Robust Sliding Mode Control for Robot Manipulator Tracking Problem using a Proportional-Integral Switching Surface

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Abstract- This paper presents the development of a Proportional-Integral sliding mode controller for tracking problem of robot manipulators. A robust sliding mode controller is derived so that the actual trajectory tracks the desired trajectory as closely as possible despite the highly non-linear and coupled dynamics. The Proportional-Integral sliding mode is chosen to ensure the stability of the overall dynamics during the entire period i.e. the reaching phase and the sliding phase. Application to a two-link planar robot manipulator is presented.

Keywords
Sliding Mode Control, Matched Uncertainties, Robot Tracking Control.

I. INTRODUCTION

Variable Structure Control (VSC) with Sliding Mode Control (SMC) has been widely applied to systems with uncertainties and/or input couplings [1], [2]. The design philosophy is to obtain a high-speed switching control law to drive the nonlinear plant's state trajectory onto a specified and user-chosen surface called the sliding surface. When a system is in the sliding mode, its dynamics is strictly determined by the dynamics of the sliding surfaces and hence insensitive to parameter variations and system disturbances. Nevertheless, the system posses no such insensitivity property during the reaching phase. Therefore insensitivity cannot be ensured throughout the entire response and the robustness during the reaching phase is normally improved by designing the system in such a way that the reaching phase is as short as possible [3].

Recently, a variety of the SMC known as Integral Sliding Mode Control (ISMC) has surfaced in the literature [4 - 8]. Different from the conventional SMC design approaches, the order of the motion equation in ISMC is equal to the order of the original system, rather than reduced by the number of dimension of the control input. Moreover, by using this approach, the robustness of the system can be guaranteed throughout the entire response of the system starting from the initial time instance.

In this paper, the problem of robust tracking for robot manipulator is considered. On the basis of sliding mode control theory, a class of VSC controllers for robust tracking of robot manipulators is proposed. It is shown theoretically and by computer simulations that for system with matched uncertainties, the tracking error is guaranteed to decrease asymptotically to zero and the system dynamics during the sliding phase can easily be shaped up using any conventional pole placement method.

II. PROBLEM FORMULATION

Consider the dynamics of the robot manipulator as an uncertain system described by

$$\dot{X}(t) = [A + \Delta A(t)]X(t) + [B + \Delta B(t)]U(t) \qquad (1)$$

where $X(t) \in R^n, U(t) \in R^m$, represent the state and input vectors, respectively. A and B are constant matrices of appropriate dimensions while ΔA and ΔB denote uncertainties present in the system and input matrices, respectively.

Define the state vector of the system as

$$X(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T \qquad (2)$$

Let a continuous function $X_d(t) \in R^n$ be the desired state trajectory, where $X_d(t)$ is defined as:

$$X_d(t) = [x_{d1}(t), x_{d2}(t), \ldots, x_{dn}(t)]^T \qquad (3)$$

Define the tracking error, $Z(t)$ as

$$Z(t) = X(t) - X_d(t) \qquad (4)$$

In this study, the following assumptions are made:
A1) The state vector $X(t)$ can be fully observed;
A2) There exist continuous functions $H(t)$ and $E(t)$ such that for all $X(t) \in R^n$ and all t:

$$\begin{aligned} \Delta A(t) &= BH(t)\ ; \quad \|H(t)\| \le \alpha \\ \Delta B(t) &= BE(t)\ ; \quad \|E(t)\| \le \beta \end{aligned} \qquad (5)$$

A3) There exist a Lebesgue function $\Omega(t) \in R$, which is integrable on bounded interval such that

$$\dot{X}_d(t) = AX_d(t) + B\Omega(t) \qquad (6)$$

A4) The pair (A, B) is controllable.

