

THEORY OF BICHROMATIC WAVE GROUPS  
AMPLITUDE AMPLIFICATION  
USING IMPLICIT VARIATIONAL METHOD

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Dedicated to  
my beloved father & mother,  
and my husband.

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## ABSTRACT

This research aims to study the maximal amplification factor for the evolution of nonlinear wave groups, particularly the evolution of bichromatic wave groups governed by temporal nonlinear Schrödinger equation. A new numerical method: implicit variational method has been proposed to simulate the nonlinear wave groups' evolution. The scheme combines the implicit differences and variational techniques. When the results are compared to the exact solutions of one soliton and bisoliton wave groups, the implicit variational method proved to be better in terms of accuracy and conserving the energy property than the existing known method, the explicit forward difference method and the Crank-Nicolson implicit method. Given an initial condition, the simulation based on the implicit variational method can be used to predict the maximal amplification factor for various form of wave groups at any location. Two analytical approximate models, namely the low-dimensional model and the optimization model with conserved properties, are developed to further exploit the amplification factor. The low-dimensional model takes into account the primary mode and the third order mode which are most relevant for bichromatic waves with small frequency differences. Given an initial condition, an analytical expression for the maximal amplitude of the evolution of bichromatic wave groups within this truncated model can be readily obtained. Good agreement is observed between the analytical and numerical solutions: when the initial amplitude is not too large, or when the difference of frequencies is not too small. The optimization model with conserved properties can predict the maximal amplification factor for all forms of wave groups' evolutions. Given an initial condition and a prescribed location, the analytical expression of a function which not only achieves the maximal amplitude at that location but is also consistent with the initial values of the conservation of energy, linear momentum and hamiltonian of the nonlinear Schrödinger equation, can be obtained. When tested with bichromatic wave groups, the model gives rather accurate prediction of the maximal amplification factor compared to the numerical simulations. The results obtained from numerical simulations and the two analytical approximate models are motivated and relevant in the generation of waves in hydrodynamic laboratories.

## ABSTRAK

Penyelidikan ini bertujuan untuk mengkaji faktor amplifikasi maksimum bagi perambatan kumpulan gelombang, terutamanya kumpulan gelombang dwikromat, yang diuraikan oleh persamaan Schrödinger tak linear. Satu kaedah berangka baru, iaitu kaedah ubahan tersirat yang menggabungkan teknik pembezaan tersirat dan ubahan telah dicadangkan. Apabila dibandingkan dapatan ini dengan penyelesaian tepat satu soliton dan dua soliton, ternyata kaedah berangka ini lebih baik dari segi kejituan dan pengekalan sifat keabadian tenaga, daripada kaedah berangka yang sedia ada, iaitu kaedah beza ke depan tak tersirat dan kaedah tersirat Crank-Nicolson. Diberi satu syarat awal, simulasi berangka berasaskan kaedah ubahan tersirat boleh digunakan untuk meramalkan faktor maksimum amplifikasi pada mana-mana kedudukan untuk pelbagai kumpulan gelombang. Dua model analisis, iaitu model matra rendah dan model optimum dengan sifat keabadian dihasilkan untuk mengkaji faktor maksimum amplifikasi dengan lebih mendalam. Model matra rendah mengambilkira mod utama dan mod peringkat ketiga, yang mana mod ini sangat berhubungkait dengan gelombang dwikromat yang perbezaan frekuensinya kecil. Di dalam model terpancang ini, ungkapan analisis amplitud maksimum bagi perambatan kumpulan gelombang dwikromat boleh didapati jika syarat awal diberi. Tidak banyak perbezaan didapati antara penyelesaian analisis dan penyelesaian simulasi, apabila amplitud awal tidak terlalu besar, atau apabila perbezaan frekuensi tidak terlalu kecil. Model optimum dengan sifat keabadian boleh menelaah faktor amplifikasi maksimum perambatan kumpulan gelombang yang pelbagai bentuk. Diberi syarat awal serta satu lokasi tertentu, ungkapan analisis boleh didapati bagi fungsi yang bukan sahaja mencapai amplitud maksima di lokasi tersebut, tetapi juga mengekalkan sifat keabadian tenaga, momentum linear serta hamiltonan bagi persamaan Schrödinger tak linear. Apabila model ini diuji dengan kumpulan gelombang dwikromat, model ini memberikan telahan yang baik bagi faktor amplifikasi maksimum berbanding dengan penyelesaian simulasi berangka. Dapatan daripada simulasi berangka serta kedua-dua model analisis hampiran ini adalah menggalakkan dan berhubungkait dengan penjanaan gelombang di makmal hidrodinamik.

