THREE-DIMENSIONAL CRANIOFACIAL SURFACE MEASUREMENT: LINEAR DISTANCE, ANGLE AND AREA COMPUTATION

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Abstract

Accurate and precise measurement of three-dimensional surface in terms of linear distance, area occupied and angles between surfaces have become key criteria particularly in reconstruction of craniofacial appearance where the main purpose is to restore a distorted or damage facial area according to surgeons and patients requirements from aesthetic point of view. This paper describes three techniques used in three-dimensional surface measurement using practical mathematical computation together with the usability requirements descriptions. These techniques are described in terms of its purposes of usage, implementation process and mathematical calculation. We summarized in the end that all the above discussed approaches are practically useful in three-dimensional surface measurement and computation.

1.0 INTRODUCTION

Craniofacial pre-operative planning and post-operative prediction applications are gaining more and more attention since the introduction of computer-aided devices in medical realm. This is due to that applications associated with this field are crucial for many tasks in diagnosis and treatment planning, as for example in the pre-operative and post-operative of craniofacial surgery. Currently, it is common practice in radiology to use 2D measurement tools for the definition of distance, diameters, arcas or angles in planar slices of radiological data. This, however, gives only rough estimation for spatial measurements such as the extent of a 3D object. In particular, the volume of 3D objects can only be estimated roughly. Therefore, tools are required that integrate measurements in 3D visualizations. The development of measurement tools to be used in the context of complex 3D visualization is difficult because the user has to be provided with enough depth cues to access the position and orientation of such a measurement tool. A simple transition of existing line-based 2D measurement tools into 3D is not sufficient.

An important aspect of medical 3D visualization is that 3D visualizations are derived by the analysis of slices of radiological data. Therefore it is desirable to combine 3D views with 2D views of the original slices. For measurements tasks this implies that measurement point should be visible and modifiable within the slice data. While 3D views have the advantage of showing the overall relations, in 2D views each and every voxel of the original Computer Tomography (CT) or Magnetic Resonance Imaging (MRI) data may be selected precisely.

The interactive use of measurement tools is the most flexible approach, however, it requires a certain effort from the users' part and might be inaccurate. Therefore, we carefully analyzed which interaction tasks are of primary importance in order to reduce the interaction effort. Most of the measurement facilities described in this paper are primarily used for craniofacial surgery planning. The involved mathematical computations for the techniques used are depicted in detail.

2.0 THREE-DIMENSIONAL SURFACE MEASUREMENT

2.1 Usability Requirements

The usability of measurement tools depend on a number of presentation parameters. Among them are font parameters (size, color) and line parameters (line width). Generally, the selection of presentation parameters is guided by the following four requirements:

a) **Distinct assignment of measurement numbers to objects.** It should be clearly recognizable to which object or region a measurement refers.

b) **Distinct assignment of measurement numbers to measurement tools.** If several measurements are included in visualization it is necessary that the affiliation of a number to a measurement tool is visualized unambiguously. The placement of numbers and the choice of presentation parameters such as color are important for this goal.

c) **Flexibility.** Due to the large variety of the spatial relations to be analyzed and due to personal preferences it is important that the default values concerning font and line parameters as well as units of measurement tools are adjustable.

d) **Precision.** Direct manipulation exhibits a lack of precision which is an essential drawback for measurement tasks. Therefore, additional facilities are required to overcome limited precision. Incremental transformations by means of arrow keys (two additional keys are used for 6 degrees of freedom (DOF)) interaction are provided to support the fine-grained modifications. Furthermore, transformations of measurement tools may be specified by numbers. The header information of medical data is employed to define its resolution (size of a mesh or voxel) and to guide the precision of measurement numbers.

2.2 Selection Surface Measurement Methods, Purposes and Usability

In this section, three useful surface measurement methods are determined and discussed here. They are: linear distance measurement, area measurement and angle measurement.

2.2.1 Linear Distance Measurement

Purpose:
Linear distance measurements are employed to define distance between objects or diameters in linear form. Such measurements are crucial for craniofacial surgery planning, where it is used particularly in measurement of direct distance between two or more facial landmark points selected. For example, it is used to measure the distance between exocanthion (ex) left and right. Figure 2.1 show a linear distance measurement between exocanthion (ex) left and right.