Assumption ii) assures that all uncertain portions $\Delta A(t)$ and $\Delta B(t)$ are contained in the range space of the nominal input matrix B. This structural condition on the uncertainty is termed matching condition [9]. The continuous functions $H(t)$ and $E(t)$ exist if and only if the following rank conditions are satisfied:

$$rank\,[B] = rank\,[B, \Delta A(X, \xi, t)] \qquad (7)$$

$$rank\,[B] = rank\,[B, \Delta B(X, \xi, t)] \qquad (8)$$

Equation (5) or the rank conditions (7) and (8) are essentially related to the structure of the matrices B, $\Delta A(t)$ and $\Delta B(t)$, and not to the values of their elements. These conditions impose constraints on the structure of the system matrix uncertainty $\Delta A(t)$, and the input matrix uncertainty $\Delta B(t)$ to lie within the range space of the input matrix B. This assumption is needed so that the control, $U(t)$, which enters the system through B can compensates the uncertainty in the system.

In view of equations (4), (5) and (6), equation (1) can be written as:

$$\dot{Z}(t) = [A + BH(t)]Z(t) + BH(t)X_d(t) - B\Omega(t) + [B + BE(t)]U(t) \qquad (9)$$

Define the Proportional-Integral (PI) sliding surface as

$$\sigma(t) = CZ(t) - \int_0^t [CA + CBK]Z(\tau)d\tau \qquad (10)$$

The structure of matrix C is as follows:

$$C = diag\,[c_1 \quad c_2 \quad \cdots \quad c_{n_i}] \qquad (11)$$

where n_i is the nth state variable associated to the ith input of the MIMO system. The matrix C is also chosen such that $CB \in R^{m\times m}$ is nonsingular. The matrix K satisfies

$$\lambda_{\max}(A + BK) < 0 \qquad (12)$$

The control problem is to design a controller using the PI sliding mode given by equation (10) such that the system's state trajectory $X(t)$ tracks the desired state trajectory $X_d(t)$ as closely as possible for all t in spite of the uncertainties and non-linearities present in the system.

III. SYSTEM DYNAMICS DURING SLIDING MODE

The concept of equivalent control was first mentioned in [10]. The equivalent control is only a mathematically derived tool for the analysis of a sliding motion rather than a real control law generated in practical systems. In fact it is not realizable in the real controller. With the equivalent control, one can predict the system behaviour during the sliding mode.

The equivalent control of the error system (9) can be found by differentiating equation (10), substituting equation (9) into it, and equating the resultant equation to zero. It can be shown that the equivalent control is [11]:

$$u_{eq}(t) = -[I_n + E(t)]^{-1}\{(H(t) - K)Z(t) - \Omega(t) + H(t)X_d(t)\} \qquad (11)$$

The system dynamics during sliding mode can be found by substituting the equivalent control of equation (11) into the system error dynamics of equation (9):

$$\dot{Z}(t) = [A + BK]Z(t) \qquad (12)$$

Hence if the matching condition is satisfied (equation (5) holds), the system's error dynamics during sliding mode is independent of the system uncertainties and couplings between the inputs, and, insensitive to the parameter variations.

IV. CONTROLLER DESIGN

In the last section, the control system has been shown to be stable during the sliding phase. In this section, it will be shown that the system trajectory during the reaching phase is guaranteed to be attracted into the switching surface (10).

The manifold of equation (8) is asymptotically stable in the large, if the following hitting condition is held:

$$(\sigma^T(t)/\|\sigma(t)\|)\dot{\sigma}(t) < 0 \qquad (13)$$

As a proof, let the positive definite function be

$$V(t) = \|\sigma(t)\| \qquad (14)$$

Differentiating equation (14) with respect to time, t yields

$$\dot{V}(t) = (\sigma^T(t)\dot{\sigma}_i(t))/\|\sigma(t)\| \qquad (15)$$

Following the Lyapunov stability theory, if equation (13) holds, then the sliding manifold $\sigma(t)$ is asymptotically stable in the large.

