PROPORTIONAL INTEGRAL SLIDING MODE CONTROL OF A TWO-WHEELED BALANCING ROBOT

MOHD FAIRUS BIN ABDOLLAH

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Universiti Teknologi Malaysia

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ABSTRACT

The objectives of this thesis are to formulate a complete mathematical model of a two-wheeled balancing robot and to control this robot focusing only on balancing using Proportional Integral Sliding Mode Control (PISMC) approach. This robot which exhibits a nonlinear and unstable system dynamics will be modelled based on the inverted pendulum theory. Three types of disturbances will be exerted to the robot of which one of them is applied to the centre gravity of the robot whereas the other two are applied to the centre of both right and left wheels of the robot. The research work is undertaken in the following development stages. The development of the mathematical model of the robot, the design of the PISMC and finally the comparison results with the Statefeedback controller using MATLAB/Simulink as its platform. Through this comparison, it can be shown that the PISMC gives better performance as compared to the Statefeedback controller.

ABSTRAK

Tesis ini bertujuan untuk memformulasikan model matematik lengkap bagi robot pengimbang dua-roda dan juga bertujuan untuk mengawal robot ini yang berfokuskan hanya kepada imbangan dengan menggunakan pendekatan Pengawal Mod Gelincir Berkadaran-Kamiran. Robot ini yang mempamerkan sistem dinamik yang tidak linear dan tidak stabil akan dimodelkan berdasarkan kepada teori bandul tersongsang. Terdapat tiga jenis gangguan yang akan dikenakan ke atas robot tersebut yang mana salah satu daripadanya dikenakan kepada pusat graviti robot manakala dua yang lainnya dikenakan kepada kedua-dua kiri dan kanan pusat roda robot tersebut. Kajian penyelidikan ini telah dibahagikan kepada beberapa peringkat berikut. Pembangunan model matematik bagi robot tersebut, mereka bentuk Pengawal Mod Gelincir Berkadaran-Kamiran dan akhirnya membuat perbandingan dari sudut keputusan simulasi dengan pengawal suapbalik keadaan dengan menggunakan MATLAB/Simulink sebagai pelantar. Melalui perbandingan ini, Pengawal Mod Gelincir Berkadaran-Kamiran telah dapat memberikan prestasi yang lebih baik berbanding dengan pengawal suapbalik keadaan.

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LIST OF SYMBOLS

SYMBOL **DESCRIPTION**

1. Uppercase

$A(x,t)$	-	6×6 system matrix of the two-wheeled balancing robot
$B(x,t)$	-	6×2 input matrix of the two-wheeled balancing robot
C_L	-	Torque of left DC motor
C_R	-	Torque of right DC motor
D	-	Distance between the contact patches of the wheels
$F(x,t)$	-	6×1 uncertainty matrix of the two-wheeled balancing robot
H_R	-	x-component of reaction force between the left wheel and the chassis
H_R	-	x-component of reaction force between the right wheel and the chassis
H_{tL}	-	Friction force between left wheel and the ground
H_{tR}	-	Friction force between right wheel and the ground
J_{aR}	-	Moment inertia of the right DC motor armature
J_{aL}	-	Moment inertia of the left DC motor armature
J_P	-	Moment inertia of the robot chassis
$J_{p\delta}$	-	Chassis inertia during rotation
J_{wL}	-	Moment inertia of left wheel

J_{wR}	-	Moment inertia of right wheel
L	-	Self inductance of the armature winding
M_P	-	Mass of the chassis
M_{wR}	-	Mass of right wheel
M_{wL}	-	Mass of left wheel
OS	-	Overshoot
$Q(x,t)$	-	6×3 disturbance matrix of the two-wheeled balancing robot
R	-	Lumped armature winding resistance
T_s	-	Settling time
V_a	-	Applied terminal voltage
V_{aL}	-	Applied terminal voltage for left DC motor
V_{aR}	-	Applied terminal voltage for right DC motor
V_{eR}	-	Back emf of the DC motor
V_L	-	y-component of reaction force between the left wheel and the chassis
V_R	-	y-component of reaction force between the right wheel and the chassis

