Application of Proportional-Integral Sliding Mode Tracking Controller to Robot Manipulators

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Abstract— This paper presents the development of a Proportional-Integral sliding mode controller for tracking problem of robot manipulators. A robust sliding mode controller is derived so that the actual trajectory tracks the desired trajectory as closely as possible despite the highly non-linear and coupled dynamics. The proposed controller is designed using the centralized and decentralized approaches. The Proportional-Integral sliding mode is chosen to ensure the stability of overall dynamics during the entire period i.e. the reaching phase and the sliding phase. Application to a two-link planar robot manipulator is considered.

***Index Terms*—Sliding mode control, robot tracking controller, robust proportional-integral sliding mode control.**

I. Introduction

Variable Structure Control (VSC) with Sliding Mode Control (SMC) has been widely applied to system with uncertainties and/or input couplings [1], [2]. The design philosophy is to obtain a high-speed switching control law to drive the nonlinear plant's state trajectory onto a specified and user-chosen surface called the sliding surface. When a system is in the sliding mode, its dynamics is strictly determined by the dynamics of the sliding surfaces and hence insensitive to parameter variations and system disturbances. Nevertheless, the system posses no such insensitivity property during the reaching phase. Therefore insensitivity cannot be ensured throughout the entire response and the robustness during the reaching phase is normally improved by designing the system in such a way that the reaching phase is as short as possible [3].

Recently, a variety of the SMC known as Integral Sliding Mode Control (ISMC) has surfaced in the literature [4], [5]. Different from the conventional SMC design approaches, the order of the motion equation in ISMC is equal to the order of the original system, rather than reduced by the number of dimension of the control input. Moreover, by using this approach, the robustness of the system can be guaranteed throughout the entire response of the system starting from the initial time instance.

In this paper, the problem of robust tracking for robot manipulator is considered. On the basis of sliding mode control theory, a class of VSC controllers for robust tracking of robot manipulators is proposed under centralized and decentralized approaches. It is shown theoretically that for system with matched uncertainties, the tracking error is guaranteed to decrease asymptotically to zero and the system dynamics during the sliding phase can easily be shaped up using any conventional pole placement method.

II. Problem Formulation

Centralized Controller Design

Consider the dynamics of the robot manipulator as an uncertain system described by [7]

$$\dot{X}(t) = [A + \Delta A(t)]X(t) + [B + \Delta B(t)]u(t) \quad (1)$$

where $X(t) \in R^n, u(t) \in R^m$, represent the state and input vectors, respectively. A and B are constant matrices of appropriate dimensions while ΔA and ΔB denote uncertainties present in the system and input matrices, respectively.

Define the state vector of the system as

$$X(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T \quad (2)$$

Let a continuous function $X_d(t) \in R^n$ be the desired state trajectory, where $X_d(t)$ is defined as:

$$X_d(t) = [x_{d1}(t), x_{d2}(t), \ldots, x_{dn}(t)]^T \quad (3)$$

Define the tracking error, $Z(t)$ as

$$Z(t) = X(t) - X_d(t) \quad (4)$$

In this study, the following assumptions are made:

A1) The state vector $X(t)$ can be fully observed;

A2) There exist continuous functions $H(t)$ and $E(t)$ such that for all $X(t) \in R^n$ and all t:

$$\Delta A(t) = BH(t)\ ; \quad \|H(t)\| \le \alpha$$
$$\Delta B(t) = BE(t)\ ; \quad \|E(t)\| \le \beta \quad (5)$$

A3) There exist a Lebesgue function $\Omega(t) \in R$, which is integrable on bounded interval such that

$$\dot{X}_d(t) = AX_d(t) + B\Omega(t) \quad (6)$$

A4) The pair (A, B) is controllable.

In view of equations (4), (5) and (6), equation (1) can be written as an error dynamic system:

$$\dot{Z}(t) = [A + BH(t)]Z(t) + BH(t)X_d(t) - B\Omega(t) + [B + BE(t)]u(t) \quad (7)$$

Define the Proportional-Integral (PI) sliding surface as

$$\sigma(t) = CZ(t) - \int_0^t [CA + CBK]Z(\tau)d\tau \quad (8)$$

where $C \in R^{m \times n}$ and $K \in R^{m \times n}$ are constant matrices. The matrix K satisfies

$$\lambda_{\max}(A + BK) < 0 \quad (9)$$

and C is chosen such that $CB \in R^{m \times m}$ is nonsingular.

