# Analysis and Design of An Unbalanced Input Three Phase Controlled Rectifier

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***Abstract*** Controlled Rectifier like all other power electronics converters are usually designed to work in balanced input condition. Evaluation of real operating conditions, however, shows that this assumption is not true in many cases. It is, therefore, necessary that converter analysis should be carried out considering unbalanced condition.

This paper discusses a novel technique to produce near perfect dc output from a three phase controlled rectifier when the three input phase voltages are amplitude unbalanced. This paper also focuses on the analysis, design and performance characteristics of the three phase controlled rectifier. Finally analytically predicted results are verified by simulation.

## 1. Introduction

Developments in power semiconductors, fast solid state switches and automation in manufacturing process have made possible widespread use of controlled rectifier. The controlled rectifiers are designed to work under balanced input condition. In general, the unbalanced ( phase and amplitude) input voltage produces various problems in the operation of static converters such as signal interference, relay malfunctioning, over voltages and excessive currents as a result of resonance due to harmonic voltages or currents in the network, excessive losses in terms of heat in rotating machines, an increase in distortion of output voltage (dc, in case of rectifier) and errors in induction-type kWh meters. The input voltage may be unbalanced in phase and amplitude. The works so far reported [1]-[4] deal with phase and amplitude unbalance. Some of these methods use symmetrical component technique to find the appropriate switching functions for correcting the phase and amplitude unbalance. Complex mathematical formula are used and their implementation need complicated logic circuits.

This paper focuses on the analysis of a three phase controlled rectifier which produces balanced output voltage when the input is unbalanced in amplitude. This voltage balancing technique [5]-[6] states that the output voltage is balanced when the fundamental components of the switching functions are equal to inverse of the amplitude of the corresponding unbalanced input phase voltages.

## 2. Fundamentals

The basic rectifier operation is shown in Figs. 1-4. Fig.1 shows the basic configuration of the proposed three phase controlled rectifier. Fig.3 shows input/output voltage waveform describing the basic operation of the proposed rectifier.

In particular, the direct multiplication ( Eqn. (1)) of the amplitude unbalanced input phase voltages $V_{an}$, $V_{bn}$ and $V_{cn}$ by the corresponding switching function components $F_1$, $F_2$ and $F_3$ yields the near perfect dc output voltage $V_{dc}$ (Fig.3). Unbalanced input currents are similarly obtained using Eqn.4.

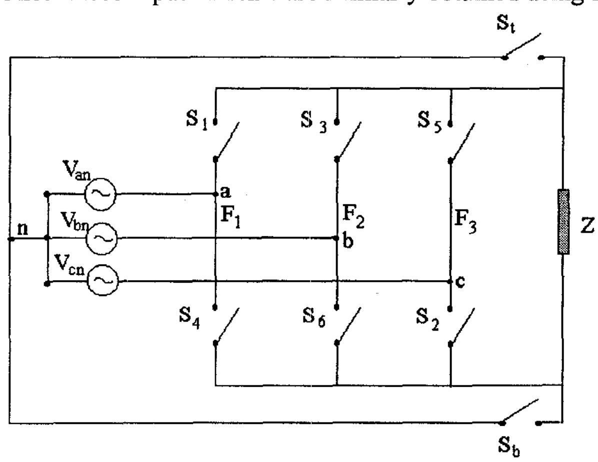


Fig. 1: Proposed three-phase controlled rectifier structure

## 3. Analysis

The theoretical input and output quantities for a three phase controlled rectifier (CR) with amplitude unbalanced input can be derived using spectrum multiplication technique [5] as follows :

$$[V_0(\omega_0 t)] = [F_d(\omega_s t)].[V_i(\omega_i t)]$$

$$= \begin{bmatrix} F_1 & F_2 & F_3 \end{bmatrix} . \begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix}$$

$$= \left[A_1 \cos(\omega_s t) \quad B_1 \cos(\omega_s t - 120^0) \quad C_1 \cos(\omega_s t - 240^0)\right] \cdot \begin{bmatrix} A\cos(\omega_i t) \\ B\cos(\omega_i t - 120^0) \\ C\cos(\omega_i t - 240^0) \end{bmatrix} \quad (1)$$

