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & H_n(s) \end{bmatrix} \quad (4.18)$$

for large  $s$

At high frequencies, the  $K_i/s$  term are negligible compared to  $K_p$ . In order to give a good asymptotic tracking at high and low frequencies, the scalar tuning  $\rho$  and  $\varepsilon$  must be properly tuned.

### 4.6.3 Maciejowski method

The technique suggested by Maciejowski is to diagonalise the system near the bandwidth,  $w_b$ , Maciejowski, J. M (1989). The controller is given by:

$$K = \left( K_p + K_i \frac{1}{s} + K_d s \right) \quad (4.19)$$

where,

$$K_p = \rho G^{-1}(jw_b),$$

$$K_i = \varepsilon G^{-1}(jw_b), \quad K_d = \delta G^{-1}(jw_b),$$

where  $\rho$ ,  $\varepsilon$  and  $\delta$  are scalar tuning parameters. The  $G^{-1}(jw_b)$  will produce a complex gain. In order to realize such a controller, it is necessary to employ a real approximation for  $G^{-1}(jw_b)$  so that:

$$J(K, \Theta) = (G(jw_b)K - e^{j\Theta})^T (G(jw_b)K - e^{j\Theta}),$$

$$\Theta = \text{diag}(\theta_1, \dots, \theta_n) \quad (4.20)$$

Therefore, in this case, we try to create nearly-decoupled unity gain open-loop transfer function from a coupled transfer function matrix. Thus, the product of  $G(jw_b)$  and  $K$  will be as close to diagonal matrix as possible with unity magnitude elements.

---

Clearly, the constant  $K$  in (20) applies only to proportional mode. Similar expressions must also be created for the integral and derivative terms.

#### 4.6.4 Combined method

In this method, we will confine our discussion to control systems that require zero offset and to controller tuning constant values that provide good performance over a reasonable range of operating conditions. The extension of three studied controller tuning methods is to combine all together;

$$K_p = \rho G^{-1}(jw_b), K_i = \varepsilon G^{-1}(0), K_d = \delta(CB)^{-1} \quad (4.21)$$

The proposed method is try to use Maciejowski proportional mode in  $K_p$ . The approach introduced here is to provide great insight into the influence of the frequency of the input changes on the effectiveness of controller. Also, a more complex disturbance can often be well presented by a combination of sines, Kraniuskas, P (1992). Thus, in proportional term with Maciejowski method can produce decoupling through a large range of frequencies by a particular choose of the right bandwidth. The system is diagonalised in a wide frequencies range and try to eliminate interaction around the bandwidth, P. Martin and R.Katebi (2005). Meanwhile, the steady-state gain inverse is of important at zero frequencies where  $K_i$  term is use to produce complete decoupling and to ensures zero steady-state offset. In the derivative mode, the proportional term in Penttinen–Koivo method is then used to prevent undesirable high-frequency variation in control variables, for example, attempt to remove coupling at very high frequencies. The motivation is each gain is design to suit each PID term in frequency domain characteristics.

---

## 4.7 RESULTS AND DISCUSSIONS

In this section, the performance of the multivariable PID controller using the four different tuning techniques are evaluated.

### 4.7.1 Parameterization

The controllers resulting from each of the four design techniques were tuned to the best of the author's abilities.

*Davison method:* In this method, the closed loop system with satisfactory closed-loop performance for  $0 < \varepsilon \leq 1$  with zero steady- state error. Letting  $\varepsilon = 1$  , this yield:

$$K = \frac{K_i}{s} = \varepsilon \frac{G(0)^{-1}}{s} = \frac{1}{s} \begin{bmatrix} 0.0014 & 0.0033 \\ 0.5971 & 28.67 \end{bmatrix} \quad (4.22)$$

By reducing the value of  $\varepsilon$  would increased the phase lag which is the system is coupled at low frequencies. On the other hand, increasing  $\varepsilon$  would lead to oscillation. The reason is that only integral action is included in this design, therefore is of no use in this study.

*Penttinen-Koivo method:* This method is essentially an extension of Davison method and is altered slightly to get a proper tuning at both low and high frequencies.

$$K = K_p + \frac{K_i}{s} = \rho(CB)^{-1} + \varepsilon \frac{G(0)^{-1}}{s} \\ = \begin{bmatrix} 0.0075 & 0 \\ 0 & 14.2980 \end{bmatrix} + \frac{1}{s} \begin{bmatrix} 0.0014 & 0.0033 \\ 0.5971 & 28.67 \end{bmatrix} \quad (4.23)$$

Small values of  $\varepsilon$  produces undesirable interactions at low frequencies. Therefore, a non-zero value of  $\varepsilon$  will be used to produce steady-state decoupling. The initial tuning for  $\varepsilon = 1.125$  and  $\rho$  is set to 1.