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The control problem is to design a controller $u(t)$ such that the system state trajectory $X(t)$ tracks the desired state trajectory $X_d(t)$ as closely as possible for all t in spite of the uncertainties and non-linearities present in the system and the system dynamics slide on the sliding surface as defined by equation (8).

Decentralized Controller Design

The robot dynamics of equation (1) can also be represented as an uncertain composite system S defined by an N interconnected sub-systems $S_i, i = 1,2,..., N$ with each sub-system described by [8]

$$S_i: \dot{x}_i(t) = [A_i + \Delta A_i(t)]x_i(t) + [B_i + \Delta B_i(t)]u_i(t) + \sum_{j=1, j\neq i}^{N} [A_{ij} + \Delta A_{ij}(t)]x_j(t) + \sum_{j=1, j\neq i}^{N} [B_{ij} + \Delta B_{ij}(t)]u_j(t) \quad (10)$$

where $x_i(t) \in R^{n_i}, u_i(t) \in R^{m_i}$ represent the state and input of sub-system S_i, respectively. A_i, B_i, A_{ij} and B_{ij} are constant nominal matrices. ΔA_i, ΔA_{ij}, ΔB_i and ΔB_{ij} representing uncertainties present in the system, interconnection, input and coupling matrices, respectively. The following assumptions are introduced:

(B1) Every state vector $x_i(t)$ can be locally observed;

(B2) There exist continuous functions $H_i(t)$, $H_{ij}(t)$, $E_i(t)$ and $E_{ij}(t)$ such that for all $X \in R^N$ and all t:

$$\begin{aligned} \Delta A_i(t) &= B_i H_i(t) \ ; \ \|H_i(t)\| \leq \alpha_{ii} \\ \Delta A_{ij}(t) &= B_i H_{ij}(t) \ ; \ \|H_{ij}(t)\| \leq \alpha_{ij} \\ \Delta B_i(t) &= B_i E_i(t) \ ; \ \|E_i(t)\| \leq \beta_{ii} \\ \Delta B_{ij}(t) &= B_i E_{ij}(t) \ ; \ \|E_{ij}(t)\| \leq \beta_{ij} \end{aligned} \quad (11)$$

(B3) There exist a Lebesgue function $\Omega_i(t) \in R$:

$$\dot{x}_{di}(t) = A_i x_{di}(t) + B_i \Omega_i(t) \quad (12)$$

where A_i and B_i are the i-th subsystem nominal system and input matrices, respectively;

(B4) The pair (A_i, B_i) is controllable.

The state vector of the composite system S is defined as

$$X(t) = [x_1^T(t), x_2^T(t),..., x_N^T(t)]^T \ ; \ x_i(t) \in R^{n_i} \quad (13)$$

Let $X_d(t) \in R^{n_i N}$ be the desired state trajectory:

$$X_d(t) = [x_{d1}^T(t), x_{d2}^T(t),..., x_{dn}^T(t)]^T; \ x_{di}(t) \bullet R^{n_i} \quad (14)$$

Define the tracking error, $z_i(t)$ as

$$z_i(t) = x_i(t) - x_{di}(t) \quad (15)$$

In view of equations (11), (12) and (15), equation (10) can be written as

$$\begin{aligned} \dot{z}_i(t) = {} & [A_i + B_i H_i(t)]z_i(t) + BH_i(t)x_{di}(t) \\ & - B_i\Omega_i(t) + [B_i + B_i E_i(t)]u_i(t) \\ & + \sum_{j=1, j\neq i}^{N} [A_{ij} + B_i H_{ij}(t)]x_j(t) \\ & + \sum_{j=1, j\neq i}^{N} [B_{ij} + B_i E_{ij}(t)]u_j(t) \end{aligned} \quad (16)$$

