Detection of Binary Data for FSK Digital Modulation Signals Using Spectrum Estimation Techniques

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Abstract - FSK is widely used for the transmission of data over passband channels due to the simplicity of implementation using noncoherent detection schemes. Better BER performance can achieved with accurate phase synchronization using coherent detection scheme. By using the fast Fourier transform (FFT), it is possible to implement spectrum based detection : complex sinusoid and complex square wave. The BER performance is slightly lower than coherent detection. Unlike coherent detection, phase synchronization is not critical for the spectrum based detection. Further simplification in hardware implementation is achievable using spectrum based detection-complex square wave since the use of multipliers is not required.

1.0 Introduction

Digital modulation using FSK (Frequency Shift-Keying) is one of the basic and widely used method for transmitting binary data over a passband channel such as the PSTN (Public Switched Telephone Network), HF (High Frequency) circuits and UHF (Ultra High Frequency) digital cellular communication systems [1]-[3]. Although not as spectrum efficient as PSK (Phase Shift-Keying), the detection process at the receiver is simplified by noncoherent detection. However, at the expense of suboptimal BER (bit-error rate) performance compared to coherent detection. The proposed method that is referred as spectrum based detection is a compromise between coherent and noncoherent detection in terms of implementation and BER performance.

2.0 Signal Model

The received signal is within a bit-duration is given as

$$y(t) = x(t) + w(t) \tag{1}$$

where $x(t)$ is the true signal, and $w(t)$ is the interference due to additive white Gaussian noise with zero mean and power σ_p^2. The true signal $x(t)$ is [4]

$$\begin{aligned} x(t) = x_1(t) &= A\cos 2\pi f_1 t \quad \text{bit '1'} \\ x_0(t) &= A\cos 2\pi f_0 t \quad \text{bit '0'} \; t_0 \le t \le t_0 + T_b \end{aligned} \tag{2}$$

where A is the signal amplitude, f_1 and f_0 are the frequencies of the signal, t_0 is any arbitrary time instant, and T_b is the bit-duration. For simulation purposes, the subcarrier frequencies used are 1400 and 1800 Hz, bit-rate of 100 bits/sec and sampling frequency of 8000 Hz. The time and frequency domain representation of the signal for transmitting a binary sequence '1110010' is shown in Figure 1.

3.0 Coherent and Noncoherent Detection

There are two ways for detecting data : coherent and noncoherent detection. Coherent detection is more optimum in terms of the bit-error rate (BER) performance but is more complex compared to noncoherent detection due to the carrier recovery circuit for phase syncronization.

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3.1 Coherent Detection

Coherent detection of data requires exact synchronization in phase between the received signal and the possible signal form. The output of the receiver is defined as [4]

$$z(t) = \int_{t_0}^{t_0+T_b} y(t)x_1(t)dt - \int_{t_0}^{t_0+T_b} y(t)x_0(t)dt \qquad t_0 \le t \le t_0 + T_b \tag{3}$$

where $x_0(t)$ and $x_1(t)$ are reference signal used at the receiver. The reference signals are required to be similar in form to the received signal.

By sampling $z(t)$ at intervals nT_b, the detection is performed as follows

$$\begin{aligned} z(n) &= +ve\text{, then binary bit "1" is detected} \\ &= -ve\text{, then binary bit "0" is detected} \end{aligned} \tag{4}$$

The BER for coherent detection is

$$BER = Q\left(\sqrt{\frac{A^2 T_b}{4\sigma_w^2}}\right) \tag{5}$$

where $Q(\,)$ is

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp\left(-\frac{z^2}{2}\right) dz \tag{6}$$

3.2 Noncoherent Detection

For noncoherent detection, the same methodology for detection is applied except that the output of the receiver is [4]

$$z(t) = \int_{t_0}^{t_0+T_b} |y(t) * h_1(t)|^2 dt - \int_{t_0}^{t_0+T_b} |y(t) * h_0(t)|^2 dt \tag{7}$$

where * indicates a convolution operation, and $h_1(t)$ and $h_0(t)$ are the impulse response of bandpass filters centered at f_1 and f_0 respectively with bandwidth $1/T_b$.