---

*Maciejowski method:* The initial selection of bandwidth was 0.02 rad/s. This value corresponds to the lowest process settling time. Since  $K$  must be a real gain:

$$K \approx G((j0.02))^{-1} = \begin{bmatrix} 0.0021 & 0.0033 \\ 1.2955 & 28.6797 \end{bmatrix} \quad (4.24)$$

When  $\rho = 1$ , the integral action scaling may take values of  $0 < \varepsilon < 3.12$  to produce acceptable closed-loop response. Thus,  $\varepsilon$  is set to 0.312 to achieve the minimum settling time for the closed-loop system.

*Combined Method:* The closed-loop system is stable with less oscillation for  $0 \leq \varepsilon \leq 3.57$ . Therefore, let  $\rho = 1$ , and  $\varepsilon = 0.357$  for a starting value of controller tuning.

$$K = K_p + \frac{K_i}{s} = \rho K + \varepsilon \frac{G(0)^{-1}}{s} \quad (4.25)$$

#### 4.7.2 Frequency analysis

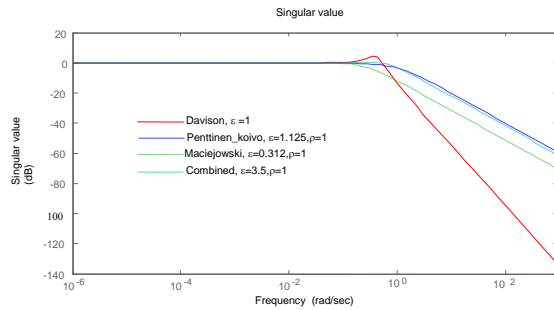
The singular value Bode plot of the closed-loop transfer function matrices  $T(s)$  play an important role in the robust multivariable design. In general the complimentary sensitivity function,  $T(s)$  is defined as:

$$T = GK(I + GK)^{-1} = I - S, \quad S = (I + GK)^{-1} \quad (4.26)$$

As shown in the Figure 4.5, which shows the complimentary sensitivity function,  $T(s)$  for the four different methods, the control system would provide good set point tracking (*i.e.* an amplitude ratio close to 1.0) for a wide range of the frequencies. The resonant peak occurred at intermediate frequencies for the Davison method, proving that the transient response is more oscillatory in comparison with the other methods. The Penttinen-Koivo method and the combined method have similar responses. The complimentary sensitivity function

---

corresponding to the method proposed by Maciejowski indicates that the method may need to be well-tuned to be considered adequately robust for this system.



**Figure 4.5** System sensitivity

To demonstrate the effectiveness of the four controller design techniques studied in this paper a series of simulation experiments were carried out. For each of the methods the following simulation scenarios were investigated:

- A step change (at t=10h) in the set-point for the substrate concentration, from 41 mg/l to 51 mg/l.
- A step change (at t=10h) in the set-point for the dissolved oxygen concentration, from 6 mg/l to 4 mg/l.

Simulation results were obtained using Simulink. The initial conditions and parameters used during these simulations are presented in Tables 4.2 and 4.3.

**Table 4.2** Initial conditions

$X(0) = 215\text{mg/l}$	$X_r(0) = 400\text{mg/l}$
$S(0) = 55\text{mg/l}$	$S_{in} = 200\text{mg/l}$
$C(0) = 6\text{mg/l}$	$C_{in} = 0.5\text{mg/l}$

**Table 4.3 Parameter values**

---

$\beta = 0.2$	$K_c = 2\text{mg/l}$
$r = 0.6$	$K_s = 100\text{mg/l}$
$\alpha = 0.018$	$K_o = 0.5$
$Y = 0.65$	$C_s = 0.5\text{mg/l}$
$\mu_{\max} = 0.15\text{h}^{-1}$	

---

The results from the simulation experiments are shown in Figure 4.6. From the simulation graphs, it can be seen that the Penttinen-Koivo method and the proposed combined method tend to exhibit somewhat faster responses than the controllers resulting from the other design techniques. The Penttinen-Koivo method furthermore exhibit only small overshoots. The combined method provides slightly faster responses to set-point changes, with only small overshoots. Proportional control action is evident for both the combined method and the Penttinen-Koivo method. The response corresponding to the Davison method is fairly oscillatory, with typically large overshoots.