Define the local PI sliding surface for S_i as

$$\sigma_i(t) = C_i z_i(t) - \int_0^t [C_i A_i + C_i B_i K_i] z_i(\tau) d\tau \quad (17)$$

where $C_i \in R^{m_i \times n_i}$ and $K_i \in R^{m_i \times n_i}$ are constant matrices. The matrix K_i satisfies

$$\lambda_{\max}(A_i + B_i K_i) < 0 \quad (18)$$

and C_i is chosen such that C_iB_i is nonsingular. For this class of system, the sliding manifold can be described as

$$\sigma(t) = [\sigma_1^T, \sigma_2^T, ..., \sigma_N^T]^T \quad (19)$$

The control problem is to design a decentralized controller $u_i(t)$ for each sub-system such that the system state trajectory $X_i(t)$ tracks the desired state trajectory $X_{di}(t)$ as closely as possible for all t in spite of the uncertainties and non-linearities present in the system, and each subsystem's dynamics approaches and remains on the PI sliding surface S_i as defined by equation (17) thereafter.

III. SYSTEM DYNAMICS DURING SLIDING MODE

The concept of equivalent control was first mentioned in [6]. The equivalent control is only a mathematically derived tool for the analysis of a sliding motion rather than a real control law generated in practical systems. In fact it is not realizable in the real controller. With the equivalent control, one can predict the system behaviour during the sliding mode.

Centralized Control System

The equivalent control of the error system (7) can be found by differentiating equation (8), substituting equation (7) into it, and equating the resultant equation to zero. It can be shown that the equivalent control is [7]:

$$u_{eq}(t) = -[I_n + E(t)]^{-1}\{(H(t) - K)Z(t) - \Omega(t) + H(t)X_d(t)\} \quad (20)$$

The system dynamics during sliding mode can be found by substituting the equivalent control of equation (20) into the system error dynamics of equation (7):

$$\dot{Z}(t) = [A + BK]Z(t) \quad (21)$$

Hence if the matching condition is satisfied (equation (11) holds), the system's error dynamics during sliding mode is independent of the system uncertainties and couplings between the inputs, and, insensitive to the parameter variations.

Decentralized Control System

Differentiating equation (17), substitute equation (16) into it, and equating the resulting equation to zero gives the equivalent control, $u_{eqi}(t)$:

$$\begin{aligned} u_{eqi}(t) = {} & -[I_{n_i} + E_i(t)]^{-1}\{(H_i(t) - K_i)z_i(t) \\ & - \Omega_i(t) + H_i(t)x_{di}(t) \\ & + \sum_{j=1, j\neq i}^{N} (C_iB_i)^{-1}C_i[A_{ij} + B_i H_{ij}(t)]x_j(t) \\ & + \sum_{j=1, j\neq i}^{N} (C_iB_i)^{-1}C_i[B_{ij} + B_i E_{ij}(t)]u_j(t)\} \end{aligned} \quad (22)$$

The system dynamics during sliding mode can be found by substituting the equivalent control (22) into the system error dynamics (16):

$$\begin{aligned} \dot{z}_i(t) = {} & [A_i + B_i K_i]z_i(t) \\ & + [I_{ni} - B_i(C_iB_i)^{-1}C_i]\{ \sum_{j=1, j\neq i}^{N} [A_{ij} + B_i H_{ij}(t)]x_j(t) \\ & + \sum_{j=1, j\neq i}^{N} [B_{ij} + B_i E_{ij}(t)]u_j(t)\} \end{aligned} \quad (23)$$

Define $P_{s_i} \underline{\Delta} [I_{n_i} - B_i(C_iB_i)^{-1}C_i]$ (24)

where P_{s_i} is a *projection operator* and satisfies the following two equations [3]:

$$C_i P_{s_i} = 0 \quad \text{and} \quad P_{s_i} B_i = 0 \tag{25}$$

In view of assumption (B2), then it follows that by the projection property, equation (14) can be reduced as

$$\dot{z}_i(t) = [A_i + B_i K_i] z_i(t) \tag{26}$$

Hence if the matching condition is satisfied, the system error dynamics during sliding mode are independent of the interconnection between the subsystems and couplings between the inputs, and, insensitive to the parameter variations. Equation (26) shows that the error dynamics during sliding mode can be specified by the designer through appropriate choice of the matrix K_i.

IV. SLIDING MODE TRACKING CONTROLLER DESIGN

In the last section, the centralized and decentralized control systems have been shown to be stable during the sliding phase. It this section, it will be shown that the systems during the reaching phase are practically stable.