The resulting bit-error rate [4] is

$$BER = \frac{1}{2}\exp\left(-\frac{A^2}{8\sigma_p^2}\right) \tag{8}$$

where σ_p^2 is

$$\sigma_p^2 = \frac{2\sigma_w^2}{T_b} \tag{9}$$

For both detection schemes, the signal-to-noise ratio (SNR) in decibels is

$$SNR_{dB} = 10\log\left(\frac{A^2 T_b}{2\sigma_w^2}\right) \tag{10}$$

In general, the performance degradation for the noncoherent over the coherent detection is 6 dB for a constant BER of 10^{-4}.

3.3 Phase Synchronization Error in Coherent Detection

Synchronization is critical to ensure that the bit-error rate performance is optimal for coherent detection. Given that a received signal within a bit-duration is

$$y(t) = A\cos(2\pi f_1 t + \varphi) \qquad t_0 \le t \le t_0 + T_b \tag{11}$$

where ϕ is the phase present in the signal. If the reference signal is

$$x_1(t) = A\cos(2\pi f_1 t) \qquad t_0 \le t \le t_0 + T_b \tag{12}$$

then the output signal is

$$\begin{aligned} z(t) &= \int_{t_0}^{t_0+T_b} y(t)x_1(t)dt \\ &= \int_{t_0}^{t_0+T_b} A\cos(2\pi f_1 t + \varphi) y(t) A\cos(2\pi f_1 t) dt \\ &= \frac{A^2}{2}\int_{t_0}^{t_0+T_b} [A\cos(4\pi f_1 t + \varphi) + \cos(-\varphi)]dt \\ &= \frac{A^2 T_b}{2}\cos(\varphi) \end{aligned} \tag{13}$$

The results show how critical the phase shift affects the magnitude of the output signal. Thus, it is desired to have a detection scheme a compromise between robustness to phase synchronization error and BER performance.

4.0 Spectrum Based Detection

Since the binary information is represented in frequency, then the power spectrum can be used to detect the binary information present in the

FSK signal. The robustness of this method to phase error will be demonstrated since phase is not represented in the power spectrum. In addition, the Fast Fourier Transform (FFT) and the present microelectronic technology make implementation of spectrum estimation feasible in real-time.

4.1 Power Spectrum Definition

If $x(t)$ is defined within a bit-duration T_b, then its power spectrum $S_{xx}(f)$ can be estimated as [5]

$$S_{xx}(f) = \frac{1}{T_b}|X(f)|^2 = \frac{1}{T_b}\left|\int_{t_0}^{t_0+T_b} x(t)b^*(t,f)dt\right|^2 \qquad (14)$$

where $X(f)$ is the amplitude spectrum and $b(t,f)$ is the basis function. Two basis function are evaluated : the complex sinusoid and complex square wave. The Fourier transform is based on the complex exponential that is

$$b(t,f) = \exp(j2\pi ft) \qquad (15)$$

The basis function based on the complex square wave defined within a period $T=1/f$ is

$$\begin{aligned} b(t,f) &= 1 && -\frac{1}{4f} \le t \le \frac{1}{4f} \\ &= -1 && \frac{1}{4f} \le |t| \le \frac{1}{2f} \\ &= j1 && 0 \le t \le \frac{1}{2f} \\ &= -j1 && \frac{1}{2f} \le t \le \frac{1}{2f} \end{aligned} \qquad (16)$$

The complex square wave transform is proposed to optimize implementation of hardware such as using ASIC (Application Specific Integrated Circuit) and FPGA (Field Programmable Gate Arrays) by avoiding the use of multipliers. A multiplier is required to calculate the signal-basis function product $x(t)b^*(t,f)$ in Equation (14). If the complex square wave is used as a basic function, then the product for a given time instant t_0 within a bit-duration is

$$\begin{aligned} real[x(t_0)b^*(t_0,f)] &= real[x(t_0)] \\ &\quad \text{if } real[b^*(t_0,f)] = 1 \\ &= -real[x(t_0)] \\ &\quad \text{if } real[b^*(t_0,f)] = -1 \end{aligned}$$

$$\begin{aligned} imag[x(t_0)b^*(t_0,f)] &= imag[x(t_0)] \\ &\quad \text{if } imag[b^*(t_0,f)] = 1 \\ &= -imag[x(t_0)] \\ &\quad \text{if } imag[b^*(t_0,f)] = -1 \end{aligned} \qquad (17)$$

The overall result is

$$x(t_0)b^*(t_0,f) = real[x(t_0)b^*(t_0,f)] + j[imag[x(t_0)b^*(t_0,f)]] \qquad (18)$$

The spectrum is estimated by integrating over all time as shown Equation (14). Thus, no multiplier is required to compute the product signal-basis function.

