# Optimal Input Shaping For Vibration Control of a Flexible Manipulator Using GA

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# Abstract

This paper presents optimization of input shaping technique using genetic algorithms (GAs) for a planar single link flexible manipulator system with varying payload. An unshaped bang-bang torque is shaped by convolving a sequence of impulses with an input shaper. The time locations of the impulses are determined using GA. The simulation work is designed in Matlab based environment and the responses are presented in the time and frequency domains. The performance of optimal input shaper is compared to the bang-bang torque in terms of maneuver speed, transient and steady state responses, computational complexity and level of reduction at resonances mode.

# **1. Introduction**

Robotic manipulators are usually designed to be rigid to maximize stiffness, thus minimizing system vibration to achieve accurate position. This leads to usage of heavy material for the system and limits the speed operation. The disadvantages are high energy consumption, increase in actuator size and overall cost [1].

As a consequence, system that once consisted of heavy materials are now being constructed using lighter materials. This lighter weight material not only lessens the mass and can increase the speed of a system, but also introduces flexible modes into system [2]. Flexible robot manipulator exhibits many advantages over their rigid counterparts: they require less material, are lighter in weight, have higher manipulation speed, lower power consumption, require small actuators, are more maneuverable and transportable, have less overall cost and higher payload to robot weight ratio [3].

However, the control of flexible robot manipulators to maintain accurate positioning is a challenging problem. Due to the flexible nature and distributed characteristic of the system, the dynamics are highly non-linear and complex. Problems arise due to precise positioning requirement, vibration due to system flexibility, the difficulty in obtaining accurate model of the system and non-minimum phase characteristics of the system [4,5]. The implication of the reduction of vibration in these studies enables it to be introduced in the space structures, flexible aircraft wings, robotic manipulator, disk drive and overhead cranes.

Input shaping is a feed-forward control technique

for reducing residual vibration in computer-controlled machines. Input shaping is implemented by convolving a sequence of impulses, an input shaper, with a desired system command to produce a shaped input that is then used to drive the system [6]. The process of shaping a bang-bang input is demonstrated in Fig. 1 [7]. Instead of using the bang-bang input, a staircase command resulting from the convolution is used as the command signal.

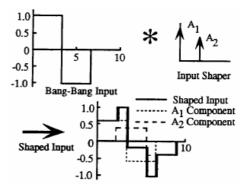


Fig. 1: Shaping a bang-bang input.

The exact values of the input shaper have to be obtained in order to cancel the vibrations in the system. This can be done by analyzing the natural frequencies and the damping ratio of the system. However, for a flexible manipulator the dynamic of the system changes due to the changes in the payload. Thus, a new value of natural frequency is needed in order to generate an effective input shaper. This problem can be overcome by introducing GA to determine the optimal input shaper for minimizing the residual vibration in the system with varying payload.

This paper presents optimization of input shaping technique using GA for a planar single link flexible manipulator system with varying payload. An unshaped bang-bang torque is shaped with an input shaper where the amplitude and time locations of the impulses are determined using GA. The simulation work designed in Matlab and the performance of optimal input shaper is compared to the bang-bang torque in terms or maneuver speed, transient and steady state responses, computational complexity and level of reduction at resonances mode.

# 2. The flexible manipulator system

A single-link flexible manipulator system shown in Fig. 2 is considered, where XOY and POQ represents the stationary and moving co-ordinates frames respectively,  $\tau$  represents the applied torque at the hub.  $E, I, \rho, A, I_h$  and  $M_p$  represents the Young modulus, area moment of inertia, mass density per unit volume, cross-section area, hub inertia and payload mass of the manipulator respectively, where an aluminium type flexible manipulator of dimensions  $900 \times 19.008 \times 3.2004 mm^3$ ,  $E = 71 \times 10^9 N/m^2$ ,  $I = 5.1924m^4$  and  $\rho = 2710 kg/m^3$  is considered [1].

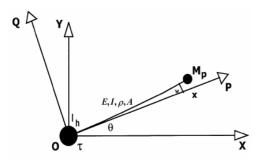


Fig. 2: Description of the flexible manipulator system.