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Therefore,

$V_{AN} = AA_1\cos(\omega_s t)\cos(\omega_i t) + BB_1\cos(\omega_s t-120^0)$
$\cdot\cos(\omega_i t-120^0) + CC_1\cos(\omega_s t-240^0)\cos(\omega_i t-240^0)$
$= 3/2$ (2)
when $AA_1 = 1$, $BB_1 = 1$, and $CC_1 = 1$

therefore,

$$A=1/A_1 \quad B=1/B_1 \quad C=1/C_1 \tag{3}$$

and where

$\omega_i$ is the frequency of the input voltage ( i.e. 50 Hz)
$\omega_S$ is the operating frequency of the frequency changer (e.g. 50 Hz)
and therefore, $\omega_o = \omega_S - \omega_I = 0$

It can be concluded from Eqns. (2) and (3) that " **the output voltage is balanced when the fundamental component of the switching functions are equal to the inverse of the amplitude of the corresponding input phase voltages**". The amount of unbalance that can be corrected depends on the choice of the proper switching function (SF). In this paper a simple single pulse modulation is considered ( Fig.3b). Maximum of 9% amplitude unbalance can be corrected by this type of switching function. However, the output contains some insignificant harmonics. The circuit configuration, switching functions and various waveforms are depicted in Figs. 1-4.

The corresponding unbalanced input currents for balanced output current can be derived as follows :

$$[I_i(\omega_i t)] = [F_d(\omega_S t)]^T \cdot [I_o]$$

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix} \cdot [I_o]$$

$$= I_D \cdot \begin{bmatrix} A_1\cos(\omega_s t) \\ B_1\cos(\omega_s t - 120^0) \\ C_1\cos(\omega_s t - 240^0) \end{bmatrix} \tag{4}$$

The input currents (Eqn.(4)) are unbalanced.

A practical controlled rectifier structure capable of producing balanced output voltage described in Eqn. (1) is shown in Fig. 1. The respective switch ON/OFF control strategy and the resulting output voltages are shown in Figs. 2-4. By comparing the ideal CR waveforms shown in Figs. 2-4, it becomes obvious that actual CR (Fig. 3b) transfer function $[F_d(\omega_s t)]$ contain additional elements as shown below:

$$[F_d(\omega_s t)] = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix}$$

where

$$F_1 = \sum_{n=1,3,5}^{\infty} A_n \cos n(\omega_s t)$$
$$F_2 = \sum_{n=1,3,5}^{\infty} C_n \cos n(\omega_s t - 240^0) \tag{5}$$
$$F_3 = \sum_{n=1,3,5}^{\infty} C_n \cos n(\omega_s t - 120^0)$$

The complete expression for the output voltage and input currents can be written as follows :

$$V_0 = [F_d(\omega_s t)].[V_i(\omega_i t)]$$
$$= \left[\sum_{n=1,3,5...}^{\alpha} A_n \cos n(\omega_s t) \ \sum_{n=1,3,5...}^{\alpha} B_n \cos n(\omega_s t - 120^0) \ \sum_{n=1,3,5...}^{\alpha} C_n \cos n(\omega_s t - 240^0)\right]$$
$$\cdot\begin{bmatrix} A\cos(\omega_i t) \\ B\cos(\omega_i t - 120^0) \\ C\cos(\omega_i t - 240^0) \end{bmatrix}$$
$$= \sum_{n=1,3,5,....}^{\infty} \{AA_n \cos n(\omega_s t).\cos(\omega_i t) + BB_n \cos n(\omega_s t - 120^0).\cos(\omega_i t - 120^0) + CC_n \cos n(\omega_s t - 240^0).\cos(\omega_i t - 240^0)\}$$