Centralized Controller

The manifold of equation (8) is asymptotically stable in the large, if the following hitting condition is held:

$$(\sigma^T(t)/\|\sigma(t)\|)\dot{\sigma}(t) < 0 \tag{27}$$

As a proof, let the positive definite function be

$$V(t) = \|\sigma(t)\| \tag{28}$$

Differentiating equation (28) with respect to time, t yields

$$\dot{V}(t) = (\sigma^T(t)\dot{\sigma}_i(t))/\|\sigma(t)\| \tag{29}$$

Following the Lyapunov stability theory, if equation (27) holds, then the sliding manifold $\sigma(t)$ is asymptotically stable in the large.

Theorem 4.1: The hitting condition (27) of the manifold given by equation (8) is satisfied if the control $u(t)$ of system (7) is given by :

$$u(t) = -(CB)^{-1}[\gamma_1\|Z(t)\| + \gamma_2\|X_d(t)\| + \gamma_3\|\Omega(t)\|]SGN(\sigma(t)) + \Omega(t) \tag{30}$$

where

$$\gamma_1 > (\alpha\|CB\| + \|CBK\|)/(1+\beta) \tag{31}$$

$$\gamma_2 > (\alpha\|CB\|)/(1+\beta) \tag{32}$$

$$\gamma_3 > (\beta\|CB\|)/(1+\beta) \tag{33}$$

Proof: See [7].

It is shown in [7] that the system (1) is stable in the sense of Lyapunov if the system is subjected to the control input (30).

Decentralized Controller

The composite manifold (19) is asymptotically stable in the large, if the following hitting condition is held [5]:

$$\sum_{i=1}^{N} (\sigma_i^T(t)/\|\sigma_i(t)\|)\dot{\sigma}_i(t) < 0 \tag{34}$$

As a proof, let the positive definite Lyapunov function be

$$V(t) = \sum_{i=1}^{N} \|\sigma_i(t)\| \tag{35}$$

Then $$\dot{V}(t) = \sum_{i=1}^{N} (\sigma_i^T(t)/\|\sigma_i(t)\|)\dot{\sigma}_i(t) \tag{36}$$

Following the Lyapunov stability theory, if equation (34) holds, then the sliding manifold $\sigma(t)$ is asymptotically stable in the large.

Theorem 4.2: The global hitting condition (34) of the composite manifold (19) is satisfied if every local control $u_i(t)$ of the error system (16) is given by :

$$u_i(t) = -(C_i B_i)^{-1}[\gamma_{i1}\|z_i(t)\| + \gamma_{i2}\|x_i(t)\| + \gamma_{i3}\|x_{di}(t)\| + \gamma_{i4}\|\Omega_i(t)\|]SGN(\sigma_i(t)) + \Omega_i(t) \tag{37}$$

where

$$\gamma_{i1} > \frac{\alpha_{ii}\|C_i B_i\| + \|C_i B_i K_i\|}{\{(1+\beta_{ii})\|C_i B_i\| + \sum_{j=1, j\neq i}^{N}[\|C_j B_{ji}\| + \beta_{ji}\|C_j B_j\|]\}(C_i B_i)^{-1}} \tag{38}$$

$$\gamma_{i2} > \frac{\sum_{j=1, j\neq i}^{N}[\|C_j A_{ji}\| + \alpha_{ji}\|C_j B_j\|]}{\{(1+\beta_{ii})\|C_i B_i\| + \sum_{j=1, j\neq i}^{N}[\|C_j B_{ji}\| + \beta_{ji}\|C_j B_j\|]\}(C_i B_i)^{-1}} \tag{39}$$

$$\gamma_{i3} > \frac{\alpha_{ii}\|C_i B_i\|}{\{(1+\beta_{ii})\|C_i B_i\| + \sum_{j=1, j\neq i}^{N}[\|C_j B_{ji}\| + \beta_{ji}\|C_j B_j\|]\}(C_i B_i)^{-1}} \tag{40}$$

$$\gamma_{i4} > \frac{\beta_{ii}\|C_i B_i\| + \sum_{j=1, j\neq i}^{N}[\|C_j B_{ji}\| + \beta_{ji}\|C_j B_j\|]}{\{(1+\beta_{ii})\|C_i B_i\| + \sum_{j=1, j\neq i}^{N}[\|C_j B_{ji}\| + \beta_{ji}\|C_j B_j\|]\}(C_i B_i)^{-1}} \tag{41}$$

Proof: See [8].

