TIME-FREQUENCY ANALYSIS OF FSK DIGITAL MODULATION SIGNALS USING THE SMOOTH LAG-WINDOWED WIGNER-VILLE DISTRIBUTION

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ABSTRACT

Digital modulation signals such as FSK are time-varying signals that can be represented in the time-frequency representation. The smooth lag-windowed Wigner-Ville distribution (SLWWVD) is proposed as a method to obtain accurate time-frequency representation of FSK signals used in HF (High Frequency) radio communications. Unlike existing time-frequency distributions such as the windowed Wigner-Ville distribution (WWVD) or the smooth windowed Wigner-Ville distribution (SWWVD), the proposed time-frequency distribution does not require priori knowledge of the signal modulation parameters such as the bit-duration and subcarrier frequencies. The algorithm generates the time-frequency representation directly based on the signal characteristics. Comparison based on mainlobe width and signal-to-interference ratio shows the SLWWVD generate accurate time-frequency representation for simulated and actual communication signals.

1. INTRODUCTION

The use of time-frequency distributions in radio communications are described in [1]-[4]. In jammer excision for spread spectrum communication application, the extended discrete-time Wigner distribution [1] is used to characterize the jammer characteristics. Similar work described in [2] applied the discrete evolutionary transform that is based on Gabor transform. The smooth pseudo Wigner-Ville distribution is used to evaluate the quality assessment of UMTS signals [3]. Its main objective is to simplify testing procedures with minimal equipment. Time-frequency distribution is used in HF spectrum monitoring specifically in signal classification [4]. The modulation parameters are used as inputs to a classifier network.

This paper describes the application of time-frequency analysis in spectrum monitoring. The objective is to use the smooth lag-windowed Wigner-Ville distribution (SLWWVD) to obtain accurate time-frequency representation for FSK (Frequency Shift-Keying) digital modulation signals with minimum interference due to cross terms. Other signals such as ASK (Amplitude Shift Keying) not considered since the problem of interference due to cross terms is less significant compared to FSK signals [5]. The signal parameters that estimated from the time-frequency representation can be used for classification purposes.

2. SIGNAL MODEL

Since data is modulated in frequency for FSK signal, the actual signal is time-varying that can be modeled as

$$z(t) = c\exp(j2\pi \int_{-\infty}^{t} f_i(\lambda)d\lambda + \phi) + n(t) \qquad (1)$$

where c is the amplitude, $f_i(t)$ is the instantaneous frequency, ϕ is the phase and $n(t)$ represents the interference due to noise.

Within a bit duration T_b, a binary bit '1' and '0' are assigned frequencies f_1 and f_0 respectively. All the signals used for analysis are defined in Table 1. Actual HF data communication signals are included to demonstrate the practicality and limitations of the time-frequency distributions. An example of such signal is FSK4 where the time domain representation and power spectrum are shown in Figure 1. In general, the power spectrum can represent the frequency contents but not the time instant they occur.

Table 1 The various FSK signals that are described within the bit duration. [Signals FSK0 to FSK2 are simulated signals while the rest are actual HF data communication signals]

Signal Name	Binary data	Freq (Hz)	Bit-rate (bits/sec)	Bit duration (msec)
FSK0	1	1800	75	13.33
	0	1400		
FSK1	1	1800	50	20
	0	1400		
FSK2	1	2100	100	10
	0	1900		
FSK3	1	2360	75	13.33
	0	1520		
FSK4	1	2265	50	20
	0	1430		

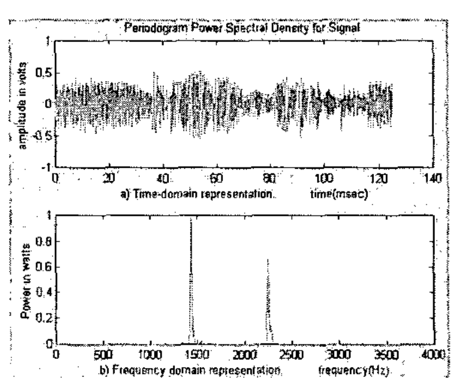

Figure 1 Time and frequency representation of signal FSK4.