$$= \frac{3}{2} + \frac{1}{2}[\sum_{n=3,5...}^{\alpha} AA_n \{\cos(n\omega_s + \omega_i)t + \cos(n\omega_s - \omega_i)t\}$$
$$+ \sum_{n=3,5...}^{\alpha} BB_n \{\cos((n\omega_s + \omega_i)t - (n+1)120^0) + \cos((n\omega_s - \omega_i)t - (n-1)120^0)\}$$
$$+ \sum_{n=3,5...}^{\alpha} CC_n \{\cos((n\omega_s + \omega_i)t - (n+1)240^0) + \cos((n\omega_s - \omega_i)t - (n-1)240^0)\}] \tag{6}$$

and the corresponding input current expression becomes :

$$I_i(\omega_i t) = [F_d(\omega_s t)]^T.[I_0]$$

$$\text{or,} \quad \begin{bmatrix} I_a(\omega_i t) \\ I_b(\omega_i t) \\ I_c(\omega_i t) \end{bmatrix} = [I_0].\begin{bmatrix} \sum_{n=1,3,5...}^{\alpha} A_n \cos n(\omega_s t) \\ \sum_{n=1,3,5...}^{\alpha} B_n \cos n(\omega_s t - 120^0) \\ \sum_{n=1,3,5...}^{\alpha} C_n \cos n(\omega_s t - 240^0) \end{bmatrix} \tag{7}$$

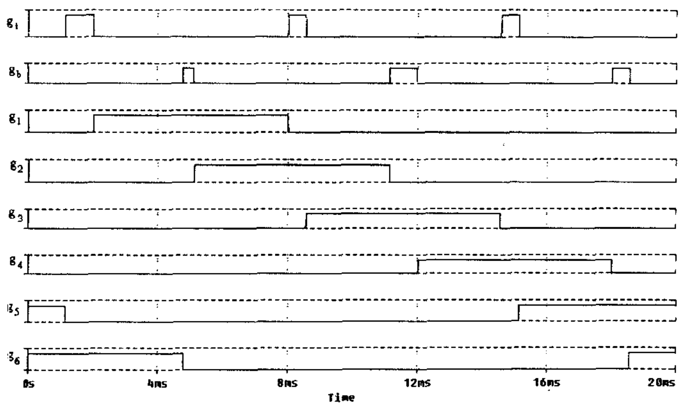
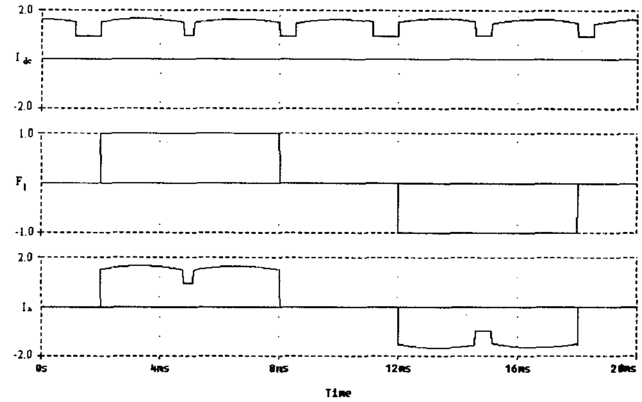



Fig 2: Gating signals for the switches $S_1$-$S_6$ and $S_t$ and $S_b$ of the proposed controlled rectifier structure