4.2 Proposed Detection Method

The peaks of the power spectrum occur at the frequency of the signal. Thus, the decision rule for detecting the transmitted sequence in an FSK signal within a bit-duration is as follows

$$\begin{aligned} S_{xx}(f_1) &\ge S_{xx}(f_0) && \text{bit “1” is detected} \\ S_{xx}(f_0) &\ge S_{xx}(f_1) && \text{bit “0” is detected} \end{aligned} \qquad (19)$$

4.2 Robustness to Phase Synchronization Error

Using the signal defined in Equation (11), the Fourier transform for the signal is

$$X(f) = \frac{AT_b}{2}\exp(j\varphi)\left[\frac{\sin\pi(f_1-f)T_b}{\pi(f_1-f)T_b}\right] + \frac{AT_b}{2}\exp(-j\varphi)\left[\frac{\sin\pi(f_1+f)T_b}{\pi(f_1+f)T_b}\right] \qquad (20)$$

The power spectrum calculated using Equation (14) is

$$S_{xx}(f) = \frac{1}{T_b}|X(f)|^2 = \frac{1}{T_b}X(f)X^*(f) = \frac{1}{T_b}\left[\frac{AT_b}{2}\left[\frac{\sin\pi(f_1-f)T_b}{\pi(f_1-f)T_b}\right]\right]^2 + \frac{1}{T_b}\left[\frac{AT_b}{2}\left[\frac{\sin\pi(f_1+f)T_b}{\pi(f_1+f)T_b}\right]\right]^2 \qquad (21)$$

The results show that the phase is not present in the power spectrum. Thus, the spectrum based detection is robust to phase synchronization error.

5.0 Results

Simulation is performed in the present additive white Gaussian noise based on Equation (1). A random phase terms is included in the simulation that is based on a uniform distribution where the phase range is $0 \leq \phi \leq \pi$. This is to determine the effect of phase synchronization error on the spectrum based detection method. For a given BER of 10^{-4}, simulation result shows that the performance downgrades by 1.5 dB compared to the coherent detection but better than noncoherent detection by 3 dB. There is no significant difference in terms of the BER performance for the spectrum based detection whether a complex sinusoid or complex square wave is used.

6.0 Conclusions

Spectrum based techniques are evaluated to detect binary data in FSK signals. By simulation, the performance is less optimal in terms of the BER performance compared to the coherent detection but is better than noncoherent detection. However, the main advantage is the robustness to phase synchronization error and the implementation can be made simpler compared to the coherent detection.

7.0 References

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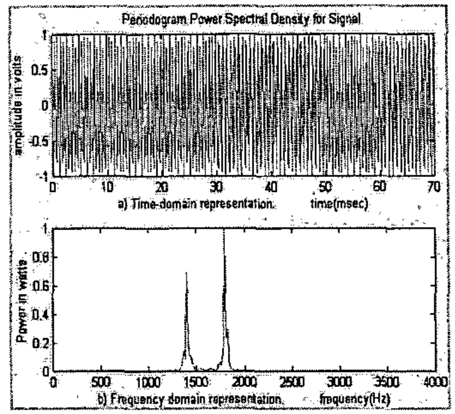

Figure 1 Time and frequency representation of an FSK signal.

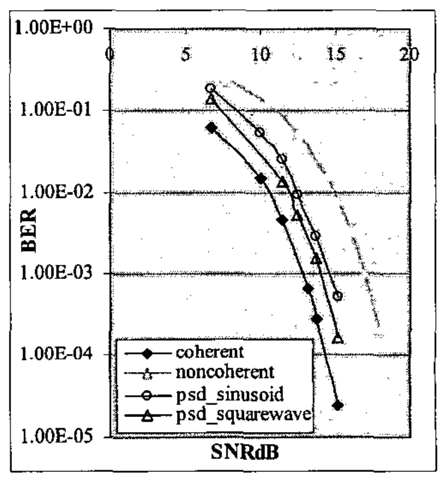

Figure 2 Comparison in the BER performance for the spectrum based detection – complex sinusoid and complex square wave.