An Observer Design for Active Suspension System

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Abstract

The purpose of this paper is to construct an active suspension for a quarter car model with observer design. The proportional-integral sliding mode is chosen as a control strategy, and the road profile is estimated by using an observer design. The performance of the proposed controller will be compared with the linear quadratic regulator by performing extensive computer simulation.

1. Introduction

Comfort and road-handling performance are the characteristics have to be considered in order achieving good suspension system. Ideally the suspension should isolate the body from road disturbance and inertial disturbances associated with cornering and braking or acceleration [2]. The suspension must also be able to minimize the vertical force transmitted to the passengers for their comfort. This can be achieved by minimizing the vertical car body acceleration. The objective of the active suspension system is to improve the suspension system performance by directly controlling the suspension forces to suit with the performance characteristics.

There are various linear control strategies have been established by researchers in the design of the active suspension system. Amongst them are a fuzzy reasoning [4], robust linear control [7], H_∞ [8], and adaptive observer [9]. The obtained active suspension systems provide more effective performance in the vibration isolation of the car body.

The purpose of this paper is to utilise the concept of proportional integral sliding mode in active suspension system [2, 5, 6, 10, 11]. Beside that, the road disturbance will be estimated by using an observer design. Therefore, the estimated road disturbance will be another state for the system.

2. Dynamic model of the suspension

The quarter car active suspension systems are developed based on Figure 1. From the Figure 1 the state space equation of the system can written as

$$\dot{x}(t) = Ax(t) + Bu(t) + Dw(t) \tag{1}$$

$$\begin{bmatrix} \ddot{x}_b(t) \\ \ddot{x}_w(t) \\ \dot{x}_b(t) \\ \dot{x}_w(t) \\ \dot{w}(t) \end{bmatrix} = \begin{bmatrix} -\frac{c_b}{m_b} & \frac{c_b}{m_b} & -\frac{k_b}{m_b} & \frac{k_b}{m_b} & 0 \\ \frac{c_b}{m_w} & -\frac{c_b}{m_w} & \frac{k_b}{m_w} & -\frac{(k_w+k_b)}{m_w} & \frac{k_w}{m_w} \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{x}_b(t) \\ \dot{x}_w(t) \\ x_b(t) \\ x_w(t) \\ w(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{m_b} \\ -\frac{1}{m_w} \\ 0 \\ 0 \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} w(t) \tag{2}$$

where m_b is a mass of car body and m_w is a mass of wheel. x_b and x_w are the displacements of car body and wheel respectively, k_b and k_w are the spring coefficients. c_b is the damper coefficient and *w (t)* is the road disturbance. The control force, *u(t)* from the hydraulic system is assumed as the control input to the suspension system. Equation (1) shows that the disturbance input is not in phase with the system input, therefore the system suffers from mismatched condition [3].

Hence, the proposed controller must be robust enough to overcome the mismatched condition so that the disturbance would not have significant effect on the performance of the system [2]. Equation 1 can be written as

$$\dot{x}(t) = Ax(t) + Bu(t) + f(x,t) \tag{3}$$

Where $x(t) = \Re^{n}$ is the state vector, $u(t) = \Re^{m}$ is the control input, and the continuous function *f(x,t)* represents the uncertainties with mismatched condition, i.e $rank[B|f(t)] \neq rank[B]$. The following assumptions are taken as standard:

Assumption 1. There exists a known positive constant such that $\|f(t) \leq \beta\|$, where $\|\bullet\|$ denotes the standard Euclidean norm.

Assumption 2. The pair (*A, B*) is controllable and the input matrix *B* has full rank.

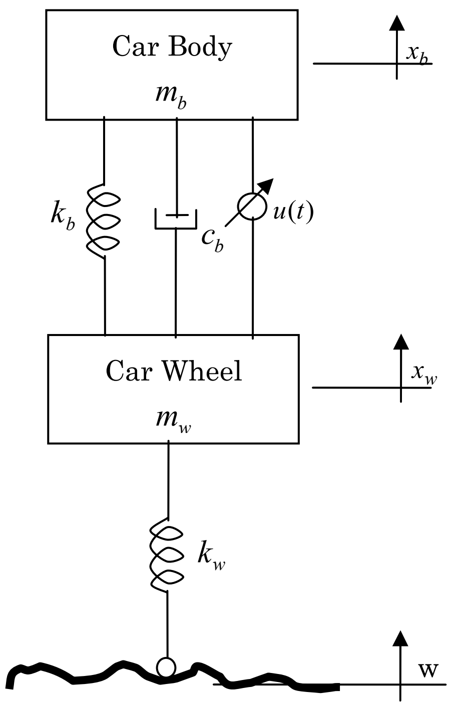

Figure 1: A quarter-car active suspension system

3. Switching surface and controller design

In this study proportional integral sliding mode is chosen based its ability to overcome the steady state error in the system. The PI sliding surface is defined as follows [1, 2, 3];