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## LIST OF SYMBOLS

$A$	-	complex amplitude function of the first order harmonic mode for the wave envelope
$\check{A}$	-	complex amplitude function $A$ in terms of $x$ and $t$ variables
$A^*$	-	complex amplitude function of the first order harmonic mode for the wave envelope corresponds to spatial-nonlinear Schrödinger equation
$\mathbf{a}$	-	vector consists of the complex amplitude function from first mode to $(2N - 1)$ mode
$\mathbf{a}_\tau$	-	vector consists of the first order derivative of complex amplitude function from mode-1 till mode- $(2N - 1)$ with respect to time variable
$a$	-	amplitude parameter
$a_0$	-	initial amplitude of bichromatic wave groups
$a_j$	-	complex amplitude function for the $j$ th mode in higher dimensional model
$a(\tau)$	-	complex amplitude function for the primary mode in low-dimensional model
$\mathbf{b}$	-	vector with the element at $j$ th row defined as $(2j - 1)^2 a_{(2j-1)}$
$B$	-	complex amplitude function of the second order non-harmonic mode for the wave envelope
$b(\tau)$	-	complex amplitude function for the third order mode in low-dimensional model
$\mathbf{c}$	-	vector with the element at row- $j$ defined as $a_{ij}$
$C$	-	complex amplitude function of the second order harmonic mode for the wave envelope
$\mathbb{C}$	-	set of all complex numbers
$c$	-	arbitrarily constant
$c.c.$	-	abbreviation of complex conjugate

$\mathbf{D}$	-	matrix resulting from the linearization of the difference equation for Crank-Nicolson implicit method
$\widehat{\mathbf{D}}$	-	matrix resulting from the linearization of the difference equation for implicit variational method
$E_j$	-	$j$ th function of $\tilde{\tau}$ associate with approximating function in numerical scheme
$\mathbf{e}^n$		error vector at $n$ th time
$e$	-	scaling coefficient
$\mathbf{F}$	-	matrix resulting from the nonlinear terms of the difference equation for Crank-Nicolson implicit method
$\widehat{\mathbf{F}}$	-	matrix resulting from the nonlinear terms of the difference equation for implicit variational method
$f$	-	real function of spatial variable defined in the Ansatz of the optimization model with conserved properties
$G$	-	functional of the wave energy properties for nonlinear Schrödinger equation
$G_{i,j}$	-	the nonlinear terms defined at point $(\tilde{\xi}_i, \tilde{\tau}_j)$ from the difference equation of implicit variational method
$g$	-	gravitational constant
$\mathcal{H}$	-	Hamiltonian function of the temporal nonlinear Schrödinger equation
$\mathcal{H}^*$	-	Hamiltonian function in the low-dimensional model
[Hz]	-	unit in Hertz
$h$	-	the water depth at equilibrium state in the hydrodynamic wave tank
$\Im$	-	imaginary part
$i$	-	imaginary unit where $i^2 = -1$
$J$	-	the absolute difference of the conserved value of energy between time $\tilde{\tau}_0$ and $\tilde{\tau}_n$
$J_n$	-	the conserved value of energy at time $\tilde{\tau}_n$
$K$	-	perturbed wavenumber
$k$	-	wavenumber of regular wave
$k_0$	-	central wavenumber

$k_1$	-	wavenumber of the first monochromatic wave
$k_2$	-	wavenumber of the second monochromatic wave
$k_{crit}$	-	critical wavenumber
$L$	-	functional of linear momentum for nonlinear Schrödinger equation
$\mathcal{L}$	-	actional functional of the temporal nonlinear Schrödinger equation
$\mathcal{L}^*$	-	actional functional defined in low-dimensional model
$L_\infty$	-	the maximal absolute difference between numerical simulation results and the exact solutions
$M$	-	functional of maximal amplitude for the wave group elevation
[m]	-	unit in meter
[min]	-	unit in minutes
$n\{E\}$	-	number of elements in set $E$
$P$	-	matrix resulting from the nonlinear term in higher dimensional model
$q$	-	amplitude parameter of the bichromatic wave groups
$R$	-	pseudo differential operator
$\mathbb{R}$	-	set of all real numbers
$\hat{R}$	-	Fourier transform from $R$
$\mathbf{R}$	-	matrix resulting from the first variation of the Hamiltonian function with the entry at row- $i$ and column- $j$ as $R_{ij} = \phi_i \phi_j$
$r_1$	-	modulus of function of $\tau$ for the primary mode in low-dimensional model
$r_2$	-	modulus of function of $\tau$ for the third mode in low-dimensional model
$r_j$	-	modulus of function of $\tau$ for the $j$ -th mode in higher dimensional model
$\Re$	-	real part
$\mathbf{S}$	-	matrix resulting from the first variation of the Hamiltonian function with the entry at row- $i$ and column $j$ as $\frac{d\phi_i}{d\xi} \frac{d\phi_j}{d\xi}$
[s]	-	unit in seconds