It is shown in [8] that the system (10) is stable in the sense of Lyapunov if the system is control by the input (37).

V. SIMULATION EXAMPLE

Consider a two-link manipulator with rigid links of nominally equal length l and mass m shown in Figure 1. The dynamics of the manipulator is [9]:

$$\ddot{\theta}_1 = \frac{(\frac{2}{3}+\cos\theta_2)\sin\theta_2.\dot{\theta}_1 + \frac{2}{3}\sin\theta_2.(2\dot{\theta}_1+\dot{\theta}_2).\dot{\theta}_2}{\frac{16}{9}-\cos^2\theta_2} + \frac{\frac{4}{3}T_1 - 2(\frac{2}{3}+\cos\theta_2)T_2}{\frac{16}{9}-\cos^2\theta_2}$$

$$\ddot{\theta}_2 = \frac{-2(\frac{5}{3}+\cos\theta_2)\sin\theta_2.\dot{\theta}_1}{\frac{16}{9}-\cos^2\theta_2} - \frac{(\frac{2}{3}+\cos\theta_2)\sin\theta_2.(2\dot{\theta}_1+\dot{\theta}_2).\dot{\theta}_2}{\frac{16}{9}-\cos^2\theta_2} - \frac{2(\frac{2}{3}+\cos\theta_2)T_1 - 4(\frac{5}{3}+\cos\theta_2)T_2}{\frac{16}{9}-\cos^2\theta_2}$$

Define

$$X(t)\underline{\Delta}\begin{bmatrix}x_1 & x_2 & x_3 & x_4\end{bmatrix}^T = \begin{bmatrix}\theta_1 & \dot{\theta}_1 & \theta_2 & \dot{\theta}_2\end{bmatrix}^T$$

$$U(t)\underline{\Delta}\begin{bmatrix}u_1 & u_2\end{bmatrix}^T = \begin{bmatrix}T_1 & T_2\end{bmatrix}^T$$

Then the plant can be represented in the form of

$$\dot{X}(t) = A(t)X(t) + B(t)u(t)$$

with

$$A = \begin{bmatrix}0 & 1 & 0 & 0\\ 0 & a_{22} & 0 & a_{24}\\ 0 & 0 & 0 & 1\\ 0 & a_{42} & 0 & a_{44}\end{bmatrix};\ B = \begin{bmatrix}0 & 0\\ b_{21} & b_{22}\\ 0 & 0\\ b_{41} & b_{42}\end{bmatrix}$$

$a_{22} = ((2/3)+\cos x_3)\sin x_3 \cdot x_2/((16/9)-\cos^2 x_3)$

$b_{24} = (2/3)\sin x_3 \cdot (2x_2 + x_4)\cdot x_4/((16/9)-\cos^2 x_3)$

$a_{42} = -2((5/3)+\cos x_3)\sin x_3 \cdot x_2/((16/9)-\cos^2 x_3)$

$a_{44} = -((2/3)+\cos x_3)\sin x_3(2x_2 + x_4)/((16/9)-\cos^2 x_3)$

$b_{21} = (4/3)/((16/9)-\cos^2 x_3)$

$b_{22} = -2((2/3)+\cos x_3)/((16/9)-\cos^2 x_3)$

$b_{42} = b_{22}$

$b_{42} = 4((5/3)+\cos x_3)/((16/9)-\cos^2 x_3)$

Suppose that the bounds of $\theta_i(t)$ and $\dot{\theta}_i(t)$ are:

$$-150^\circ \le \theta_1 \le 150^\circ,\ 0^\circ s^{-1} \le \dot{\theta}_1 \le 50^\circ s^{-1},$$

$$-35^\circ \le \theta_2 \le 100^\circ,\ 0^\circ s^{-1} \le \dot{\theta}_2 \le 30^\circ s^{-1}$$

It is assumed that each sub-system is required to track a pre-specified cycloidal function of the form:

$$\theta_{di}(t) = \begin{cases}\theta_i(0) + \frac{\Delta_i}{2\pi}[\frac{2\pi t}{\tau} - \sin(\frac{2\pi t}{\tau})], & 0 \le t \le \tau\\ \theta_i(\tau), & \tau \le t\end{cases}$$

where $\Delta_i = \theta_i(\tau) - \theta_i(0)$, $i = 1,2$. In this example, the input trajectory data used are as follows:

- Start time, $t(0) = 0.0\ s$
- Final time, $\tau = 10.0\ s$
- Start positions, $\theta_1(0) = 10^\circ$; $\theta_2(0) = 15^\circ$
- Final positions, $\theta_1(\tau) = 50^\circ$; $\theta_2(\tau) = 60^\circ$

Centralized PI Sliding Mode Control

With the given bounds, the plant can be represented in the form of equation (1) with the nominal value of A and B calculated as:

$$A = \begin{bmatrix}0 & 1 & 0 & 0\\ 0 & 1.169 & 0 & 0.972\\ 0 & 0 & 0 & 1\\ 0 & -2.650 & 0 & -2.431\end{bmatrix};B = \begin{bmatrix}0 & 0\\ 1.509 & -1.949\\ 0 & 0\\ -1.949 & 8.415\end{bmatrix}$$

with the uncertainties for system and input matrices calculated as

$$\Delta A = \begin{bmatrix}0 & 0 & 0 & 0\\ 0 & 0.974 & 0 & 0.972\\ 0 & 0 & 0 & 0\\ 0 & 4.208 & 0 & 2.431\end{bmatrix};\Delta B = \begin{bmatrix}0 & 0\\ 0.209 & 0.337\\ 0 & 0\\ 2.3368 & 5.299\end{bmatrix}$$

Using equation (5), the bounds of $H(t)$ and $E(t)$ can be computed:

$$\|H(t)\| \le \alpha = 2.6046\ ;\ \|E(t)\| \le \beta = 1.9617$$

Define the gains:

$$K = \begin{bmatrix}2.0125 & 3.4291 & 0.0919 & 0.8735\\ 0.3235 & 0.4080 & 0.6868 & 0.4838\end{bmatrix}$$ such that

$\lambda(A+BK) = \{-1,-2,-2,-3\}$

and $C = \begin{bmatrix}3 & 1 & 0 & 0\\ 0 & 0 & 0.2 & 1\end{bmatrix}$,

the controller parameter γ's can therefore be computed from equation (31)-(33) as:

$$\gamma_1 > 1.1731;\ \gamma_2 > 0.8742;\ \gamma_3 > 0.6584$$

For comparison purposes, two sets of the controller parameters are chosen:

Set 1: $\gamma_1 = 0.5;\ \gamma_2 = 0.2;\ \gamma_3 = 0.2$

Set 2: $\gamma_1 = 1.5;\ \gamma_2 = 1.0;\ \gamma_3 = 1.0$

In Set 1, the controller parameter is selected to study the performance of the system if the gain conditions of equations (31)-(33) are violated; while in Set 2 the controller parameters is selected to represent a situation where the conditions imposed on the controller are met. The simulation results for both sets are shown in Figure 2. If the controller parameter conditions are not met (Set 1), the actual output positions fail to track the desired positions (Figure 2a and Figure 2b). This is due to the fact that the control inputs are not capable of switching fast enough and hence the sliding mode fails to materialized. On the contrary, when the controller parameter conditions are met (Set 2), the position tracking is satisfactory (Figure 2c and 2d).

Decentralized PI Sliding Mode Control

With the given bounds, the plant can be represented in the form of equation (10). Each joint of the robot is treated as a sub-system with the nominal value of A_i, A_{ij}, B_i and B_{ij} is calculated as:

$$A_1 = \begin{bmatrix}0 & 1\\ 0 & 1.168\end{bmatrix};A_2 = \begin{bmatrix}0 & 1\\ 0 & -2.431\end{bmatrix};A_{12} = \begin{bmatrix}0 & 0\\ 0 & 0.972\end{bmatrix};$$

$$A_{21} = \begin{bmatrix}0 & 0\\ 0 & -2.650\end{bmatrix};B_1 = \begin{bmatrix}0\\ 1.506\end{bmatrix};B_2 = \begin{bmatrix}0\\ 8.415\end{bmatrix}$$

$$B_{12} = \begin{bmatrix}0\\ -1.949\end{bmatrix},\ B_{21} = \begin{bmatrix}0\\ -1.949\end{bmatrix}$$