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3. BILINEAR TIME-FREQUENCY DISTRIBUTION

The generalized formulation for bilinear time-frequency distributions is defined as [6]

$$\rho_z(t,f)=\int_{-\infty}^{\infty} g(t,\tau)\underset{(t)}{*}\left[z(t+\tau/2)z^*(t-\tau/2)\right]\exp(-j2\pi f\tau)d\tau \qquad (2)$$

where $g(t,\tau)$ is the time-lag kernel function and $z(t)$ is the signal in analytical form. In Equation (2), the product $z(t+\tau/2)z^*(t-\tau/2)$ is the bilinear product. Based on this formulation, the Wigner-Ville distribution (WVD) and windowed Wigner-Ville distribution (WWVD) are

$$W_z(t,f)=\int_{-\infty}^{\infty} z(t+\tau/2)z^*(t-\tau/2)\exp(-j2\pi f\tau)d\tau \qquad (3)$$

$$W_{z,w}(t,f)=\int_{-\infty}^{\infty} g(\tau)z(t+\tau/2)z^*(t-\tau/2)\exp(-j2\pi f\tau)d\tau \qquad (4)$$

The window function in the WWVD is defined such that $g(\tau)\neq 0$ for the interval $-\tau_g/2<\tau<\tau_g/2$. For this application, the window length is chosen such that it is less than the bit-duration [5].

By making the time-lag kernel separable

$$g(t,\tau)=h(t)g(\tau),\ g(\tau)\neq 0,\ -\tau_g/2<\tau<\tau_g/2,$$
$$h(t)\neq 0,\ \text{for } 0<t<T_h \qquad (5)$$

the smooth windowed Wigner-Ville distribution (SWWVD) [5] is defined as

$$W_{z,sw}(t,f)=\int_{-\infty}^{\infty} h(t)\underset{(t)}{*}\left[z(t+\tau/2)z^*(t-\tau/2)\right]g(\tau)$$
$$\times\exp(-j2\pi f\tau)d\tau \qquad (6)$$

where $g(\tau)$ windows the bilinear product in lag and $h(t)$ is the time-smooth (TS) function. Further accuracy in the time-frequency representation is possible by setting the length T_h in the TS function. If a raised cosine pulse is used, the length of the TS function can be determined by [5]

$$T_h\geq\frac{3}{2(f_1-f_0)} \qquad (7)$$

4. LAG-WINDOWED WIGNER-VILLE DISTRIBUTION

Both the WWVD and SWWVD require priori knowledge of the signal parameters such as the bit-duration and frequency to obtained accurate time-frequency representation. Since this is not known in practice, the lag-windowed Wigner-Ville distribution (LWWVD) [7] was developed to generate time-frequency representations directly based on the signal characteristics. Further improvement in the time-frequency representation by smoothing the LWWVD results in the smooth LWWVD (SLWWVD). Both distributions are defined as

$$W_{z,lw}(t,f)=\int_{-\infty}^{\infty} g(t,\tau)z(t+\tau/2)z^*(t-\tau/2)\exp(-j2\pi f\tau)d\tau \qquad (8)$$

$$W_{z,slw}(t,f)=\int_{-\infty}^{\infty} h(t)\underset{(t)}{*}g(t,\tau)z(t+\tau/2)z^*(t-\tau/2)$$
$$\times\exp(-j2\pi f\tau)d\tau \qquad (9)$$

where $g(t,\tau)$ is the time-dependent window function defined over the lag interval $-\tau_{win}(t)/2<\tau<\tau_{win}(t)/2$ and $h(t)$ is the time smooth function similar to that defined in Equation (5).

Since the signal parameters are unknown, the T_h is estimated based on the peaks of the power spectrum and the window width for the $g(t,\tau)$ from the local lag correlation (LLC) function.

4.1 Power Spectrum Estimation

The power spectrum describes the distribution of signal power in frequency. It can be obtained from the periodogram

$$S_{xx}(f)=\frac{1}{T}\left|\int_{T/2}^{T/2} x(t)\exp(-j2\pi ft)dt\right|^2 \qquad (10)$$

where $x(t)$ is the signal of interest. The signal frequencies f_0 and f_1 can be estimated from the

$$\langle f_0,f_1\rangle=\max[S_{xx}(f)] \qquad (11)$$

Once estimated, the T_h is derived based on Equation (7).