$T$	-	central period
$T_1$	-	period of the first monochromatic wave
$T_2$	-	period of the second monochromatic wave
$t$	-	time variable
$u$	-	complex function in optimization model with conserved properties
$\check{u}$	-	complex function obtained after numerical simulation in optimization model with conserved properties
$V$	-	real functional space
$V_0$	-	group velocity
$v$	-	the opposite velocity of the two solitons in bisoliton solution
$\vec{v}$	-	fluid velocity vector
$\mathbf{v}_s$	-	eigenvectors
$x$	-	the horizontal coordinate of a two-dimensional plot in Cartesian coordinate
$y$	-	the vertical coordinate of a two-dimensional plot in Cartesian coordinate
$Z$	-	complex amplitude function of the first order harmonic mode for the wave envelope in normalized coordinate
$\mathbb{Z}$	-	set of all integers
$Z_0$	-	complex amplitude function $Z$ at initial state
$Z_{m,n}$	-	numerical approximation to the exact solution $Z$ at $(\tilde{\xi}_m, \tilde{\tau}_n)$
$\mathbf{Z}^{n+1}$	-	vector containing the unknown value of $Z$ at time $\tilde{\tau}_{n+1}$
$z$	-	vertical coordinate of a three-dimensional plot in Cartesian coordinate

### Greek Symbols

$\alpha$	-	amplitude parameter in the solution of optimization model with conserved properties
$\beta$	-	coefficient of the linear term in nonlinear Schrödinger equation
$\beta^*$	-	coefficient of the linear term in spatial nonlinear Schrödinger

		equation
$\chi$	-	amplification factor
$\chi_{low-dim}$	-	amplification factor of low-dimensional model
$\chi_s$	-	amplification factor from numerical simulation results
$\chi_1$	-	amplification factor of optimization model with conserved properties
$\chi_2$	-	amplification factor of optimization model with conserved properties and after numerical simulation
$\delta(\tilde{\xi} - \hat{\xi})$	-	Dirac delta function
$\epsilon$	-	perturbation parameter
$\eta$	-	function describing the free surface elevation
$\tilde{\eta}$	-	arbitrarily function
$\Delta T$	-	difference of period for bichromatic waves
$\Delta \kappa$	-	difference of wavenumber when compare to central wavenumber
$\Delta \psi$	-	phase difference between two solitons of the bisoliton solutions
$\Delta \tilde{\tau}$	-	grid spacing in time variable used in numerical scheme
$\Delta \tilde{\xi}$	-	grid spacing in spatial variable used in numerical scheme
$\gamma$	-	coefficient of the nonlinear term $ A ^2 A$ in nonlinear Schrödinger equation
$\gamma^*$	-	coefficient of the nonlinear term $ A ^2 A$ in spatial nonlinear Schrödinger equation
$\kappa$	-	difference of wavenumber of bichromatic waves in terms of slow variable
$\bar{\kappa}$	-	difference of wavenumber of bichromatic waves in terms of $x$ and $t$
$\lambda$	-	coefficient in the difference formula of the implicit Crank-Nicolson method
$\hat{\lambda}$	-	coefficient in the difference formula of the implicit variational method
$\lambda_i$	-	Lagrange multiplier constants
$\lambda_s$	-	eigenvalues
$\nu$	-	difference of frequency of bichromatic waves in terms of slow



		variable
$\bar{\nu}$	-	difference of frequency of bichromatic waves in terms of $x$ and $t$
$\omega$	-	regular wave frequency
$\omega_0$	-	central regular wave frequency
$\omega_1$	-	frequency of the first monochromatic wave
$\omega_2$	-	frequency of the second monochromatic wave
$\Omega$	-	exact dispersion relation
$\Phi$	-	velocity potential
$\phi_m$	-	spline hat function defined in the implicit variational method
$\psi_{ijkl}$	-	defined as $\cos((2i-1)\kappa\xi)\cos((2j-1)\kappa\xi)\cos((2k-1)\kappa\xi)\cos((2l-1)\kappa\xi)$
$\rho$	-	maximal amplitude at each spatial variable over all time defined in optimization model with conserved properties
$\tau$	-	slowly varying time variable
$\tau^*$	-	slowly varying time variable corresponds to spatial nonlinear Schrödinger equation
$\tilde{\tau}_n$	-	discrete time variable in numerical scheme
$\tilde{\theta}$	-	phase parameter
$\theta_0$	-	phase for the carrier wave
$\theta_1$	-	amplitude of function of $\tau$ for the primary mode in low-dimensional model
$\theta_2$	-	amplitude of function of $\tau$ for the third order mode in low-dimensional model
$\varphi$	-	difference of $\theta_1$ and $\theta_2$ in low-dimensional model
$\xi$	-	spatial variable in the moving frame of reference $\xi = x - \Omega'(k_0)t$
$\xi^*$	-	spatial variable corresponds to spatial-nonlinear Schrödinger equation
$\hat{\xi}$	-	spatial position when wave groups elevation achieves maximal amplitude
$\tilde{\xi}_m$	-	discrete spatial variable in numerical scheme