2. Lowercase

$d(t)$	-	Disturbances of the system
f_{dlL}	-	Disturbance force applied to the center of left wheel
f_{drR}	-	Disturbance force applied to the center of right wheel
f_{dp}	-	Disturbance force applied to the center of gravity of the robot
g	-	Gravity force
i_L	-	Armature current of left DC motor
i_R	-	Armature current of right DC motor
k_{eL}	-	Back emf constant of left DC motor

k_{eR}	-	Back emf constant of right DC motor
k_{fL}	-	Frictional constant of left DC motor
k_{fR}	-	Frictional constant of right DC motor
k_{mL}	-	Torque constant of left DC motor
k_{mR}	-	Torque constant of right DC motor
l	-	Distance between the centre of the wheel and the robots' centre gravity
m	-	Number of input matrix
n	-	Number of order of the system matrix
r	-	Radius of the robots' wheels
$u(t)$	-	Inputs of the system
u_1	-	Control inputs 1 of the system
u_2	-	Control inputs 2 of the system
x	-	Horizontal displacement of the robot
$\dot{x}$	-	Displacement velocity of the robot
$x(t)$	-	State variables of the system

3. Greek symbol

Φ	-	Flux per pole due to the field winding of DC motor
ω_L	-	Speed rotation of left DC motor
ω_R	-	Speed rotation of right DC motor
$\sigma(t)$	-	Switching surface or switching line of sliding mode controller
θ	-	Pendulums' angle
$\dot{\theta}$	-	Pendulum's angular velocity
θ_L	-	Left wheel rotational angle
$\omega_L, \dot{\theta}_L$	-	Left wheel rotational angular velocity

θ_R	-	Right wheel rotational angle
$\omega_R, \dot{\theta}_R$	-	Right wheel rotational angular velocity
δ	-	Rotational angle
$\dot{\delta}$	-	Rotational angular velocity
$\Delta\theta$	-	Small angle from the vertical upward direction of the pendulum
$\%OS$	-	Percent overshoot

LIST OF ABBREVIATIONS

CG	-	Centre Gravity
DC	-	Direct Current
LQR	-	Linear Quadratic Regulator
PID	-	Proportional-Integral-Derivative
PISMC	-	Proportional Integral Sliding Mode Control
SMC	-	Sliding Mode Control
VSC	-	Variable Structure Control
3-DOF	-	Three Degrees of Freedom

CHAPTER 1

INTRODUCTION

1.1 Overview

Nowadays, applications involving robots have gained momentum due to their functionality and reliability when completing certain tasks as compared to human. These advantages have attracted many researchers to dedicate themselves into this area of research. One of the many types of research is the two-wheeled balancing robot.

This two-wheeled balancing robot has the exact behaviour of the inverted pendulum system which is known for its nonlinear and unstable system. The only difference between these two systems is the capability to freely move around for the two-wheeled balancing robot. Basically, the robot consists of two driving wheels which attached on each side of the robot chassis. These two wheels will be controlled by two dc motors that are coupled to a planetary gearbox as their actuator. Just as the inverted pendulum, this robot will balance itself by controlling the rotation of the wheels or in other words the rotation of the dc motors. This can be achieved by controlling the amount of voltages that applied to the dc motor.

This robot has three degrees of freedom (3-DOF) where it is able to rotate around the z-axis or pitch, a movement described by the angle θ with the corresponding angular velocity $\dot{\theta}$. The linear movement of the robot is characterized by the position x and the speed $\dot{x}$. Additionally, the vehicle also able rotates around its vertical axis or yaw with the associated angle δ and angular velocity $\dot{\delta}$. As described in Grasser *et al.* (2002), three types of disturbances are applied to the robot. One of the disturbances is applied to the centre gravity of the robot while the other two are applied to the centre of both left and right wheels. These disturbances will indicate the movement made by the driver on his seat as described in the paper.

In this thesis, two references are chosen as its main guidance. Based on these two references, the mathematical model of the two-wheeled balancing robot will be derived and then a Proportional Integral Sliding Mode Control as described in Sam (2004) will be designed in order to control the system.

1.2 Objective

The objectives of this research are as follows:

1. To formulate a complete mathematical model in state-space form of a two-wheeled balancing robot.

2. To control the robot focusing only on balancing using Proportional Integral Sliding Mode Control (PISMC) approach. The theoretical verification of the controller on its stability and reachability will be accomplished by using Lyapunov's second method.

3. To simulate the mathematical model of the two-wheeled balancing robot using MATLAB/Simulink in order to validate the derived controller.