#### 3. Modeling of the flexible manipulator

The system is modeled using the finite element (FE) method with 10 elements for feed forward control strategies. The flexible manipulator is assembled in n element and represented in a state space form. For a small angular displacement  $\theta(t)$  and a small elastic deflection w(x,t), the total displacement y(x,t) of a point along the manipulator at a distance x from the hub can be described as a function of both rigid body motion  $\theta(t)$  and elastic deflection w(x,t) measured from the line OX as:

$$y(x,t) = x\theta(t) + w(x,t)$$

Using the FE method, kinetic and potential energies of an element yields the element mass matrix,  $M_n$  and stiffness matrix,  $K_n$  as [8]:

$$M_{n} = \frac{\rho A l}{420} \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} & m_{15} \\ m_{21} & 156 & 22l & 54 & -13l \\ m_{31} & 22l & 4l^{2} & 13l & -13l^{2} \\ m_{41} & 54 & 13l & 156 & -22l \\ m_{51} & -13l & -13l^{2} & -22l & 4l^{2} \end{bmatrix}$$
$$K_{n} = \frac{EI}{l^{3}} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 12 & 6l & -12 & 6l \\ 0 & 6l & 4l^{2} & -6l & 2l^{2} \\ 0 & -12 & -6l & 12 & -6l \\ 0 & 6l & 2l^{2} & -6l & 4l^{2} \end{bmatrix}$$

where

$$m_{11} = 140l^{2} (3n^{2} - 3n + 1)$$
  

$$m_{12} = m_{21} = 21l(10n - 7)$$
  

$$m_{13} = m_{31} = 7l^{2} (5n - 3)$$
  

$$m_{14} = m_{41} = 21l(10n - 3)$$
  

$$m_{15} = m_{51} = -7l^{2} (5n - 2)$$

- 1

where l is the elemental length of the manipulator and n is the number of elements. Assembling the element mass and stiffness matrices utilising the Lagrange equation of motion, the desired dynamic equations can be obtained as:

$$M\ddot{Q}(t) + D\dot{Q}(t) + KQ(t) = F(t)$$
<sup>(1)</sup>

where M, D and K are global mass, damping and stiffness matrices of the manipulator respectively. F(t)is a vector of external force and Q(t) is a nodal displacement vector given as:

$$Q(t) = \begin{bmatrix} \theta & w_0 & \theta_0 & \cdots & w_n & \theta_n \end{bmatrix}^1$$

where  $w_n(t)$  and  $\theta_n(t)$  are the flexural and angular deflections at the end point of the manipulator respectively. The flexural and angular deflections, velocity and acceleration are all zero at the hub at  $\tau$  and the external force as the manipulator is considered pinned-free arm. Equation (1) can be represented in state space as

$$\dot{v} = Av + Bu$$
$$v = Cv$$

where

$$A = \begin{bmatrix} 0_m & | & I_m \\ -M^{-1}K & | & -M^{-1}D \end{bmatrix},$$
$$B = \begin{bmatrix} 0_{m \times 1} \\ M^{-1} \end{bmatrix}, \qquad C = \begin{bmatrix} I_{2m} \end{bmatrix}$$

 $0_m$  is an  $m \times m$  null matrix,  $I_m$  is an  $m \times m$ identity matrix,  $0_{m \times 1}$  is an  $m \times 1$  null vector,  $u = [\tau \quad 0 \quad \cdots \quad 0]^T$ ,

 $v = \begin{bmatrix} \theta & w_1 & \theta_1 & \cdots & w_n & \theta_n & \dot{\theta} & \dot{w}_1 & \dot{\theta}_1 & \cdots & \dot{w}_n & \dot{\theta}_n \end{bmatrix}^{\mathrm{T}}$ Solving the state-space matrices gives the vector of states v, that is, the angular, nodal flexural and angular displacements and velocities.

#### 4. Input Shaping

The bang-bang torque input for the system is then convolved with a sequence of impulses. The system can be modeled in a superposition of second order system with a transfer function of

$$G(s) = \frac{\omega^2}{s^2 + 2\xi\omega s + \omega^2}$$
(2)

where  $\omega$  is the natural frequency and  $\xi$  is the damping ratio of the system. The amplitude and the time location of the impulses are the most important criteria in designing impulse sequences. The first two impulses are sufficient to reduce vibration in the system drastically, but in this work, four impulses are used to cancel one vibration mode and thus increase the robustness of the input shaper to error in natural frequencies. Utilizing the input shaping technique, the parameters can be obtained as:

$$A_{1} = \frac{1}{1+3K+3K^{2}+K^{3}}, t_{1} = 0$$
(3)  

$$A_{2} = \frac{3K}{1+3K+3K^{2}+K^{3}}, t_{2} = \frac{\pi}{\omega}$$
  

$$A_{3} = \frac{3K^{2}}{1+3K+3K^{2}+K^{3}}, t_{3} = 2t_{2}$$
  

$$A_{4} = \frac{K^{3}}{1+3K+3K^{2}+K^{3}}, t_{4} = 3t_{2}$$
  

$$-\frac{\xi\pi}{2}$$

where  $K = e^{\sqrt{1-\xi^2}}$  and  $\omega$  is natural frequency of the system  $t_i$ , is the time location and  $A_i$  is the amplitude of the impulse sequence [1].