4.2 Local Lag Correlation Function

In the bilinear product, the desired signal terms lie within lag τ=0 while the cross terms lie at larger lag values [7]. The lag variation in the bilinear product is detected by the localized correlation function and is used to estimate the lag window width for all time instants. From the bilinear product definition in Equation (2), the local lag correlation (LLC) function is

$$R_z(t,\lambda)=\int_{-T/2}^{T/2}|w(\tau)|^2K_z(t,\tau)K_z(t,\tau-\lambda)d\tau \qquad (12)$$

where T is the analysis duration, $w(\tau)$ is the analysis window function similar to Equation (5) and λ is the instant of interest in lag. The variation in the bilinear product is reflected by the LLC function

$$|K_z(t,0)|\neq|K_z(t,\lambda)|,\quad \lambda\in\tau, 0\leq\lambda\leq T \qquad (13)$$

Thus, the amplitude of the LLC function can be used to indicate any miscorrelation in lag to estimate the time-dependent window width $\tau_{win}(t)$.

The LLC function is amplitude dependent since the computation is based on the product average of two signals. This is overcome by using the normalized LLC function defined as

$$R_K(t,\lambda)=\frac{\int_{-T/2}^{T/2}|w(\tau)|^2 K_z(t,\tau)K_z(t,\tau-\lambda)d\tau}{\sqrt{E_K(t,0)E_K(t,\lambda)}} \tag{14}$$

where

$$E_K(t,\lambda)=\int_{-T/2}^{T/2}|w(\tau)|^2 K_z(t,\tau-\lambda)d\tau \tag{15}$$

Thus, the possible range of values for the normalized LLC function is

$$0<|R_K(t,\lambda)|<1 \tag{16}$$

The correlation is highest when $|R_K(t,0)|=1$ and is lower elsewhere.

The lag window width $\tau_{win}(t)$ is determined by varying λ such that the following inequality is satisfied

$$|R_K(t,\lambda)|<R_{K,thd}\,|R_K(t,0)| \tag{17}$$

where $R_{K,thd}$ is the correlation threshold level that is chosen as 0.5. It is used to decide the limit if the $R_K(t,\lambda)$ function at a given lag instant λ corresponds to lag variation in the bilinear product. If satisfied, then the instant λ is used as the time-dependent window width $\tau_{win}(t)$.

4.3 Performance Measure

The accuracy of the time-frequency representation is measured based on the mainlobe width (MLW) and the signal-to-interference ratio (SIR). The sharpness of the time-frequency representation is determined by the MLW. For a time-frequency representation, the MLW is estimated for given arbitrary time instant from the frequency difference based on the 50% of the peak value along the frequency axis. Averaging over several time instants will further improve this estimate.

The signal-to-interference ratio (SIR) indicates the amount of interference present in the time-frequency representation with respect to the desired signal energy. For FSK signals, it can be estimated as

$$SIR=10\log_{10}\frac{E_{sig}}{E_{int}} \tag{18}$$

where E_{sig} is the total desired signal energy that is represent within the subcarrier frequencies of the signal and E_{int} is the total energy of the interference that lies elsewhere within the time-frequency representation.

5. RESULTS

The window function $g(\tau)$ and time smooth function $h(t)$ for the WWVD and SWWVD are set based on the knowledge of the individual signal modulation parameters such as the bit-duration and the subcarrier frequencies. However, this is not necessary for the LWWVD and SLWWVD since these functions are automatically set without priori knowledge of the actual signal parameters.

The time-frequency representations for signal FSK4 based on the WWVD, SWWVD, LWWVD and SLWWVD are shown in Figure 2. The chosen signal is an actual FSK digital modulation signal used in HF radio communication that is subjected to multipath fading and interference due to noise. All the frequency components of the signal are represented in all of the time-frequency representations. The presence of cross terms is noticeable for the WWVD and LWWVD at the frequency transition regions. In general, this problem is minimal for the SWWVD and SLWWVD. Except that there is some level of smearing present on the time-frequency representation for the SLWWVD.

The comparisons for the time-frequency representation in terms of the MLW and SIR for all the signals are summarized in Table 2 and 3. For simulated signals FSK0 to FSK2, the MLW is the lowest for the LWWVD and the SLWWVD. However, this is not true for signal FSK3. The presence of smearing that is reflected by a larger MLW is due to the presence of noise in the signal that causes uncertainty in the estimation of the time-dependent window function.