**Superscripts**

— - complex conjugate

\* - linearized from

**Subscripts**

*lab* - physical laboratory coordinate

$\sim$  - normalized coordinate

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# CHAPTER 1

## INTRODUCTION

### 1.1 General Introduction

The subject of water waves has been a fascinating field being studied for a century ago. The behaviour of water waves can be appreciated by any of us in nature descriptively without any technical knowledge. Although the literature is vast, apart from a few conjectures and some numerical simulations, the phenomenon of waves with large amplitude has not yet been exploited or understood fully (Liu and Pinho, 2004; Haver, 2004; Dysthe, 2000). The causes of its occurrence and its properties are still an active area of research (Gunson et al., 2001). This kind of waves, known as extreme wave, freak wave or rogue wave is described as individual waves with exceptional wave height, steepness or abnormal shape. Monster waves, giant waves, abnormal waves or mega waves are other names used occasionally to define this huge breaking walls of water that come out of the blue. Mallory (1974) describes these waves as having a steeper forward face preceded by a deep trough, or “hole in the sea”. These waves are different from tsunamis or tidal waves. Tsunamis or tidal waves are very rare event when an earthquake or landslide displaces a large volume of water, and create a single large wave. But, extreme waves or freak waves seem to be a fundamental property of the ocean and are occurring far more regularly (Mori, 2004).

Several definitions have been proposed during the last decades, but currently there is no general consensus on a unique definition of extreme waves or freak waves (Belcher et al., 2001). The simple, commonly used definition in characterizing possible extreme waves is the ratio of maximum wave height to the significant wave height must be greater than 2 (Liu and Pinho, 2004). Skourup et al. (1996) used the similar criteria but the significant wave height is defined from the surrounding 20[*min*] wave record. Dean (1990) defined freak waves as extraordinary large water waves whose heights exceed the significant wave height of a measured wave train by a factor of 2.2. Technically the event of extreme wave could happen even when the significant wave height is small. For example when the significant wave height is 12[m], or 2[m] or even 0.5[m]. An interesting question here is would such an event with low significant wave height still be considered as freak or extreme? Thus, there exist some definitions which imposed restrictions on the extreme wave height. Paprota et al. (2003) defined the extreme waves in Baltic Sea as wave exceeding twice the significant wave height with the significant wave height must be larger than 1[m].

## 1.2 Significance of Research

There is a growing evidence suggesting that the risk of extreme waves is higher than what had been expected, but the data are far from conclusive. According to Lloyd's global database and Lloyd's casualty report (Bitner-Gregersen and Eknes, 2001) on the data of five years (1995-1999) of ship accidents due to bad weather, only a few accidents were categorized as being caused by freak waves. However, Gunson et al. (2001) concluded that this does not mean that other ship accidents were not caused by extreme waves as extreme waves is a newly introduced term. Furthermore, the definition of extreme waves is still controversial. One of the most well known ship accidents due to extreme waves is

the *Spray's* hull which was completely submerged by a giant wave well off the Patagonian coast (White and Fornberg, 1998). Kjeldsen (2000) reported that at a special occasion, two large ships M/S “NORSE VARIANT” and M/S “ANITA” disappeared at the same location in a lapse of one hour in time. The Court of Inquiry finally concluded that the loss of these two bulk ships was due to an event in which a very large wave suddenly hit and broke several hatch covers on the ships’ deck, subsequently the ships were filled with water and sank before any emergency call was given.

In view of the increasing awareness of the risks from extreme waves, the issues raised in the field of research in extreme waves include identification of the underlying mechanisms; role of refraction by sub-mesoscale currents; method of prediction; cause of occurrence rate; role of nonlinearity; spatial-temporal statistics of extremes and an acceptable definition for freak waves (Belcher et al., 2001). Furthermore, there is an increasing need to generate the deterministic extreme waves with large amplitude that do not break in a hydrodynamic laboratory for the purpose of testing floating bodies such as ships and fixed structures (Andonowati and Groesen, 2003a).

The hydrodynamic laboratory is a laboratory that provides facilities for the testing of the performance of maritime structures on a model scale. This would allow the designers of the structures to collect the hydrodynamic properties of their designs before the actual construction. In order to meaningfully conduct the tests, the model must be tested under the extreme condition apart from the normal tests under the generation of regular waves. It is therefore becoming increasingly important for the wave generator to be able to generate the extreme wave condition at a prescribed position in the water tank so that the model to be tested can be placed at that location beforehand.