Then the impulse sequences are convolved with desired system input (bang-bang torque). This yields a shaped input that drives the system to a location less vibration. The objective of the design is to cancel three modes of the natural frequencies in the system.

#### 5. Genetic algorithm

Genetic Algorithms (GAs) are adaptive methods which may be used to solve, search and optimize problems. They are based on the genetic processes of biological organisms [9]. The principles of GA are well described in many texts [10-17]. In this work, GA is applied to optimize the impulse sequences and thus minimize the vibration in the system. The basic application of GA in the system is shown in Fig. 3. This simulation work is designed in Matlab based environment with sampling frequency of 2 kHz and implemented on a Pentium 4 2.66 GHz processor and the responses are presented in time and frequency domains.

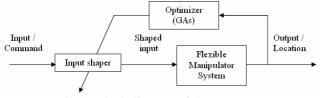


Fig. 3: Block diagram of the system

The genetic algorithm begins like any other optimization algorithm by defining the optimization parameters, cost function and the cost. It ends like other optimization algorithms by testing the convergence. The basic components of the genetic algorithm are shown as a flow chart in Fig. 4.

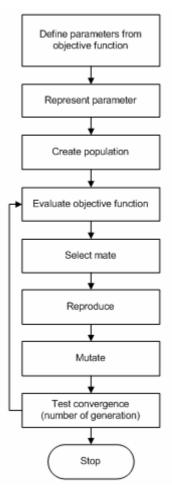


Fig. 4: Flow chart of a binary genetic algorithm

The vibration of the flexible manipulator is represented as the area under the graph of the absolute value of end point acceleration. Thus, the objective function of the optimization is the absolute area under the graph of end point acceleration. In order to attain this objective, the entire system needs to be simulated using random sequence of impulses generated by the GA by varying the 3 mode of natural frequencies  $\omega_{I}$ ,  $\omega_{2}$  and  $\omega_{3}$  utilizing Equation (3). Fig. 5 shows the absolute of end point acceleration.

In this work, 16 bits binary coding is used to represent a variable. Thus, the total length of a chromosome is 48 bits. The chromosomes represent the natural frequencies of the system as  $\omega_1$ ,  $\omega_2$  and  $\omega_3$  within the range of 5 to 15Hz, 25 to 40 Hz and 55 to 70Hz respectively. 30 initial populations  $V_i$  are randomly generated for first iteration of GA.

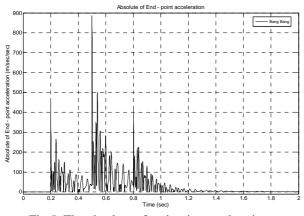


Fig 5: The absolute of end-point acceleration

Each chromosome is evaluated for fitness value by converting from binary strings into corresponding real values and the objective function  $F(V_i)$  is evaluated. In order to make the fitness value positive, the fitness of each chromosome is evaluated as:

$$Eval(V_i) = F(V_{\max}) - F(V_i)$$
(4)

New populations are created after evaluation from the current generation using three operators which are reproduction, crossover and mutation. The two chromosomes (strings) with the best fitness are allowed to live and produce offsprings in the next generation. A roulettle wheel method is constructed according to the fitness of each chromosome to decide which chromosome will be selected to crossover. This method requires the cumulative probability  $Q_i$  of each chromosome and is calculated for each chromosome using:

$$Q_i = \sum_{k=0}^{i} P_k \tag{5}$$

where

total fitness, 
$$F\_total = \sum_{i=1}^{pop\_size} Eval(V_i)$$
  
and probability,  $P_i = Eval(V_i)/F\_total$ 

The one-cut crossover method is used, which normally selects one cut point and exchanges the right parts of two parents to generate offspring. In this research, the crossover rate is set to 1.0. The parents are selected to crossover at the cut point within the length of chromosome, randomly. This process is repeated altogether 14 times to finish the whole crossover. The creation of 28 offsprings plus 2 chromosomes reproduced, maintain the same population in each generation.

Mutation is performed after crossover. Mutation alters (from 1 to 0 or vice versa) one or more genes with a probability equal to the mutation rate (mutation rate = 0.01) within the number of the bits in the whole population (30x40 = 1440) randomly. The chromosomes reproduced are not subjected to mutation, so after mutation, they will be restored. The GA is forced to

iterate 50 generations although it converges towards the optimal point to study the behavior of GA, as shown in Fig. 6.

