# MODELLING AND SIMULATION OF MAGNETICALLY SUSPENDED BALANCE BEAM SYSTEM

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## ABSTRACT

Magnetically Suspended Balance Beam (MSBB) system is a balancing system that uses two magnetic coils to balance the beam. The use of magnetic coils, introduce the nonlinearities which is very difficult to control with conventional controllers such as Proportional Integral Derivative (PID) controller. To overcome this problem various controllers have been put into trial. In this project, Integral Sliding Mode Controller (ISMC) is used to control the balance beam. The integral compensator is used for achieving a zero steady-state error under an external step disturbance force. Lastly, using a computer simulation, the performance of designed ISMC is evaluated and compared with pole placement technique. The result shows that, ISMC can perform better compare to pole placement technique in controlling the balanced beam.

# ABSTRAK

Sistem Gantungan Rasuk Secara Magnetik (MSBB) adalah sejenis sistem gantungan yang menggunakan dua lilitan gegelung magnetik untuk mengimbangi rasuk yang hendak dikawal tadi. Oleh kerana sistem ini menggunakan lilitan gegelung magnetik, maka akan wujudlah masalah ketidaklinearan pada keseluruhan sistem ini. Oleh sebab masalah ini timbul, maka system ini sukar untuk dikawal dengan menggunakan pengawal-pengawal yang agak konventional, contohnya Pengawal Kamiran Berkadaran (PID). Untuk mengatasi masalah ini, berbagai jenis pengawal pernah dicuba. Dalam projek ini, pengawal jenis Pengawal Ragam Menggelungsur-Kamiran (ISMC) digunakan untuk mengawal rasuk tersebut. Penggunaan pemampas kamiran adalah untuk menghasilkan keadaan yang bebas dari ralat keadaan mantap walaupun rasuk tadi telah diganggu dengan gangguan jenis fungsi langkah. Akhir sekali, dengan menggunakan simulasi komputer, prestasi ISMC dinilai dan bandingkan dengan teknik pengaturan kutub. Melalui keputusan yang diperolehi, adalah didapati ISMC telah menunjukkan prestasi yang lebih baik daripada teknik pengaturan kutub dalam mengawal rasuk tersebut.

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# LIST OF SYMBOLS

| | | |
|---|---|---|
| $A$ | - | N x N system matrix for the Magnetically Suspended Balanced Beam |
| $B$ | - | N x 1 input matrix for the Magnetically Suspended Balanced Beam |
| $C$ | - | 1 x N ouput matrix for the Magnetically Suspended Balanced Beam |
| $E$ | - | N x 1 disturbance matrix for the Magnetically Suspended Balanced Beam |
| $r$ | - | Set point (radian) |
| $x$ | - | State vectors |
| $f_d$ | - | Disturbance of the system (N) |
| $\theta$ | - | Gap displacement (radian) |
| $\dot{\theta}$ | - | Velocity of gap displacement(radian/second) |
| $\ddot{\theta}$ | - | Acceleration of gap displacement (radian/seconds$^2$) |
| $i'$ | - | Ovearall instantaneous current (A) |
| $i_o$ | - | Steady current (A) |
| $e`$ | - | Control voltage (V) |
| $K_x$ | - | Magnetic bearing open loop stiffness (N/m) |
| $K_i$ | - | Actuator current gain (N/A) |
| $L_a$ | - | Half bearing span (m) |
| $J$ | - | Mass moment of Inertia about the pivot point (kg/m$^2$) |
| $g_0$ | - | Steady gap (m) |
| $\sigma$ | - | Sliding surface |
| $\rho$ & $\delta$ | - | Design parameter |

## LIST OF ABBREVIATIONS

| | |
|---|---|
| MSBB | Magnetically Suspended Balance Beam |
| SMC | Sliding Mode Controller |
| ISMC | Integral Sliding Mode Controller |
| PID | Proportional-Integral-Derivative |
| sgn | Signum |

# CHAPTER I

# INTRODUCTION

## 1.1 Introduction

The magnetically suspended balance beam (MSBB) is a balancing system that used two magnetic coils to balance the beam. These two magnetic coils are placed at each end of the beam. One at the right hand side and one in the left hand side. It can be easily described as a small see-saw.

The whole system is needed to be modeled first by using a state space equation. It has been found that this system is having a non linear model. From this nonlinear model, the linearization process has to be done. After the linearized model has been acquired, the next task to do is to control the beam until it become stable.

In this project, the main task is to control the displacement angle of the beam. If the displacement angle is equal to the set point, it can be concluded that the designed controller is successful in controlling the angle and make the beam become stable. In this project, there are 2 types of controllers that have been used. First, it is the pole placement technique and another one is Integral Sliding Mode Controller (ISMC).

After the model has been acquired, the pole placement technique is first to be used to control the beam. Then followed by ISMC. In both evaluations, the presence of step disturbance is also included in the system.