The motivations of this project stem from the problem of generating the extreme waves in the hydrodynamics laboratory for testing the performance of the marine structure on scaled models. In a realistic situation involving large spatial and temporal interval, such a generation is not easy due to the physical limitation of the wave makers as well as the nonlinear behavior that dominates the deformation of propagating signals from the wave maker. The dominant nonlinear effects in large amplitude wave generation can be seen from the previous theoretical, numerical as well as experimental investigation on bichromatic waves (Stansberg, 1990; van Groesen et al., 1999; Westhuis et al., 2001). A bichromatic wave group is resulting from the linear superposition of two regular waves with different frequencies. A well known model for such collective behaviour of wave groups evolution is the nonlinear Schrödinger (NLS) equation.

In a complete realistic investigation, the model of full surface equations with 3-dimensional effects has to be taken into account. In this report, we would like to investigate the amplitude amplification of the evolution of wave groups, in particular the bichromatic wave groups, within the nonlinear Schrödinger equation framework. This thesis present various ways to predict the maximal amplitude for different forms of wave groups when an initial condition is given. The results are motivated in the generation of deterministic gravity waves in the hydrodynamic laboratory, especially in predicting the amplitude amplification for the evolution of waves. The following section outlines the aims and scopes of this research in more detail. This is followed by the literature reviews in Section 1.4. Finally, we wrap up this chapter by giving an outlines of this thesis.

### 1.3 Objectives and Scopes

The main aim of the research is to study the amplitude amplification of the nonlinear deterministic gravity waves. The scope of the investigations will be limited to the evolution of initial bichromatic wave groups generated in the hydrodynamic laboratory and governed by the (focusing) nonlinear Schrödinger (NLS) equation. In particular, we will seek to:

1. develop and implement a numerical model to simulate the evolution of wave groups.
2. determine the maximal amplification factor by numerical simulation and the approximate analytical models so as to elucidate the theoretical basis of the nonlinear evolutions.
3. provide a means to predict the maximal amplitude amplification.

### 1.4 Literature Reviews

In this section, we present the literature reviews related to our research. We begin our discussion in Subsection 1.4.1 on the general overview of different model equation and various techniques that had been applied in the analysis of the phenomenon of waves with large amplitude, inclusive of the research work related to the amplitude amplification. Since we intend to develop a numerical code to simulate the evolution of wave groups, the review on the numerical simulation of nonlinear waves is presented in Subsection 1.4.2.



### 1.4.1 Extreme Waves or Freak Waves

The early work on extreme wave or freak wave was based on linear models. Peregrine (1976) and Jonsson (1990) used interaction of waves with opposing currents to explain the existence of freak waves. Stansberg (1990) analyzed the extreme event by using the linear superposition of long waves with preceding shorter waves. Shyu and Phillips (1990) explained the freak waves as the blockage of short waves by longer waves and currents. Irvine and Tilley (1988) used conservation of wave action to analyze the Synthetic Aperture Radar data of the Agulhas current, and concluded that the extreme waves are due to the current induced caustics. White and Fronberg (1998) investigated the extreme event by using the wave ray theory, and they showed that the probability distribution for the formation of a freak wave - formed from the concentration of wave action in a caustic region did not depend on the statistics of the current. Donato et al. (1999) analyzed the phenomenon based on the interaction of surface waves with internal waves and found that the wave steepness was not necessarily associated with a particular phase of the internal wave. Various formulations on the maximal wave height of regular waves are given in Singamsetti and Wind (see Dingemans, 2000a). These formulations are based on regular linear theory with inclusion of breaking or dissipations effects. The conclusion from linear theory show that the mechanisms are related to wave focusing of frequency modulated wave groups (dispersive and geometrical focusing), and with blocking effects of spectral components on opposite current (Kharif and Pelinovsky, 2003)

Alternatively, the probabilistic approach is proposed to perform the statistical analysis of the extreme wave on the aspect related to heights, crest amplitude, trough depths and etc. (Bocotti, 1981; Phillips et al., 1993; Azais and Delmas, 2002). The Gaussian beam summation method are used to combine with stochastic processes to calculate extreme wave statistics for wave propagation

problems where caustics occurred randomly (Nair and White, 1991). Wavelet-based approaches are proposed to process the time and frequency localized freak wave (Liu and Mori, 2000; Jacobsen et al., 2001; Kim and Kim, 2003). Techniques had also been developed to detect wave groups and corresponding individual extreme wave in Synthetic Aperture Radar images (Lehner et al., 2002; Niedermeier et al., 2002).