$$\sigma(t) = Cx(t) - \int_{0}^{t} (CA + CBK)x(\tau)d\tau \tag{4}$$

where $C \in \Re^{m \times n}$ and $K \in \Re^{m \times n}$ are constant matrices. The matrix C is chosen such that $CB \in \Re^{m \times n}$ is nonsingular. The matrix *K* is chosen such that λ (A + BK) < 0. It is well known that if the system is fulfilled the sliding mode condition, hence σ(t) =0.

Then, the differential of Equation 4 gives,

$$\dot{\sigma}(t) = C\dot{x}(t) - (CA + CBK)x(t) \tag{5}$$

Substituting Equation (3) into Equation (5) gives,

$$\dot{\sigma}(t) = C[Ax(t) + Bu(t) + f(x,t)] - [CA + CBK]x(t)$$

$$= CAx(t) + CBu(t) + Cf(x,t) - CAx(t) - CBKx(t)$$

$$= CBu(t) + Cf(x,t) - CBKx(t) \tag{6}$$

Equating Equation 6 to zero gives the equivalent control, $u_{eq}(t)$ as follows,

$$u_{eq}(t) = Kx(t) - [CB]^{-1}Cf(x,t) \tag{7}$$

Substituting Equation 7 into Equation 3, gives the equivalent dynamics equation of the system in sliding mode as follows,

$$\dot{x}(t) = Ax(t) + B[Kx(t) - [CB]^{-1}Cf(x,t)] + f(x,t)$$

$$\dot{x}(t) = Ax(t) + BKx(t) - B[CB]^{-1}Cf(x,t)] + f(x,t)$$

$$\dot{x}(t) = [A + B]x(t) - [I_n - B[CB]^{-1}C]f(x,t) \tag{8}$$

During the sliding mode, the uncertain system with mismatched condition is stable provided that the following theorem is satisfied [1, 2, 3].

Theorem 1

The uncertain system equation (3) is bounded stable on the sliding surface, σ (t) =0 if,

$\|\tilde{F}(x,t)\| \leq \beta_1$, where $\beta_1 \|I_n - B(CB)^{-1}C\| \beta$

and

$$F(x,t) = \{I_n - B(CB)^{-1}C\}f(x,t) \quad (9)$$

Proof:

For simplify, let

$$\tilde{A} = (A + BK) \quad (10)$$

$$\tilde{F}(t) = \{I_n - B(CB)^{-1}C\}f(x,t) \quad (11)$$

And rewrite Equations 10 and 11 as

$$\dot{x}(t) = \tilde{A}x(t) + \tilde{F}(x,t) \quad (12)$$

Let Lyapunov function candidate for the system is chosen as

$$V(t) = x^T(t)Px(t) \quad (13)$$

Taking the derivative of *V(t)* and substituting into Equation 8, gives

$$\dot{V}(t) = x^T(t)[\tilde{A}^T P + P\tilde{A}]x(t) + \tilde{F}^T(x,t)Px(t) + x^T P\tilde{F}(x,t)$$

$$= -x^T(t)Qx(t) + \tilde{F}^T(x,t)Px(t) + x^T(t)P\tilde{F}^T(x,t) \quad (14)$$

where P is solution of $\tilde{A}^T P + P\tilde{A} = -Q$ for a given positive definite symmetric matrix Q. It can be shown that Equation (14) can be reduced to

$$\dot{V}(t) = -\lambda_{\min}(Q)\|x(t)\|^2 + 2\beta_1\|P\|\|x(t)\| \quad (15)$$

Since λmin(Q)>0, consequently $\dot{V}(t) < 0$ for all t and $x \in B^c(\eta)$, where $B^c(\eta)$ is the complements of the closed ball B(η), centered at x=0 with radius $\eta = [2\beta_1\|P\|]/[\lambda_{\min}(Q)]$. Hence the system is boundedly stable.□

Remark. For the system with uncertainties satisfy the matching condition, i.e. $rank[B|f(t)] \neq rank[B]$, then equation (8) can be reduced to $x(t) = [A + Bk]x(t)$ [10]. Thus asymptotic stability of the system during sliding mode is assured.