a) Input phase current, $I_a$ waveform

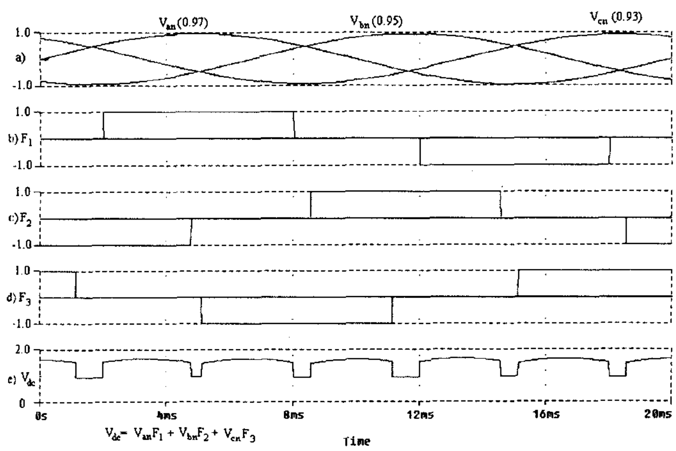
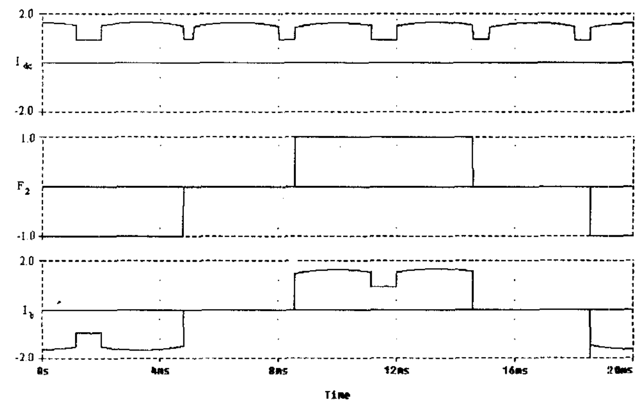
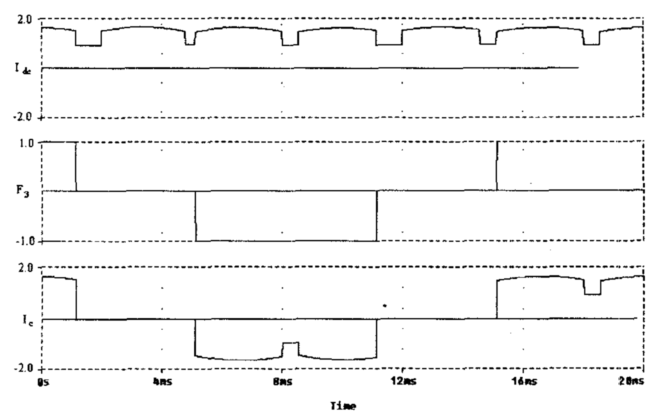



Fig 3: Output dc voltage, $V_{dc}$ waveform




b) Input phase current, $I_b$ waveform




c) Input phase current, $I_c$ waveform

Fig 4: Input phase currents of the unbalanced controlled rectifier

## 4. An Example

To verify the proposed technique a specific example of input voltage of magnitudes $V_{an}$=0.97 p.u. $V_{bn}$=0.95 p.u. and $V_{cn}$=0.93 p.u. i.e. 3, 5 and 7 percent amplitude unbalance is considered. A simple single pulse modulation ( Fig. 3b) is considered for this example. As the input phase voltages are of different amplitudes, the pulse duration will be different for the three phases (Eqn. (3)). The duration δ, of the pulses can be calculated as follows :

$$A_n = (4/n\pi)\ \mathrm{Sin}(n\delta/2) \qquad \text{for } n=1, 2, 3, 4 \ldots\ldots\ldots.$$

To get the output voltage balanced, fundamental component $A_1$ must satisfy Eqn. (3), which states that

$$A1 = \frac{1}{A} = \frac{1}{0.97} = 1.03093$$

or, $A_1 = 4/\pi\ \mathrm{Sin}\ (\delta_1/2)$

or, $1.03093 = 4/\pi\ \mathrm{Sin}\ (\delta_1/2)$

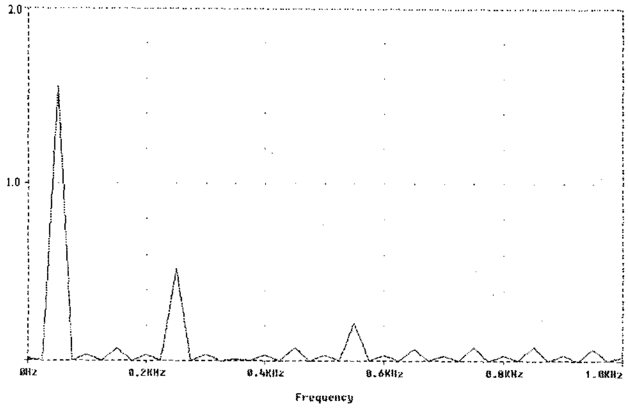
$$\therefore \delta_1 = 108^0$$

Corresponding pulse widths for b and c phases are $\delta_2 = 112^0$ and $\delta_3 = 116^0$ respectively. The gating signals $g_1$ - $g_6$ for this specific example are depicted in Fig 2. The output dc voltage $V_{dc}$ is shown in Fig.3. In order to provide a detailed description of the output voltage (Fig. 1), output voltage waveform and its spectrum has been computed and shown in Table 1. This information is essential not only for the proper evaluation of the CR performance but also for the design of input/output filters (when required). Corresponding unbalanced input currents spectra are also shown in the table.