The nonlinear mechanism was proposed as an alternative approach in investigating freak waves. Dean (1990) suggested that the nonlinear superposition of waves might produce waves larger than the waves generated under linear superposition. The resulting statistical distribution of wave heights had larger waves occurring more frequently than that predicted by Rayleigh distribution. Benjamin and Feir (1967) demonstrated theoretically that nonlinearities can also cause instability of a regular wave train. They showed that regular wave train were unstable to perturbations for weakly nonlinear surface gravity waves in deep water. Gerber (1987) found the instability of wave groups in the neighborhood of a caustic caused by a shear current based on a variation of the Benjamin-Feir instability. The approach of nonlinear wave focusing was studied in situations resembling the rough seas in natural conditions, and has been theoretically validated in several models.

Smith (1976) used the adapted nonlinear Schrödinger equation and Peregrine (1986) used the NLS method to estimate the extreme wave. The breather type solutions (Henderson et al., 1999; Dysthe and Trulsen, 1999), and the solution of the soliton on finite background (Osborne et al., 2000; Osborne, 2001) of nonlinear Schrödinger equation had been related to the analysis of extreme wave event. The phenomenon of nonlinear wave focusing was also demonstrated numerically in the framework of 2-dimensional Schrödinger equation (Onorato and Serio, 2002). Slunyaev et al. (2002) used 2 dimensional model of the Davey-

Stewartson equation to study the extreme wave event.

Some experiments had been carried out to show the effects of possible freak or extreme waves occurring in a laboratory waveflumes (Baldock and Swan, 1996; Kaldenhoff and Schlurmann, 1999; Johanessian and Swan, 2001). By introducing a spatial ordering of frequencies in a chirped wavetrain, Pelinovsky et al. (2000) proposed that this effect could produce short groups of large waves at a given position in a wave tank at the laboratory. They developed a one-dimensional model of Korteweg & de Vries equation for the analysis. However, the question of how such a situation may develop spontaneously has not been answered. Besides, the limits of the elevation have not been analyzed in details.

Another mathematical model which utilized Kadomtsev-Petviashvili (KP) equation was used to describe the nonlinear shallow water gravity waves (Kadomtsev and Petviashvili, 1970), with the exact solution can be obtained using Hirota bilinear formalism (Hirota, 1971). In this framework, the interaction of 2-dimensional two-soliton has also been analyzed (Satsuma, 1976; Freeman, 1980). Based on this model, many authors concluded that the amplitude of the water elevation at the intersection point of two solitons exceed the sum of the amplitude of the incoming solitons (Segur and Finkel, 1985; Haragus-Courcelle and Pego, 2000; Chow, 2002). Using the same framework, Peterson et al. (2003) found that the extreme surface elevations was found to be up to four times exceeding the amplitude of the incoming waves, and the maximum amplification factor is 2 in the case of the Mach reflection.

Though scientists are still unsure of the causes of the existence of extreme waves, in general they believed that the occurrence of extreme waves in the coastal waters may be explained by focusing due to refraction by bottom topography, current gradients, or even reflection from land (Lavrenov, 1998). On the other hand, the extreme waves in the open ocean may be a result of nonlinear self-

focusing mechanisms (Trulsen and Dysthe, 1997; Henderson et al., 1999; Osborne et al., 2000; Peterson et al., 2003) which causes a high concentration of wave energy (Dysthe, 2000).

It has recently been established that the nonlinear Schrödinger equation can describe many features of the dynamics of extreme waves which are found to arise as a result of nonlinear self-focusing phenomenon (Henderson et al., 1999; Osborne et al., 2000; Onorato et al., 2001). Thus, we employ the framework of NLS equation, particularly the temporal-nonlinear Schrödinger equation which is more relevant in describing the wave groups evolution in laboratory.

#### 1.4.2 Numerical Simulation of Nonlinear Waves

The numerical simulation of nonlinear waves depends on the mathematical model employed. The conservation of mass and momentum in an isothermal flow lead to the Navier-Stokes equations for the fluid velocity, pressure and density. If the fluid is inviscid, the equations are further reduced to Euler's equations for the velocity. By introducing the velocity potential based on the assumptions that the fluid is incompressible and irrotational, we have the potential flow model described by the quantities in terms of potential and the shape of the free surface. If further assumptions are imposed, we have numerous free-surface models in terms of the quantity describing the free surface as discussed in Section 2.2.

The research work on the numerical simulation of nonlinear potential flow are vast, and there are extensive reviews on this subjects. These include articles by Schwartz and Fenton (1982) on the theoretical aspect of ideal, nonlinear flow; Yeung (1982) for both linear and nonlinear flow; Tsai and Yue (1996) on the treatment of the free surface and (large) nonlinearity. All numerical methods for nonlinear potential flow model are developed under specific sets of condi-

tions, assumptions, and have different degree of approximation depend on the point of view of the application. Generally, the schemes can be categorized into boundary-discretization and volume-discretization approaches (Tsai and Yue, 1996). Boundary-discretization methods are used for inviscid irrotational flows, whereas volume-discretization scheme can be applied to both inviscid and viscous flows.