We now design the control scheme that drives the state trajectories of the system in Equation 3 onto the sliding surface *σ(t)* and the system remains in it thereafter. For the uncertain system in Equation 3 satisfying assumptions 1 and 2, the following control law is proposed: [1]

$$u(t) = Kx(t) - (CB)^{-1}Cf(x,t) - (CB)^{-1}\rho\frac{\sigma(t)}{|\sigma(t)| + \delta} \quad (16)$$

where δ is the boundary layer thickness that to be selected to reduce the chattering effect and ρ is a design parameter which will be specified by designer.

Theorem 2

The hitting condition of the sliding surface (4) is satisfied if

$$\|A + BK\|\|x(t) \geq \|f(t)\|\| \quad (17)$$

Proof.
The reaching condition is evaluated as follows,

$$\sigma(t)\dot{\sigma}(t) = \sigma(t)[C\dot{x}(t) - [CA + CBK]x(t)$$

$$= \sigma(t)[C\{Ax(t) + Bu(t) + f(x,t)\} - \{CA + CBK\}x(t)$$

$$= \sigma(t)[CBu(t) + Cf(x,t) - CBKx(t) \quad (18)$$

Substituting Equation 16 into Equation 18, gives

$$\sigma(t)\dot{\sigma}(t) = \sigma(t)[CB[Kx(t) - (CB)^{-1}Cf(x,t) - (CB)^{-1}\rho\frac{\sigma(t)}{|\sigma(t)|\delta}] + Cf(x,t) - CBKx(t)]$$

$$= \sigma(t)\left[-\rho\frac{\sigma(t)}{|\sigma(t)| + \delta}\right] \quad (19)$$

The sliding condition is established if $\rho > 0$

4. Observer Design

Assuming that the states $\dot{x}_b, \dot{x}_w, x_b, x_w$ are observable and *w* is unobservable in the state vector *x*, the algorithm of VSS observer [5, 6, 11] for estimating *w* is composed of the up-dating relation for the estimates of five state variables. It can be done by up-dating relation for the estimator of x_w and *w* only [6]

$$\dot{\hat{x}}_w(t) = F(\dot{x}_b, \dot{\hat{x}}_w, x_b, x_w, \hat{w}) + b_{21}u(t) + d_{21}(\dot{x}_w - \dot{\hat{x}}_w)$$

$$d_{22}(\dot{x}_w - \dot{\hat{x}}_w)/(|\dot{x}_w - \dot{\hat{x}}_w| + \varepsilon_2) \quad (20)$$

$$\hat{w}(t) = d_{51}(\dot{x}_w - \hat{\dot{x}}_w) + d_{52}(\dot{x}_w - \hat{\dot{x}}_w)/(|\dot{x}_w - \hat{\dot{x}}_w| + \varepsilon_5)$$

(21)

where d_{21},d_{22},d_{51}, and d_{52} are constants, and ε_2 and ε_5 are small positive constants to compensate the chattering. The state of w is replaced by the estimate $\hat{w}$ in the state vector. The state vector x is replaced by $\hat{x}$ defined as

$$\hat{x} = [\hat{\dot{x}}_b, \hat{\dot{x}}_w, x_b, x_w, \hat{w}]^T$$

Then $u(t)$ in equation 16 are respectively can be replaced as,

$$u(t) = K\hat{x}(t) - (CB)^{-1} Cf(x,t) - (CB)^{-1} \rho \frac{\sigma(t)}{|\sigma(t)| + \delta}$$

(22)

5. Simulation and discussion

The mathematical models of the system as defined in equation 1 and proposed proportional-integral sliding mode controller (PISMC) in equation 22 were simulated on computer. The system assumed that the road disturbance as an input to the system. The parameter for the spring and the damper for the active suspension system are considered to be linear.