Theorem 4.1: The hitting condition (13) of the manifold given by equation (10) is satisfied if the control $U(t)$ of system (9) is given by :

$$U(t) = -(CB)^{-1}[\gamma_1\|Z(t)\| + \gamma_2\|X_d(t)\| + \gamma_3\|\Omega(t)\|]SGN(\sigma(t)) + \Omega(t) \qquad (16)$$

where

$$\gamma_1 > (\alpha\|CB\| + \|CBK\|)/(1+\beta) \qquad (17)$$

$$\gamma_2 > (\alpha\|CB\|)/(1+\beta) \qquad (18)$$

$$\gamma_3 > (\beta\|CB\|)/(1+\beta) \qquad (19)$$

Proof: See [11].
It is shown in [11] that the system (1) is stable in the sense of Lyapunov if the system is subjected to the control input (16).

As shown in equation (16), the controller consists of two parts. The first part is designed based on the nominal system, interaction, input and coupling matrices. Beside that, it utilizes the pre-defined system closed-loop locations that will give the desired constant gains matrix, K, as well as the pre-specified proportional-integral sliding surface constants matrix, C. The second part of the controller, $\Omega(t)$, is basically the control component to ensure asymptotic tracking of the nonlinear uncertain system.

The sign function $SGN(\sigma(t))$ used in equation (16) is an $m\times 1$ vector of discontinuous functions described as follows:

$$SGN(\sigma(t)) = [SGN(\sigma_1(t)) \ldots SGN(\sigma_m(t))]^T \quad (20)$$

The discontinuous nature of the control input signal described by equation (16) due to sign function $SGN(\sigma(t))$ described by equation (20), gives rise to the control input chattering and direct application of such control to the plant may be impractical. To obtain a continuous control signal such that the chattering is eliminated, each element of the discontinuous sign function $SGN(\sigma(t))$ in equation (20) can be replaced by a proper continuous function as follows [12]:

$$S_{\delta_i}(t) = \frac{\sigma_i(t)}{|\sigma_i(t)| + \delta_i} \quad (21)$$

where δ_i is a positive constant. This replacement is shown in Figure 1, when δ_i is equal to zero, $S_\delta(t)$ becomes $SGN(\sigma(t))$ and when δ_i is large, $S_\delta(t)$ goes to zero.

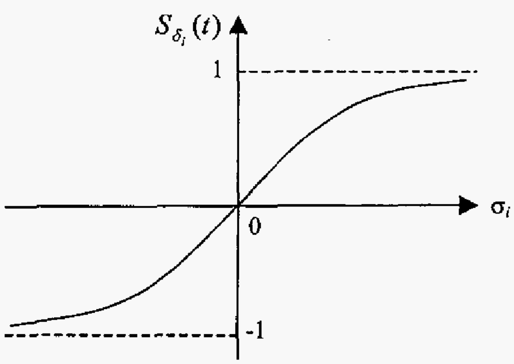

Fig. 1. Continuous Function $S_{\delta_i}(t)$.

Therefore, in order to eliminate the, the discontinuous function vector described by equation (20) can be replaced by the following continuous function vector:

$$S_\delta(t) = \begin{bmatrix} \dfrac{\sigma_1(t)}{|\sigma_1(t)| + \delta_1} \\ \vdots \\ \dfrac{\sigma_i(t)}{|\sigma_i(t)| + \delta_i} \\ \vdots \\ \dfrac{\sigma_m(t)}{|\sigma_m(t)| + \delta_m} \end{bmatrix} \quad ; i = 1, 2, \ldots, m \quad (22)$$

The continuous function described by equation (22) guarantees that the chattering problem encountered in the control input signal $U(t)$ is removed. This will make the control more sensible from practical point of view. However, tracking accuracy may be lost.

V. APPLICATION TO ROBOT TRACKING CONTROL

Consider a two-link planar manipulator with rigid links of nominally equal length l and mass m as shown in Fig. 2.

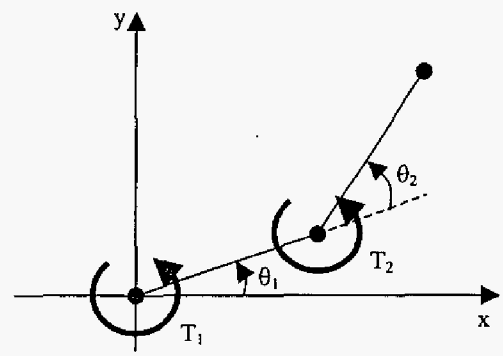

Fig. 2. A Two-Link Planar Manipulator.