In boundary-discretization approaches, the boundary integral equation is used in approximating the boundary value problems. Longuet - Higgins & Cokelet (1976) were the first to simulate the dynamics of nonlinear waves by employing the Eulerian-Lagrangian method in separating the elliptic boundary value problems from the dynamics equations at the free surface. The boundary integral equation is then discretized using boundary element method, among are the work by Dommermuth et al. (1988) with 2d constant panel method, Romate (1989) with 3d second-order panel method, Grilli et al. (1989) with 2d higher order boundary elements and Celebi et al. (1998) with 3d desingularised boundary integral.

In volume-discretization scheme, the boundary value problems of the potential flow models are solved by finite difference and finite element methods. In finite difference method, the geometry is usually mapped to a rectangular domain, for example the work by Chan (1977) and DeSilva et al. (1996). Finite element method is more widely used in viscous flow problems and linearised free surface problems (Washizu, 1982). Not until recently, it is used in potential flow problems, for example, in the work by Wu and Eatock Taylor (1994), Cai et al. (1998).

By posing various and additional assumptions to the potential flow models, we have the nonlinear uni-directional free surface model with the KdVKorteweg & de Vries equation as the governing equation. Another widely used model equation for nonlinear wave propagation is the nonlinear Schrödinger equation which

describes the dynamic evolution of the slowly varying complex wave envelope. For the initial value problem involving either the Korteweg & de Vries equations or nonlinear Schrödinger equations as the model equations, the numerical schemes can be roughly categorized into two categories: namely the finite difference method and the finite fourier/pseudospectral method.

In finite difference approaches, the schemes could be calculated explicitly or implicitly. The explicit finite difference method include the classical method (see Taha and Ablowitz, 1984a), the Zabusky and Kruskal scheme (1965) which utilized the explicit leapfrog scheme in approximating the solutions of Korteweg & de Vries equations. For implicit approach, Greig and Morris (1976) proposed a hopscotch scheme which leads to a quasi-tridiagonal system of equations to be solved at each time step. Goda (1975) proposed an implicit scheme for Korteweg & de Vries equation which requires solving a quasi-pentagonal system of equations at each time level. Various implicit approaches have also been suggested (see Taha and Ablowitz, 1984b).

For the category of finite fourier transform or pseudospectral methods, Tappert (see Taha and Ablowitz, 1984b) introduced a split step Fourier method which utilize the discrete fast Fourier transform. Fornberg and Whitham (1978) proposed the pseudospectral methods which use the fast Fourier transform method combined with the leap-frog time step.

The new numerical scheme, namely implicit variational method proposed in this thesis differs from the previous reported schemes as the proposed method combines the idea of finite difference approach and pseudospectral methods.

## 1.5 Thesis Outline

This section gives the main contents of the thesis and serves as an outline for quick reference to the appropriate section. The thesis is divided into 6 chapters including this introductory chapter. Chapter 2 discusses some fundamentals in the linear and nonlinear wave groups evolutions, particularly the derivation of the model equation, the temporal-nonlinear Schrödinger equation which governs the evolution of wave groups for the deterministic free surface waves generated in the hydrodynamic laboratory. In Chapter 3, we develop a new numerical approach: implicit variational numerical method to approximate the solutions of the normalized nonlinear Schrödinger equation. The implicit variational method is used to predict the maximal amplification factor of the nonlinear Schrödinger equation in Chapter 4 and Chapter 5. These results are then compared with two analytical approximate models; namely low-dimensional model and optimization model with conserved properties, in Chapter 4 and Chapter 5 respectively. Chapter 6 contains the concluding remarks and the recommendations for further research. In the following, detail contents from Chapter 2 to Chapter 6 are summarized.

Chapter 2 concerns with the model equation that we used to describe the evolution of wave groups in the hydrodynamic laboratory. In Section 2.1, we begin with a literature review of the nonlinear focusing, which has been recently established as one of the possible causes that leads to the wave with large amplitude. In particular, we discuss the instability of the nonlinear wave groups evolutions governed by the nonlinear Schrödinger equation. This is then followed by Section 2.2 on the derivation of the field equations describing the potential flow, and the assumptions leading to the equations describing the free-surface dynamics of the unidirectional waves. We then introduce the improved Korteweg & de Vries equation which is known to be more relevant in representing the evolutions of free-surface waves in the hydrodynamics laboratory compared to

the original Korteweg & de Vries equation. Next, the equation governing the free-surface wave groups evolutions, namely the temporal-nonlinear Schrödinger equation is derived from the improved Korteweg & de Vries equation. Materials from Chapter 2 are mainly extracted from the references and provides the fundamentals for the research work carried out in this thesis. However, we observe that there are some errors for the given coefficients in the work related to the derivation of the temporal-nonlinear Schrödinger equation and thus the improvements have been done in Section 2.4. The chapter ends with some brief introduction of the properties of temporal-nonlinear Schrödinger equation. For the remaining discussion of this thesis, the abbreviations of KdV equation and NLS equation will be used to denote Korteweg & de Vries equation and nonlinear Schrödinger equation respectively.