For comparison purposes, the performance of the PISMC is compared to the linear quadratic regulator (LQR) control approach. Assumed a quadratic performance index in the form of

$$J = \frac{1}{2}\int_0^t (x^T Qx + u^T Ru)dt \qquad (23)$$

Where the matrix Q is symmetric positive semi-definite and R is positive symmetric definite. Then the optimal linear feedback control law is obtained as

$$u(t) = -Kx(t) \qquad (24)$$

Where K is the designed matrix gain.

The typical road disturbance be in the form of

$$w(t) = \begin{cases} a(1 - \cos 8\pi t), 2.25\,\text{sec} \le t \le 2.5\,\text{sec} \\ 0, otherwise \end{cases} \qquad (25)$$

Figure 2 shows the typical road disturbance that generated to the system.

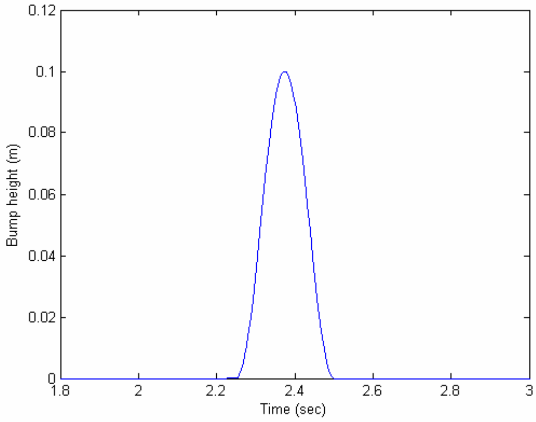

Figure 2: Typical Road disturbance

Numerical value for the model parameters are as follows:

- m_b = 290 kg
- m_w = 59 kg
- K_b = 16812 N/m
- K_w = 190,000 N/m
- C_b = 1000 N/(m/s)

In the design of the LQR controller, weighting matrices Q and R are selected as Q = diag($q1,q2,q3,q4,q5$) where q1 = q2 = q3 = q4 = q5 = $1\text{x}10^2$ and R = [0.1], respectively. Thus, the designed gains of the LQR controller are K= [0.5985,-0.4819, 0.0307,-3.4870, 3.5103]. The value of the matrix K for PISMC is similar to the value of the designed gains in the LQR controller such that $\lambda(A + BK)$ = {-8.7084 ± 58.1974i, -1.4717 ± 7.2008i,-0.0000}. In this simulation the following value are selected for the PISMC: C = [200 10 7500 8000 3500], δ = 10 and ρ = 100.

Figure 3 and 5 shows the estimated road disturbance for the system when using LQR and PISMC controller. Figure 4 and 6 shows the error between typical road disturbances with estimated road disturbance. The result shows that the error for purposed controller is less than LQR controller.

The objective of designing an active suspension is to increase the ride comfort and road handling, there are three parameters to be observed in the simulation. The parameters are the wheel deflection, the body acceleration and the suspension travel. Figure 7 shows the wheel deflection of both controllers for an active suspension system. The result shows the purposed controller perform better than LQR controller. Figure 8 illustrates clearly how the PISMC technique can effectively absorb the vehicle vibration in comparison to the LQR method. The body acceleration in the PISMC design system performs better, which guarantee better ride comfort. Figure 9 shows the suspension travel of both controllers for an active suspension system. The result shows that the active suspension utilizing the PISMC technique performs better as compared to the others. Therefore it can conclude the active suspension system with PISMC improves the ride comfort while retaining road handling characteristics, as compared to the LQR method.

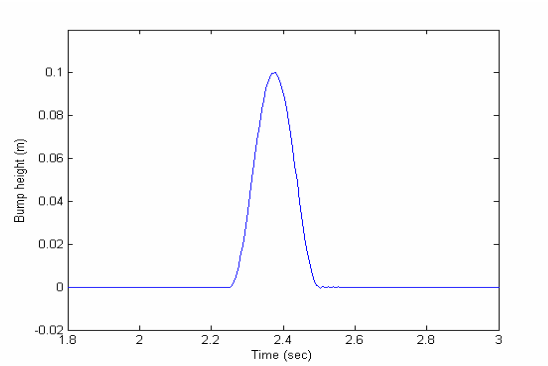

Figure 3: The estimated road disturbance using LQR controller

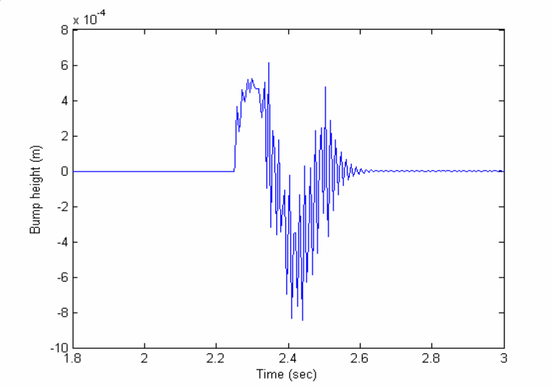

Figure 4: The error between typical road disturbance and estimated road disturbance when using LQR controller