The dynamics of the manipulator is [13]:

$$\ddot{\theta}_1 = \frac{(\frac{2}{3} + \cos\theta_2)\sin\theta_2.\dot{\theta}_1 + \frac{2}{3}\sin\theta_2.(2\dot{\theta}_1 + \dot{\theta}_2).\dot{\theta}_2}{\frac{16}{9} - \cos^2\theta_2} + \frac{\frac{4}{3}T_1 - 2(\frac{2}{3} + \cos\theta_2)T_2}{\frac{16}{9} - \cos^2\theta_2}$$

$$\ddot{\theta}_2 = \frac{-2(\frac{5}{3} + \cos\theta_2)\sin\theta_2.\dot{\theta}_1}{\frac{16}{9} - \cos^2\theta_2} - \frac{(\frac{2}{3} + \cos\theta_2)\sin\theta_2.(2\dot{\theta}_1 + \dot{\theta}_2).\dot{\theta}_2}{\frac{16}{9} - \cos^2\theta_2} - \frac{2(\frac{2}{3} + \cos\theta_2)T_1 - 4(\frac{5}{3} + \cos\theta_2)T_2}{\frac{16}{9} - \cos^2\theta_2}$$

Define

$$X(t)\underline{\Delta}[x_1 \quad x_2 \quad x_3 \quad x_4]^T = \left[\theta_1 \quad \dot{\theta}_1 \quad \theta_2 \quad \dot{\theta}_2\right]^T$$

$$U(t)\underline{\Delta}[U_1(t) \quad U_2(t)]^T = [T_1(t) \quad T_2(t)]^T$$

Then the plant can be represented in the form of

$$\dot{X}(t) = A(t)X(t) + B(t)U(t)$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & a_{22} & 0 & a_{24} \\ 0 & 0 & 0 & 1 \\ 0 & a_{42} & 0 & 0_{44} \end{bmatrix}; \; B = \begin{bmatrix} 0 & 0 \\ b_{21} & b_{22} \\ 0 & 0 \\ b_{41} & b_{42} \end{bmatrix}$$

$$a_{22} = ((2/3) + \cos x_3)\sin x_3 \cdot x_2 /((16/9) - \cos^2 x_3)$$

$$b_{24} = (2/3)\sin x_3 \cdot (2x_2 + x_4) \cdot x_4 /((16/9) - \cos^2 x_3)$$

$$a_{42} = -2((5/3) + \cos x_3)\sin x_3 \cdot x_2 /((16/9) - \cos^2 x_3)$$

$$a_{44} = -((2/3) + \cos x_3)\sin x_3 (2x_2 + x_4)/((16/9) - \cos^2 x_3)$$

$$b_{21} = (4/3)/((16/9) - \cos^2 x_3)$$

$$b_{22} = -2((2/3) + \cos x_3)/((16/9) - \cos^2 x_3)$$

$$b_{42} = b_{22}$$

$$b_{42} = 4((5/3) + \cos x_3)/((16/9) - \cos^2 x_3)$$

Let the bounds of the $\theta_i(t)$ and $\dot{\theta}_i(t)$ be:

$$-150^\circ \le \theta_1 \le 150^\circ, \; 0^\circ s^{-1} \le \dot{\theta}_1 \le 50^\circ s^{-1},$$

$$-35^\circ \le \theta_2 \le 100^\circ, \; 0^\circ s^{-1} \le \dot{\theta}_2 \le 30^\circ s^{-1}$$

It is assumed that each sub-system is required to track a pre-specified cycloidal function of the form:

$$\theta_{di}(t) = \begin{cases} \theta_i(0) + \dfrac{\Delta_i}{2\pi}\left[\dfrac{2\pi t}{\tau} - \sin\left(\dfrac{2\pi t}{\tau}\right)\right], & 0 \le t \le \tau \\ \theta_i(\tau), & \tau \le t \end{cases}$$

where $\Delta_i = \theta_i(\tau) - \theta_i(0)$, $i = 1,2$. In this paper, the input trajectory data used are as follows:

Start time, $t(0) = 0.0\ s$
Final time, $\tau = 10.0\ s$
Start positions, $\theta_1(0) = 10$ deg ; $\theta_2(0) = 15$ deg
Final positions, $\theta_1(\tau) = 50$ deg ; $\theta_2(\tau) = 60$ deg

With the given bounds, the plant can be represented in the form of equation (1) with the nominal value of A and B calculated as:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1.169 & 0 & 0.972 \\ 0 & 0 & 0 & 1 \\ 0 & -2.650 & 0 & -2.431 \end{bmatrix}; B = \begin{bmatrix} 0 & 0 \\ 1.509 & -1.949 \\ 0 & 0 \\ -1.949 & 8.415 \end{bmatrix}$$

and the uncertainties for system and input matrices as:

$$\Delta A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.974 & 0 & 0.972 \\ 0 & 0 & 0 & 0 \\ 0 & 4.208 & 0 & 2.431 \end{bmatrix}; \Delta B = \begin{bmatrix} 0 & 0 \\ 0.209 & 0.337 \\ 0 & 0 \\ 2.3368 & 5.299 \end{bmatrix}$$

Using equation (5), the bounds of $H(t)$ and $E(t)$ can be computed:

$$\|H(t)\| \le \alpha = 2.6046 \; ; \; \|E(t)\| \le \beta = 1.9617$$

Define the gains:

$K = \begin{bmatrix} 2.0125 & 3.4291 & 0.0919 & 0.8735 \\ 0.3235 & 0.4080 & 0.6868 & 0.4838 \end{bmatrix}$ such that the desired closed-loop poles $\lambda(A + BK) = \{-1,-2,-2,-3\}$

and $C = \begin{bmatrix} 3 & 1 & 0 & 0 \\ 0 & 0 & 0.2 & 1 \end{bmatrix}$

The controller parameter γ's can therefore be computed from equation (17-19):

$$\gamma_1 > 1.1731; \; \gamma_2 > 0.8742; \; \gamma_3 > 0.6584$$

Let

$$\gamma_1 = 1.5; \; \gamma_2 = 1.0; \; \gamma_3 = 1.0$$

and compute $\Omega(t)$ using equation (6) gives:

$$\Omega(t) = [\Omega_1(t), \Omega_2(t)]^T$$

where

$$\Omega_1(t) = -0.1933\dot{x}_{d1}(t) + 0.9455\dot{x}_{d2}(t) - 0.3867\dot{x}_{d3}(t) + 0.219\dot{x}_{d4}(t)$$

$$\Omega_2(t) = 0.1933\dot{x}_{d1}(t) + 0.219\dot{x}_{d2}(t) + 0.1993\dot{x}_{d3}(t) + 0.1696\dot{x}_{d4}(t)$$

The simulation results are shown in Figures 3, 4 and 5 for the tracking responses, the control inputs, and the sliding surfaces, respectively. Figures 3a and 3b show the tracking response for joint 1 and joint 2, respectively. It can be observed that the tracking performance is satisfactory for both joints, indicating that the controller is capable of handling the nonlinearities, couplings and uncertainties present in the system. The control input for joint 1 and joint 2 is shown in Figures 4a and 4b, respectively. The control inputs generated switch indiscriminately very fast to ensure that all states are directed toward the sliding surface (Figures 5a and 5b). This however produces chattering, as expected. To overcome this problem, the continuous function vector described in equation (22) is employed. Figures 6a and 6b show the control input of joint 1 and joint 2, respectively, using the continuous functions with $\delta_1 = 0.005$, $\delta_2 = 0.001$. It can be observed from the graphs that the chattering is totally eliminated without causing much differences in the tracking performances (Fig. 7).

VI. CONCLUSIONS

In this paper, a PI Sliding Mode controller is proposed for trajectory tracking of robot manipulators. It is shown mathematically that the error dynamics during sliding mode is stable and can easily be shaped-up using the conventional pole-placement technique. Besides, the system stability is also guaranteed during the reaching phase. Application to a two-link manipulator shows that the proposed controllers are effective in tackling the uncertainties, nonlinearities and coupling exist in robotic system.