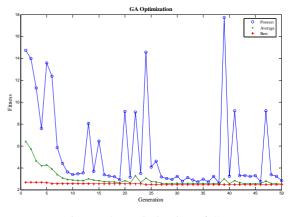


Fig. 6: The optimization of GA

Fig.6 is a plot of the best, average and poorest values of the objective function across 50 generations. Since reproduction is used to keep the best two individuals at each generation, the "best" curve is monotonically decreasing with the respect to generation numbers and thus, the GA is able to track the minimal point within a few generation due to natural frequency boundaries. The erratic behavior of the 'poorest' curve is due to mutation and crossover operators, which explores the landscape in random manner.

# 6. Implementation and results

Input shaping technique is based on feed-forward control technique where the desired input is convolved with sequence of impulses. These convolution technique acts like a low pass filter in the system and thus cancel the vibrations in the system. In this work, modeling is done using FE method where the system are divided into 10 element and the damping ratio of the system are deduced as 0.026, 0.038 and 0.040 for the first three vibration mode respectively to determine the amplitude of the impulse sequence. Note that these values were obtained experimentally in previous research [1]. The natural frequencies of the system are randomly generated by GA and the values are used to determine the amplitude and time location of the impulses. The obtained impulse sequences will be combined and convolved with the desired input (bang-bang torque) to generate a shaped input. The shaped input will be applied to the flexible manipulator system to achieve desired location. The bang-bang torque input and shaped input are compared to verify the performance of the control techniques as shown in Fig. 7.

The magnitude of vibration can be represented as the absolute area under the graph of end-point acceleration of the system as shown in Fig. 8. The simulation can be repeated for different payload of the system varying from 0 to 100g. In this paper, 0 payload is used.

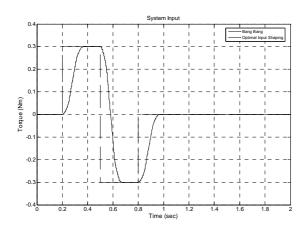


Fig. 7: The bang-bang torque input and optimal shaped input

The values of natural frequencies obtained from GA simulations are 13.827039 Hz, 39.789425 Hz, and 65.872969 Hz for the first three vibration modes respectively. Referring to Fig. 8, the vibration in the system is reduced drastically using input shaping technique with GA optimization method compared to the bang-bang torque input technique. The percentage of vibration improvement in term of area representation is 2720 %.

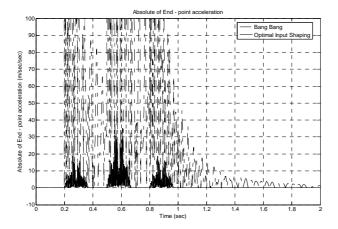


Fig. 8: The response of the absolute of End-point acceleration of the system

Fig. 9 shows the response at the end-point displacement of the flexible manipulator system. The bang-bang torque input control responses were obtained as; dead time = 0.08s, rise time = 0.328s and settling time = 0.552s. The input shaping control responses were obtained as; dead = 0.031s, rise time = 0.334s and settling time = 0.674s. The transient response of bang-bang torque input control is slightly faster than the shaped input control. The transient response of shaped input control. The steady state response of the shaped input control is better than the bang-bang torque control. The shaped input control performs similar to critically damped and the bang-bang torque control performs similar to underdamped (slight overshoot and oscillation

until 2s). The oscillation in bang-bang torque input control delays the system to achieve the desired location accurately.

Fig. 10 shows the spectrum density of the end-point displacement of the flexible manipulator system. It is noted that significant amount of the vibration is reduced in the flexible manipulator system with the shaped input.

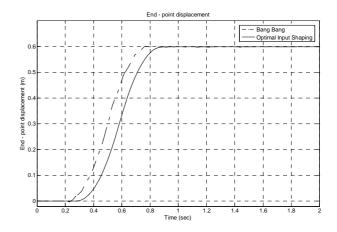


Fig. 9: The response of the system at the end-point displacement of the system

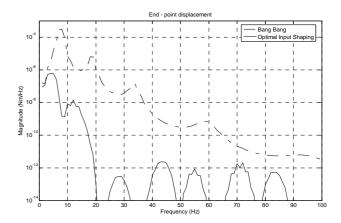


Fig. 10: The spectrum density at the end-point displacement of the system

# 7. Conclusions

The application of GA in the development of input shaping technique for vibration of a flexible manipulator has been presented in this paper. The flexible manipulator system is modeled using FE method and simulated using Matlab environment. In this work, GA is used to optimize the input shaping control technique to minimize the vibration in the flexible manipulator system. It is noted that the input shaping control technique is a better control technique compared to the bang-bang torque input control technique. It reduces a significant amount of vibration in the flexible manipulator system and achieves better accuracy in positioning compared to bang-bang torque input control technique. However, the GA with input shaping increases the time and complexity of Matlab simulation.

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