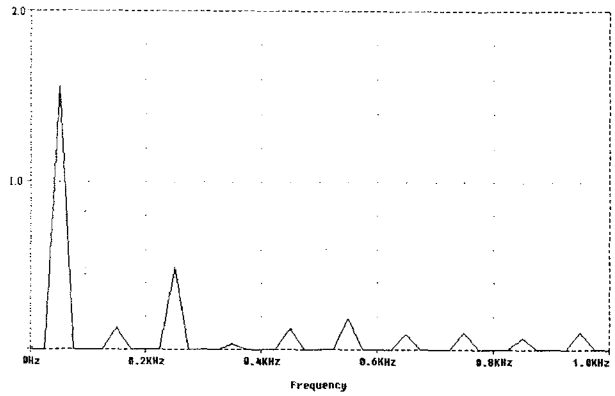
The unbalanced controlled rectifier may be implemented (Fig 6) by using simple logic blocks. The gating signals can be stored in a look-up table for different combination of input unbalance. The gating signals can be generated real time by using a Field Programmable Gate Arrays (FPGA), e.g. Xilinx 4000 series. Unbalance voltages may be sensed using standard circuit, the six switches may be gated accordingly using the look-up table

## 5. Results

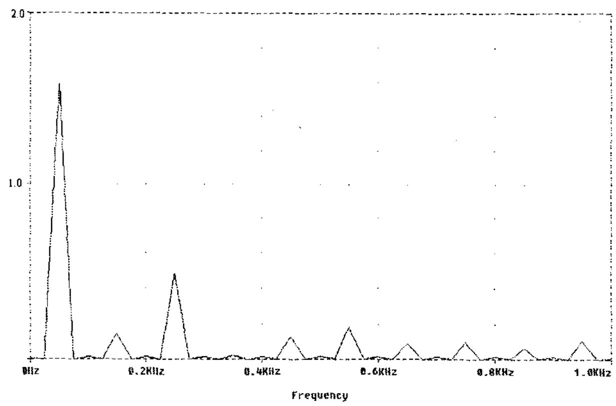
To verify the key analytical results, the discussed controlled rectifier was tested by simulation on a personal computer. A dedicated computer program simulating the precise opening and closing of the six rectifier switches is employed to generate the output voltage and input current waveforms. Further processing of these waveforms by using MATLAB package yields the respective frequency spectra. The same simulation was also done using PSPICE package and the simulation results, i.e. frequency spectra are depicted in figure 5. Comparison between analytically predicted frequency spectra (Table 1) and spectra obtained by simulation (Fig. 5) shows that they are in close agreement.