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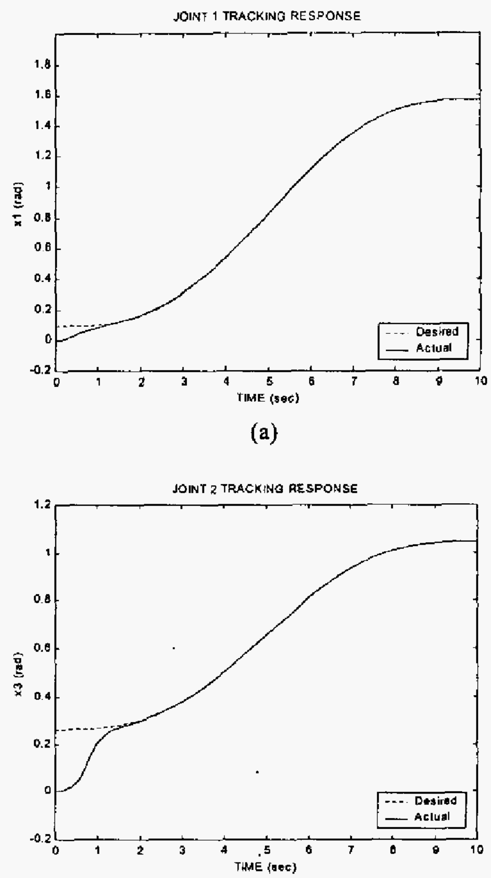
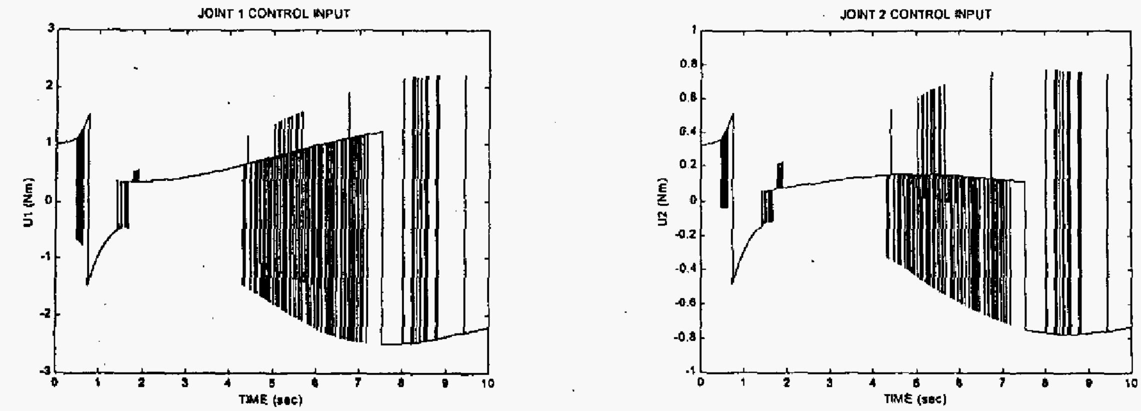
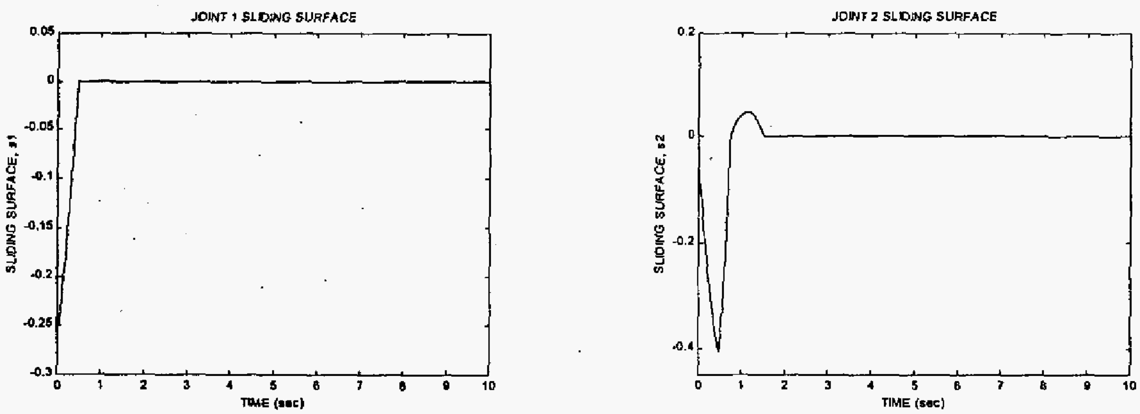