Using equation (11), the bounds of $H_i(t)$ and $E_i(t)$ can be computed:

$$\|H_1(t)\| \le \alpha_{11} = 0.6471,\ \|H_2(t)\| \le \alpha_{22} = 0.2889,$$

$$\|H_{12}(t)\| \le \alpha_{12} = 0.6458,\ \|H_{21}(t)\| \le \alpha_{21} = 0.5$$

$$\|E_1(t)\| \le \beta_{11} = 0.1385,\ \|E_2(t)\| \le \beta_{22} = 0.6297,$$

$$\|E_{12}(t)\| \le \beta_{12} = 1.5519,\ \|E_{21}(t)\| \le \beta_{21} = 0.2777$$

In this study, the gains are chosen as follows:

$K_1 = [1.3282\quad 2.7683\ ]$ so that $\lambda(A_1 + B_1K_1) = \{-1,-2\}$;

$K_2 = [0.7130\quad 0.3053]$ so that $\lambda(A_2 + B_2K_2) = \{-2,-3\}$;

$C_1 = [3\quad 1]$ and $C_2 = [4\quad 1]$.

Therefore, from equations (38)-(41):

$$\gamma_{11} > 1.40;\gamma_{12} > 1.72;\gamma_{13} > 0.25;\gamma_{14} > 1.13;$$

$$\gamma_{21} > 4.19;\gamma_{22} > 0.91;\gamma_{23} > 1.14;\gamma_{24} > 4.48$$

For simulation purposes, two sets of controller parameters are chosen:

Set 1: $\begin{cases}\gamma_{11} = 0.5;\ \gamma_{12} = 0.5;\ \gamma_{13} = 0.1;\ \gamma_{14} = 0.5;\\ \gamma_{21} = 4;\ \gamma_{22} = 0.5;\ \gamma_{23} = 0.5;\ \gamma_{24} = 4\end{cases}$

Set 2: $\begin{cases}\gamma_{11} = 3;\ \gamma_{12} = 3;\ \gamma_{13} = 2;\ \gamma_{14} = 2;\\ \gamma_{21} = 5;\ \gamma_{22} = 2;\ \gamma_{23} = 2;\ \gamma_{24} = 5\end{cases}$

Set 1 contains the controller parameter selected to study the performance of the system if equations (38)-(41) are not met; while Set 2 contains the parameters satisfying the condition imposed. The output trajectories for both subsystems 1 and 2 are shown in Figure 3. It can be seen that the tracking performance for both subsystems when Set 1 parameters were used are unsatisfactory (Figures 3a and 3b). The simulation was run again but this time with the decentralized controller parameter was supplied from Set 2 (Figures 3c and 3d). As predicted theoretically, the tracking performance is good for both subsystems.

VI. CONCLUSIONS

In this paper, a PI Sliding Mode controller is proposed for trajectory tracking of robot manipulators. The controller can be design and implemented either in centralized or decentralized fashion. In both of the approaches, it is shown mathematically that the error dynamics during sliding mode is stable and can easily be shaped-up using the conventional pole-placement technique. Besides, the system stability is also guaranteed during the reaching phase. Application to a two-link manipulator shows that the proposed controllers are effective in tackling the uncertainties, non-linearities and coupling exist in robotic system.

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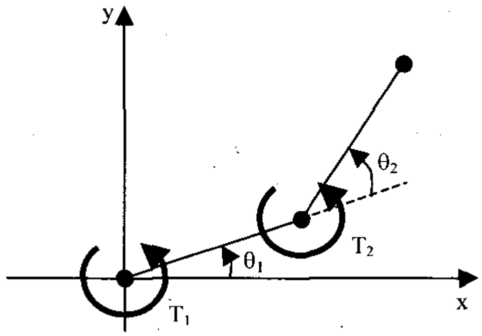
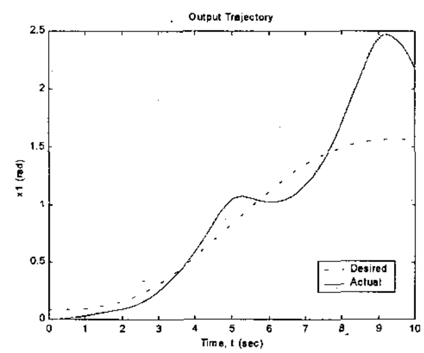
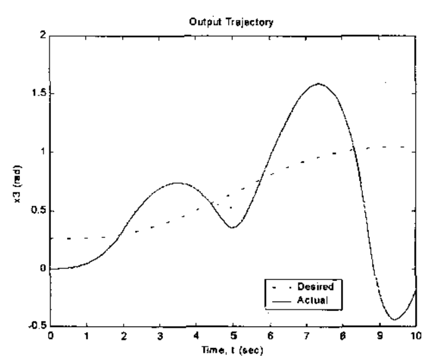