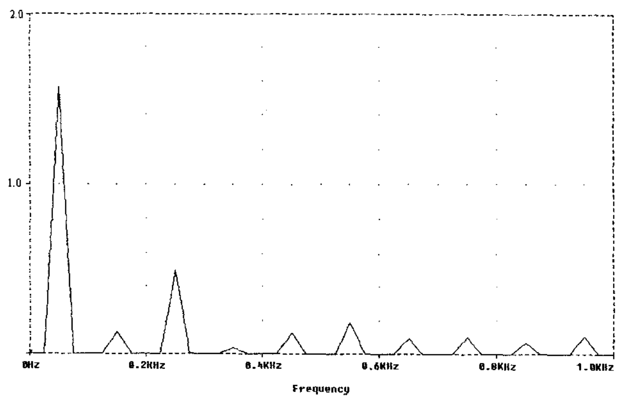

a) Output voltage, $V_{dc}$ frequency spectrum




b) Input phase current $I_a$ frequency spectrum




c) Input phase current $I_b$ frequency spectrum




d) Input phase current $I_c$ frequency spectrum

Fig 5: Frequency spectra of output voltage and input currents

| Table 1 | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Frequency spectra* of waveforms associated with unbalanced controlled rectifier Output voltage and Input currents shown in Fig. 3 - 4. The equations are described in Eqns.(6) &(7) | | | | | | | | | |
| Harmonic coefficients of switching function ( Fig. 3b-3d) | | | | Harmonic coefficients of output voltage, $V_{dc}$ (Fig. 3e ) | | Harmonic coefficients of Input currents, $I_a$, $I_b$, $I_c$, ( Fig. 4a-c) | | | |
| Order | Amplitude | | | Order | Amplitude | Order | Amplitude | | |
| (n) | $A_n$ | $B_n$ | $C_n$ | (n) | $V_{dc}$ | (n) | $I_a$ | $I_b$ | $I_c$ |
| 1 | 1.03 | 1.06 | 1.08 | 0 | 1.65 | 1 | 0.52 | 0.53 | 0.54 |
| 3 | 0.13 | 0.09 | 0.04 | 2 | 0.03 | 3 | 0.07 | 0.05 | 0.02 |
| 5 | 0.25 | 0.25 | 0.24 | 4 | 0.01 | 5 | 0.13 | 0.13 | 0.12 |
| 7 | 0.06 | 0.09 | 0.13 | 6 | 0.29 | 7 | 0.03 | 0.05 | 0.07 |
| 9 | 0.11 | 0.08 | 0.04 | 8 | 0.05 | 9 | 0.06 | 0.04 | 0.02 |
| 11 | 0.09 | 0.11 | 0.11 | 10 | 0.02 | 11 | 0.05 | 0.06 | 0.06 |
| 13 | 0.03 | 0.01 | 0.05 | 12 | 0.09 | 13 | 0.02 | 0.01 | 0.03 |
| 15 | 0.08 | 0.07 | 0.04 | 14 | 0.03 | 15 | 0.04 | 0.04 | 0.02 |
| 17 | 0.02 | 0.06 | 0.07 | 16 | 0.04 | 17 | 0.01 | 0.03 | 0.04 |
| 19 | 0.05 | 0.02 | 0.03 | 18 | 0.11 | 19 | 0.03 | 0.01 | 0.02 |

*Input phase voltage and output current magnitudes are 1 p.u.

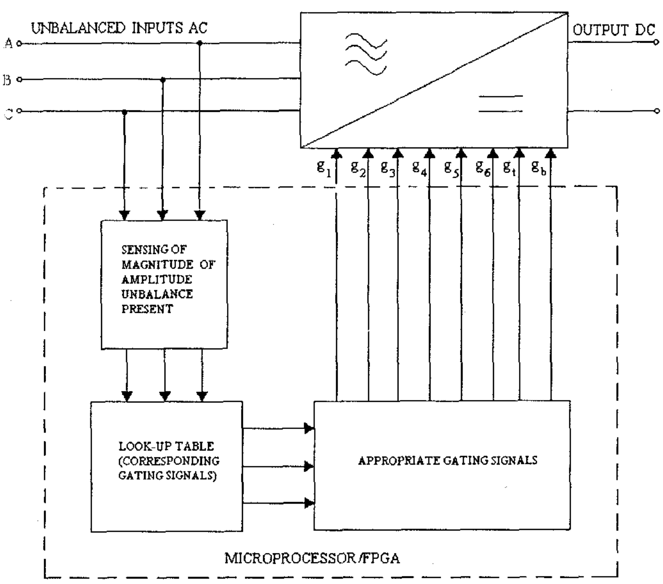



Fig 6: Microprocessor/FPGA based control circuitry

## 6. Conclusions

A comprehensive analysis of a three phase rectifier under amplitude unbalance input voltage has been presented in this paper. A simple example is used to illustrate the validity of the principle and is supported by simulation. The simple control logic circuit required makes the proposed technique attractive economically as well as practically.

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