The aim of Chapter 3 is to introduce a new numerical scheme in simulating the wave groups evolutions governed by the NLS equation. The new method combines the implicit idea and the variational idea in deriving the difference equation, and thus this new method is given the name as implicit variational method. For validation of this new numerical scheme, we check the numerical results with two established solutions: the exact analytical solution for one-soliton wave groups as well as the exact analytical solution for bisoliton wave groups. Further to that, we also conduct a comparison among the proposed method with two other existing well known methods, namely finite difference and Crank-Nicolson implicit method. For the purpose of comparisons, we calculate the absolute errors obtained from both schemes with the exact solutions for one-soliton wave groups and bisoliton wave groups. Besides that, we also carried out a study to check which numerical methods can preserve the wave energy conservation property of NLS equation. We first introduce the idea of the finite difference and Crank-Nicolson implicit method in Section 3.2. Detail working in developing our new proposed method is delivered in Section 3.3. Stability analysis has been carried



out in Section 3.4 for the three numerical schemes. Whereas, results of comparisons are treated in Section 3.5. Our comparisons show that the implicit variational method yields better results and is more superior in conserving the wave energy conservation property. Hence, the implicit variational method developed would be used to simulate the numerical results in predicting the maximal amplification factor in Chapter 4 and Chapter 5.

In order to further exploit the maximal amplification factor of the non-linear wave groups evolutions, two analytical approximate mathematical models have been constructed besides the numerical simulations. Chapter 4 discusses the low-dimensional model which investigates the dynamics of wave groups with initial bichromatic wave groups. Whereas, Chapter 5 presents another alternative approach in studying the maximal amplification factor by looking at several conserved properties of the NLS equation. The optimization model developed in Chapter 5 is capable to suit for all the different types of wave groups.

Chapter 4 begins with the literature reviews on the bichromatic wave groups, this is followed by a discussion on the scaling and the transformation used to relate the normalized dimensionless coordinates to the physical laboratory coordinates. Section 4.3 presents the mathematical set up of the low-dimensional model in the four-dimensional manifold. By exploiting the conservation of energy of the NLS equation, a reduction is obtained to a two-dimensional Hamiltonian system, and the system is further reduced to one-dimensional dynamic equation. The analytical approximate results obtained are begin discussed in Section 4.4. In Section 4.5, we relate the results obtained in the normalized coordinate to the physical laboratory coordinate, correspond to the dimensions of the High Speed Basin at Maritime Research Institute Netherlands (MARIN), and further made a comparison between the numerical simulations and the results obtained from low-dimensional model. Due to the fact that the maximal amplification factor

obtained from the low-dimensional model is at most  $\sqrt{2}$  (Tan and Andonowati, 2003), we thus extend the model to high dimensional model in Section 4.6 and give the detail working on the derivation of the actional functional in terms of  $N$  modes. However, due to the fact that the resulting actional functional is defined in a  $2N$ -dimensional manifold, the complication makes the analysis work a nontrivial task.

In order to investigate the amplification factor larger than  $\sqrt{2}$ , we introduce another analytical approximate model; the optimization model which couples the conserved properties of NLS equation in Chapter 5. This model provides a general means to study the maximal amplification factor, not only restricted to the bichromatic wave groups, but also to other type of wave groups. The model is constructed based on the normalized NLS equation which can be easily scaled or transformed to fit the physical laboratory setting. Section 5.2 discusses the set up of this new model, this is followed by the discussion of its analytical solutions in Section 5.3. In Section 5.4, the analytical results are being tested with the well known exact solution of one-soliton wave groups in Section 5.4.1 and both results agree well. We then use this model to generate the prediction of maximal amplification factor and the results are then compared with numerical simulation in Section 5.4.2. The parameters used in the comparison of bichromatic wave groups are related to the actual dimension of the High Speed Basin at MARIN.

Chapter 6 is the final and concluding chapter, which contains the concluding remarks, summary of the research findings as well as some suggestions for future research. All the references quoted are listed in the reference section at the end of Chapter 6. There is one appendix on the discussion of the numerical solver of in solving the nonlinear systems obtained from both numerical methods, the Crank-Nicolson implicit method and the implicit variational method.

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