4. To compare the performance of the Proportional Integral Sliding Mode Control with Statefeedback controller.

1.3 Scope of Project

The work undertaken in this project is limited to the following aspects:

1. The mathematical model of the two-wheeled balancing robot is derived based on Grasser *et al.* (2002) and Ooi (2003).

2. The Proportional Integral Sliding Mode Control (PISMC) will be designed as described in Sam (2004).

3. Simulation work using MATLAB/Simulink as a platform to prove the effectiveness of the designed controller.

4. Comparison results between the Proportional Integral Sliding Mode Control with Statefeedback controller.

1.4 Research Methodology

The research work is undertaken in the following four developmental stages:

1. The development of mathematical model for two-wheeled balancing robot.

2. The design of a Proportional Integral Sliding Mode Control.

3. Perform simulation for the Proportional Integral Sliding Mode Control in controlling the two-wheeled balancing robot.

4. Compare of performances between the Proportional Integral Sliding Mode Control with the Statefeedback controller.

1.5 Literature Review

The research on two-wheeled balancing robot has gained momentum over the last decade due to the nonlinear and unstable dynamics system. Various control strategies had been proposed by numerous researchers to control the two-wheeled balancing robot such that the robot able to balance itself. In this chapter, the previously done control approaches will be briefly discussed.

Grasser *et al.* (2002) had built JOE, a prototype of a revolutionary two-wheeled vehicle. The main objective of this vehicle is to balance its driver on two coaxial wheels. Each of the coaxial wheels is coupled to a dc motor. Due to its configuration with two coaxial wheels the vehicle is able to do stationary U-turns. The vehicle is controlled by applying a torque to the corresponding wheels. A control system, made up of two decoupled state space controllers, pilots the motors in order to ensure the system will always stay in equilibrium. The control system that was implemented into this system is a Statefeedback controller using pole-placement method. Three types of disturbances that indicate the movement made by the driver were also included into the system.

As a final year project student Ooi (2003) had discussed the development of a two-wheeled balancing autonomous robot based on the inverted pendulum model. The system is built as a platform to investigate the use of a Kalman filter for sensor fusion. The discussion examines the suitability and evaluates the performance of a Linear Quadratic Regulator (LQR) and a pole-placement controller in balancing the system. The LQR controller uses several weighting matrix to obtain the appropriate control force to be applied to the system while the pole-placement requires the poles of the system to be placed to guarantee stability. As the robot will be moving about on a surface, a Proportional-Integral-Derivative (PID) controller is implemented to control the trajectory of the robot.

Salerno and Angeles (2004) have made a report on a control of semi-autonomous two-wheeled mobile robots undergoing large variations of payload. These robots which being underactuated, poses some challenges when resorting to a simple controller. The latter being a linear controller, its robustness with respect to model uncertainty was investigated. A time-domain analysis was conducted in order to investigate the robustness of the foregoing controller with respect to parametric and unmodeled dynamics uncertainty. In this paper, a controller which is designed by a dominant second-order pole-technique is proved to be fragile with respect to unmodeled dynamics uncertainty. Thus, a novel controller based on Linear Quadratic Regulator (LQR) is designed for robustness with respect to the model-uncertainty.

Pathak *et al.* (2005) has analyzed the dynamic model of a wheeled inverted pendulum from a controllability and feedback point of view. The dynamic model of the underactuated system is derived with respect to the wheel motor torques as its inputs by taking the nonholonomic no-slip constraints into considerations. The partial feedback linearization of the system is obtained based on the results of the accessibility condition and the maximum relative degree of the system. The resulting equations are used to design two novel controllers of which one of them is a two-level velocity controller used for tracking vehicle orientation and heading speed set-points, while controlling the vehicle pitch. The other controller which is also a two-level controller used to stabilize the vehicle's position to the desired set-point, while ensuring the pitch is bounded between specified limits.

Shiroma *et al.* (1996) have discussed the cooperative transportation of an object by two or more wheeled inverted pendulum robot. This task requires each robot to exert an appropriate force to support and move the object, to move along the object and to maintain its attitude. A Statefeedback controller using pole-placement method is implemented into this system in order for the robots to cooperate among themselves.

1.6 Thesis Layout

Chapter 2 deals with the mathematical modelling of the two-wheeled balancing robot. The formulation of the integrated dynamic model of this robot will be presented in detail. The modelling will be separated into three subsystems which are the modelling of the dc motor, the wheels of the robot and the chassis of the robot. These obtained equations will be integrated to form a complete state space representation of the system.

Chapter 3 presents the controller design using Proportional Integral Sliding Mode Control. This controller will be designed based on the Ph.D. Thesis of Sam (2004).

Chapter 4 discusses the simulation results. The performance of the Proportional Integral Sliding Mode Controller is evaluated by simulation study using MATLAB/Simulink. For comparison purposes, the simulation study of Statefeedback controller using pole-placement technique is also presented.

Chapter 5 summarizes the works undertaken. Recommendations for future work of this project are presented at the end of the chapter.