Fig. 3. Tracking Responses



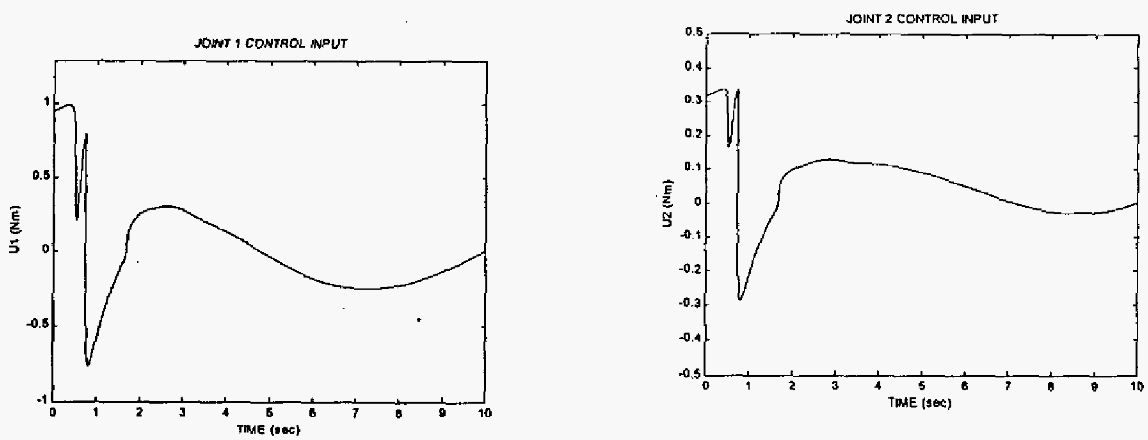
(a) (b)

Fig. 4. Control Inputs



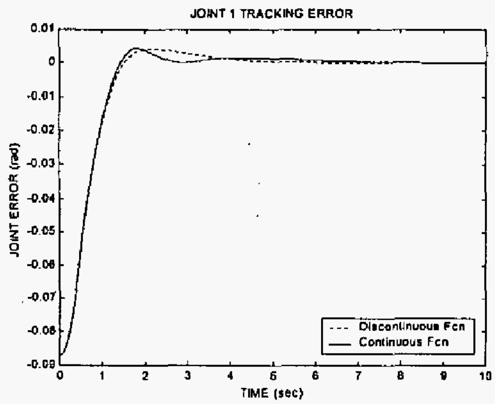
(a) (b)

Fig. 5. Sliding Surfaces

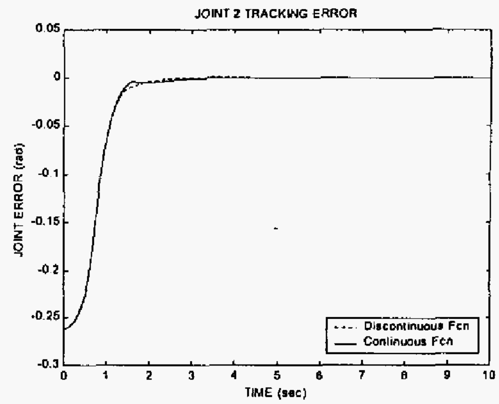


(a) (b)

Fig. 6. Control Inputs Using Continuous Function



(a)



(b)

Fig. 7. Tracking Errors