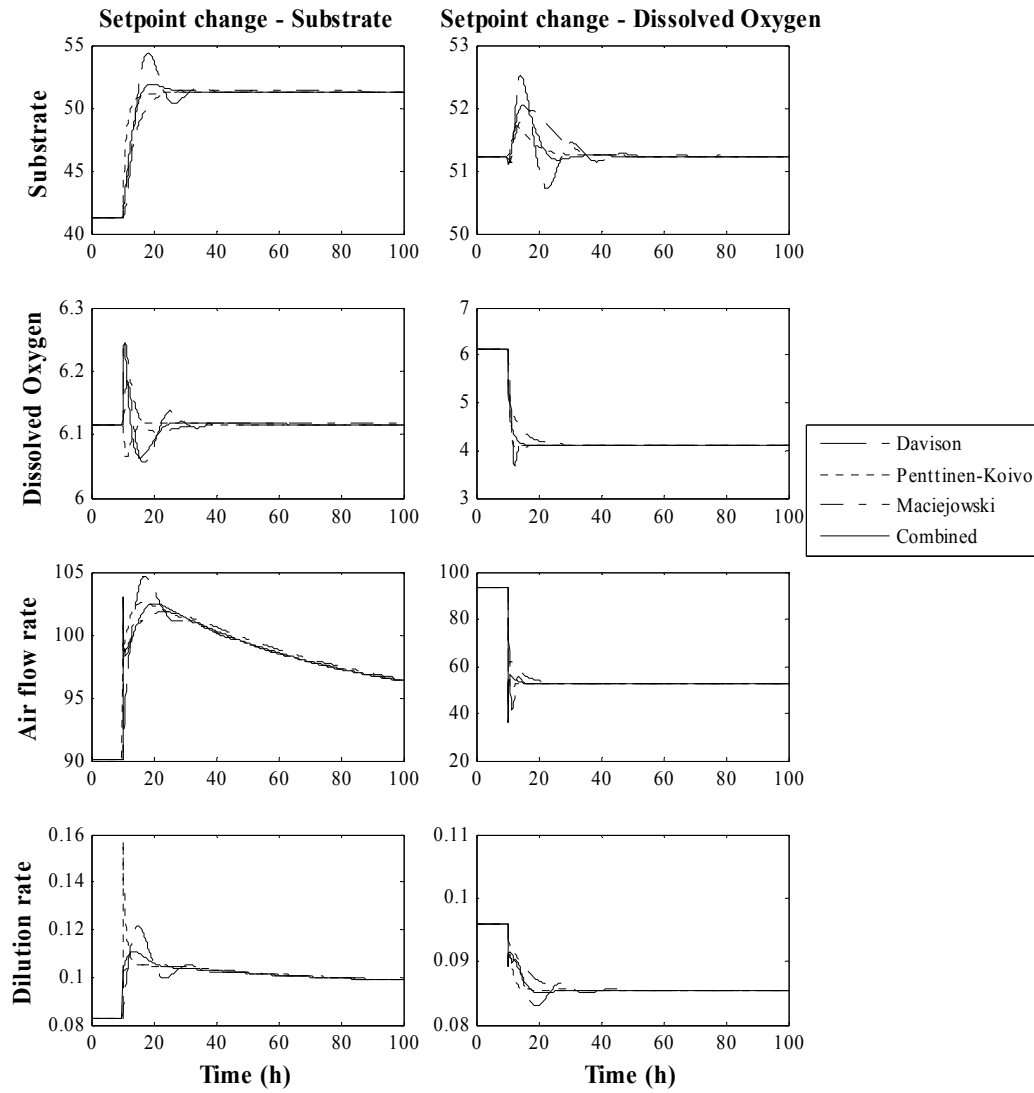


Figure 4.6. Dynamic response to set-point changes.

The response from the controller resulting from the Maciejowski method generally provided for rapid closed loop rise-times. However, strong integral action was difficult to obtain. The decoupling capabilities resulting from the four design techniques were generally good. The Davison method provided the weakest decoupling capability, while the Penttinen-Koivo method provided the strongest.

In summary, this work has presented a comparison between four multivariable PID tuning techniques, and explored their suitability as design techniques for multivariable control of an activated sludge process. The performance of the control techniques have been demonstrated using simulations, which their responses to step changes in the plant's set-points.

### **Acknowledgements**

The work has been supported financially by Universiti Teknologi Malaysia. This support is gratefully acknowledged.

### **REFERENCES**

- [1] Metcalf, Eddy and eds. (1991). *Wastewater Engineering: Treatment, Disposal and Reuse*. McGraw-Hill International Editions, Singapore.
  - [2] Nejjari F., BenYoussef C., Benhammou A, and Dahhou B., "Procedure for state and parameter estimation of a biological wastewater treatment", CESA'96 IMACS, Volume 1, pp 238-243, Lille France, July 9-12, (1996).
-

- [3] Davison, E., "Multivariable Tuning Regulator", IEEE Transaction on Automatic Control, Volume 21, Number 1, pp35-47. (1976).
- [4] Penttinen, J. and Koivo, N.H.(1980), Multivariable tuning regulators for unknown systems, Automatica, Volume 16, pp. 393-398.
- [5] Maciejowski, J. M. , "Multivariable Feedback Design", 1<sup>st</sup> edition, Addison Wesley, Wokingham, England. (1989).
- [6] Kraniuskas, P, "Transforms in Signals and Systems". Addison-Wesley Publishing Company (1992).
- [7] F.G.SHINSKEY. "Controlling Multivariable Processes", Instrument Society of America, Research Triangle Park, NC, (1981).
- [8] P. Martin and R.Katebi, "Multivariable PID Tuning of Dynamic Ship Positioning Control System", Journal of Marine Engineering and Technology, Number A7, pp 11-24. (2005).
-