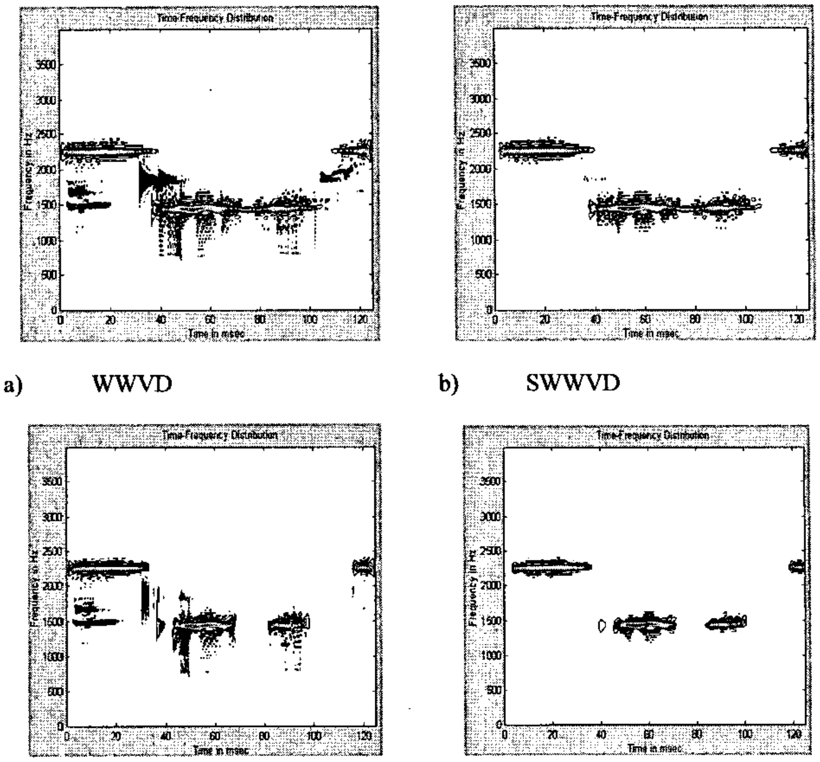
Table 2 Comparison in the mainlobe width of the time-frequency representations.

Signal Name	WWVD	SWWVD	LWWVD	SLWWVD
FSK0	203	203	110	110
FSK1	134	134	86.3	86.3
FSK2	252	252	93.75	93.75
FSK3	190	186	484	512
FSK4	140	140	109.3	109.3

Table 3 Comparison in the signal-to-interference ratio of the time-frequency representations.

Signal Name	WWVD	SWWVD	LWWVD	SLWWVD
FSK0	3.41	13.59	8.32	16.16
FSK1	5.61	15.30	10.47	17.38
FSK2	5.41	8.86	4.5	6.05
FSK3	<0	1.0	<0	3.1467
FSK4	4.3	6.83	5.9	8.92

Based on the time-frequency representations in Figure 2, the SIR shown in Table 3 is relatively higher for both the SWWVD and the SLWWVD for all signals. Thus, the most accurate time-frequency representation can be obtained for FSK signals using the SWWVD and SLWWVD. Since the actual signal parameters is unknown in practice, then the most appropriate time-frequency distribution that can be used to analyze FSK signal is the SLWWVD.



a) WWVD b) SWWVD

c) LWWVD d) SLWWVD

Figure 2 Time-frequency representation signal FSK4 based on the WWVD, SWWVD, LWWVD and SLWWVD.

6. CONCLUSIONS

Time-frequency distribution can be used to analyze FSK digital modulation signals used in HF radio communications. The SLWWVD is designed to generate accurate time-frequency representation for these signals directly based on the signal characteristics with no priori knowledge on the actual signal characteristics. This is an improvement over existing time-frequency distributions such as the WWVD and SWWVD. In addition, comparisons are made with existing time-frequency distributions such as the WWVD, SWWVD and LWWVD in terms of the mainlobe width and the signal-to-interference ratio. In general, the SLWWVD generates accurate time-frequency representations for all the FSK signals both simulated and actual. Thus, the modulation parameters for unknown FSK signals can be estimated directly from the time-frequency representation that can be used as an input to a classifier network.

7. REFERENCES

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