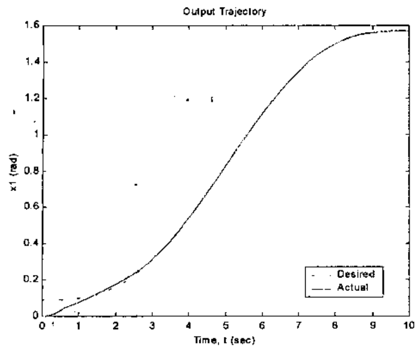
Fig. 1 A two-link manipulator.



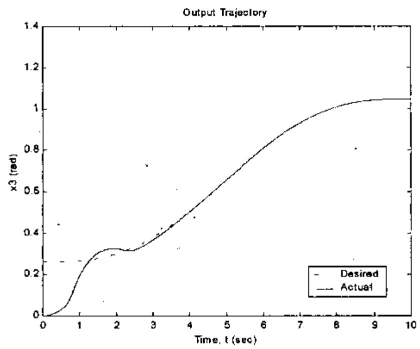
(a) State $x_1(t)$ response for Set 1



(b) State $x_3(t)$ response for Set 1

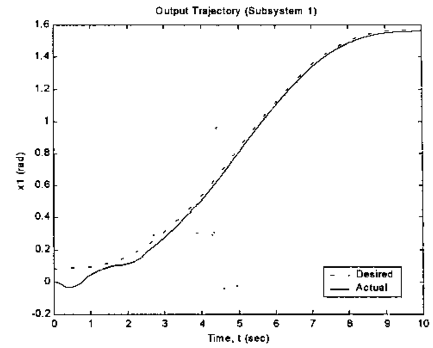


(c) State $x_1(t)$ response for Set 2

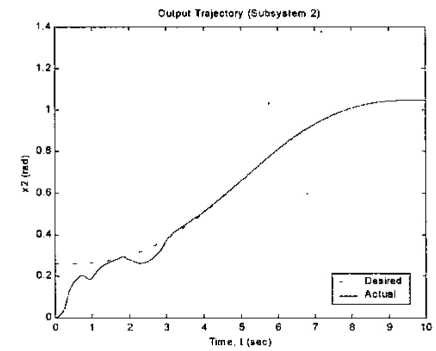


(d) State $x_3(t)$ response for Set 2

Fig. 2 Simulation results for Centralized PI Sliding Mode Control.

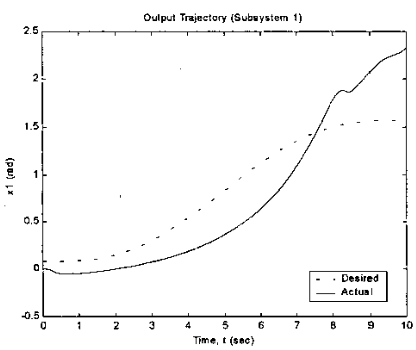


(c) State $x_1(t)$ response for Set 2

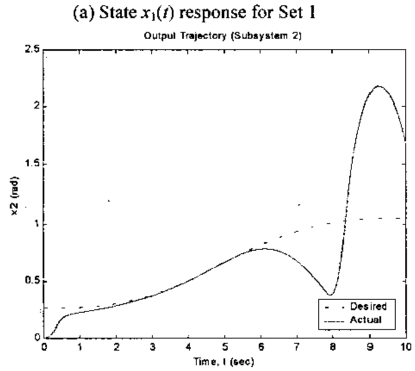


(d) State $x_3(t)$ response for Set 2

Fig. 3: Simulation results for Decentralized PI Sliding Mode Control.



(a) State $x_1(t)$ response for Set 1



(b) State $x_3(t)$ response for Set 1