The performance of both controllers in controlling the MSSB will be evaluated through extensive computer simulation using MATLAB/SIMULINK

## 1.2 Objective

The objectives of this project are as follows:

1. To formulate the complete state-space presentation of Magnetically Suspended Balance Beam (MSBB).

2. To design a controller based on the Integral Sliding Mode Control (ISMC) technique.

3. To compare the performance of the ISMC with the pole placement technique via simulation result.

## 1.3 Scope of Project

The work undertaken in this project is limited to the following aspects:

1. The nonlinear mathematical model of MSBB based on (Baloh *et al*, 1999 ), (Lee *et al,2001*) and (Hu *et al*, 2002) of MSBB is studied and the linear mathematical model is derived afterwards

2. The controller based on pole placement technique to control the MSBB is designed.

3. The Integral Sliding Mode Controller (ISMC) is designed as described in (Lee *et al,2001*).

4. Simulation work using MATLAB/SIMULINK as a platform to prove the effectiveness of the both designed controller.

5. Comparative study between the ISMC and pole placement technique will be done.

## 1.4 Research Methodology

The research work undertaken in the following five development stages:

1. The development of linear mathematical model for MSBB.

2. The design of controller base on pole placement technique

3. The design of Integral Sliding Mode Controller

4. Perform simulation using MATLAB/SIMULINK for pole placement and ISMC

5. Comparative study of both controllers is done

## 1.5 Literature Review

An artificial heart pump which employs a hybrid bearing system has five degrees of freedom to be controlled. However, by the proper use of permanent magnets it is possible to reduce the degree of freedom to be controlled to one degree of freedom. Consequently the mathematical model of the artificial heart pump which is suspended by a hybrid system and has one degree of freedom to be controlled is schematically the same as the proposed single-input–single-output (SISO) balance beam mathematical model.

As magnetic bearing applications become more complicated, the need for accurate models of the controlled bearing systems becomes more important. (Baloh *et al*, 1999) in his initial research use an adaptive estimation to identify unknown parameters and disturbances for a simple one dimensional magnetic bearing system.

(Lee *et al*. ,2001) in his latest development of a proper controller, use sliding mode control method to reduce the power consumption, a critical problem of the artificial heart pump. From this point of view, sliding mode controller gives a significant contribution in the test of magnetic bearing using a continuous function (Chern and Wu, 1992). It is experimentally evaluated for the reduction of chattering and power consumption, when studying control of the balance beam - a benchmark system for magnetic bearings. He first derived the mathematical model of the magnetically suspended balance beam, and then showed how to design a switching surface to guarantee the desired dynamic behavior of the nominal system. He also had employed an experimental evaluation rather than a complicated mathematical analysis to show the validation of the proposed continuous function. This work is a part of his long term project with his team in developing active magnetic bearings for artificial heart pumps and other applications. The balance beam used in the study is a benchmark system.

## 1.4 Layout of Thesis

This thesis contains six chapters. Chapter II contains a brief introduction of MSBB. In this chapter also, the mathematical model, which is a nonlinear model of the MSBB is presented. The linear mathematical model of the system is derived and then transforms into the state space representations.

Chapter III presents the brief introduction of pole placement technique. Then the controller is designed using pole placement technique plus integral control. The use of integral control is to eliminate the steady-state error. The controller is designed base on two cases. They are CASE NO.1 and CASE NO.2.

For CASE NO.1, the percentage overshoot, % OS and settling time, $T_s$ that have been used are:

i. %OS = 10%
ii. $T_s$ =0.1 second.

FOR CASE NO.2, the percentage overshoot, % OS and settling time, $T_s$ that have been used are:

i. %OS = 10%
ii. $T_s$=1 second.

After all of the feedback vectors, $K$ for both cases are calculated, the simulation diagram can be constructed. All of the simulation diagrams for pole placement technique are presented in this chapter.

Chapter IV presents the brief introduction of sliding mode controller (SMC). Then the controller is designed using the ISMC technique. The use of integral control is to eliminate the steady-state error. The sliding surface $\sigma = Sx$ is introduced. Then, using the MATLAB programming, the values of $S$ are calculated. Same as in Chapter III, the ISMC is also designed base on two cases. They are CASE NO.3 and CASE NO.4.

For CASE NO.3, the percentage overshoot, % OS and settling time, $T_s$ that have been used are:

iii. %OS = 10%
iv. $T_s$=0.1 second.

FOR CASE NO.4, the percentage overshoot, % OS and settling time, $T_s$ that have been used are:

iii. %OS = 10%
iv. $T_s$=1 second.

After all of the values of *S* for both cases are obtained, the simulation diagram can be constructed. All of the simulation diagrams for ISMC are presented in this chapter.

Chapter V presents both the results of pole placement technique and ISMC. For every cases there will be two graph presented. The first one is a gap displacement's graph and another one is an input voltage's graph. At the end of this chapter, the comparison between the pole placement technique and ISMC is done.

Chapter VI presents the analysis and discussions about the results that had been obtained in Chapter V. There are six sub chapters in this chapter.

Chapter VII conclude the work undertaken, suggestions for future work are also presented in this chapter.