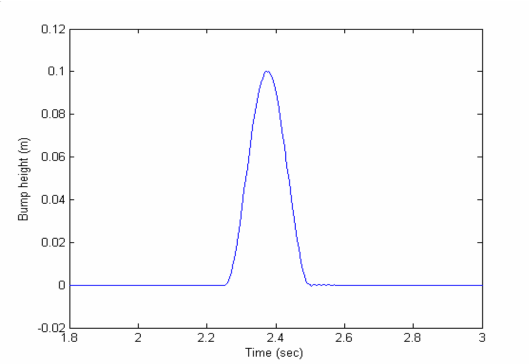

Figure 5: The estimated road disturbance using PISMC controller

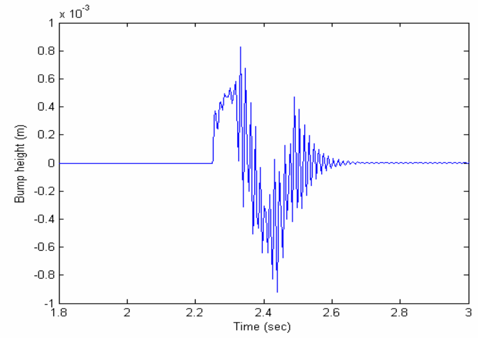

Figure 6: The error between typical road disturbance and estimated road disturbance when using PISMC Controller

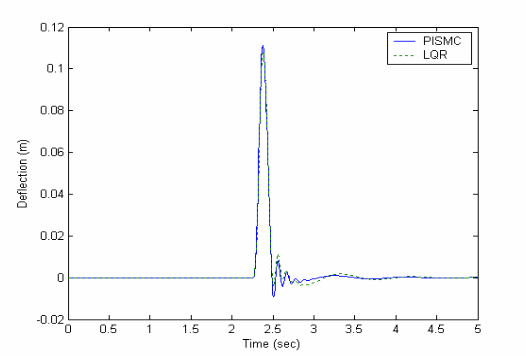

Figure 7: Wheel deflection

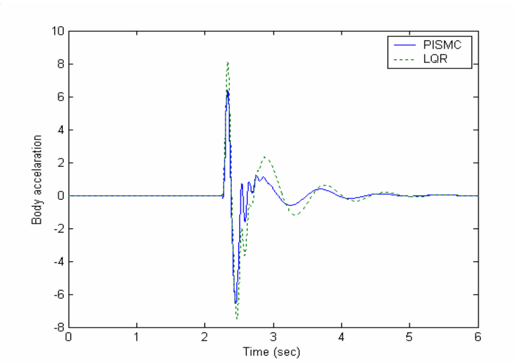

Figure 8: Body acceleration

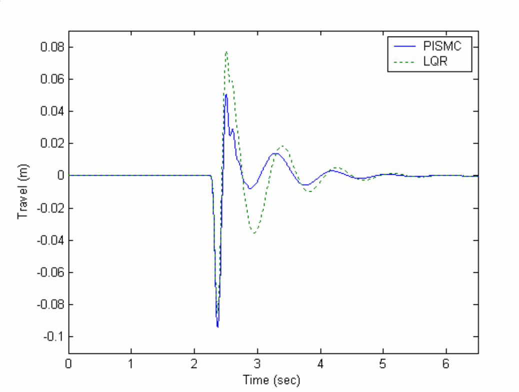

Figure 9: Suspension travel.

6. Conclusion

In this paper, the proportional integral sliding mode control is proposed to solve the mismatched condition problem. The road profile was estimated by using observer with good agreement between exact and estimated value. The performances of the active suspension system are evaluated by PISMC and LQR method and compared the result. The result shows that the purposed PISMC technique proved to be effective in controlling vehicle compared LQR method.

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