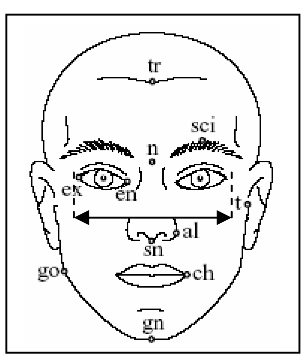


Figure 2.1: Linear distance measurement between exocanthion (ex) left and right.

Euclidean Distance:
Euclidean Distance is referred to straight line distance between two points. In a three-dimensional space plane with p_1 at (x_1,y_1, z_1) and p_2 at (x_2, y_2, z_2), it is defined as:

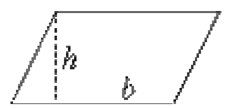
$$\text{Euclidean distance, } e = \sqrt{\left(\left(x_1 - x_2\right)^2 + \left(y_1 - y_2\right)^2 + \left(z_1 - z_2\right)^2\right)}$$

2.2.2 Surface Area Measurements

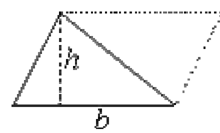
Purpose:
Area measurement is used in estimation of cross-sectional area of an object being viewed. In craniofacial surgery, area measurement is necessary when the surgeons need to know how much area to be operated for a hard tissue part especially for operation involved of skull.

Ancient Triangle Area Measurements:
Before Pythagoras, the area of the parallelogram (including the rectangle and the square) has been known to equal the product of its base times its height. Further, two copies of the same triangle paste together to form a parallelogram, and thus the area of a triangle is half of its base *b* times its height *h*. So, for these simple but commonly occurring cases, we have:

Parallelogram: $A(\text{Y}) = bh$

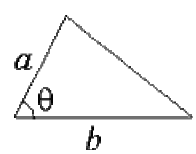


Triangle: $A(\Delta) = \frac{1}{2}bh$



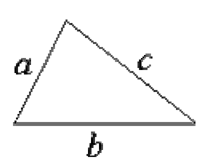
For example, if one knows the lengths of two sides, *a* and *b*, of a triangle and also the angle θ between them, then Euclid says this is enough to determine the triangle and its area. Using trigonometry, the height of the triangle over the base *b* is given by $h = a \sin \theta$, and thus the area is:

$$A(\Delta) = \tfrac{1}{2}ab\sin\theta$$



Another frequently used computation is derived from the fact that triangles with equal sides are congruent, and thus have the same area. This observation from Euclid (~300 BC) culminated in Heron's formula (~50 AD) for area as a function of the lengths of its three sides. There are some historians attribute this result to Archimedes (~250 BC); namely:

$$A(\Delta) = \sqrt{s(s-a)(s-b)(s-c)}$$
where $s = \frac{1}{2}(a+b+c)$



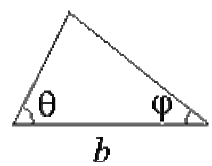
where *a*, *b*, *c* are the lengths of the sides, and *s* is the semi-perimeter. There are interesting algebraic variations of this formula. Figure 2.2 illustrated the formula that has been used in our surface area computation where we have implemented divide and conquer method in surface area computation using triangle:

$$A(\Delta) = \tfrac{1}{4}\sqrt{4a^2b^2 - \left(a^2 + b^2 - c^2\right)^2}$$

Figure 2.2: Surface area computation using algebraic variation derived from Heron's formula.

which avoids calculating the 3 square roots to explicitly get the lengths *a*, *b*, *c* from the triangle's vertex coordinates. The remaining classical triangle congruence is when two angles and one side are known. Knowing two angles gives all three, so we can assume the angles θ and φ are both adjacent to the known base b. Then the formula for area is:

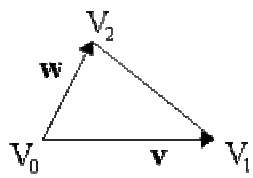
$$A(\Delta) = \frac{b^2}{2(\cot\theta + \cot\varphi)}$$



Modern Triangle Area Measurements:

More recently, starting in the 17-th century with Descartes and Fermat, linear algebra produced new simple formulas for area. In three-dimensional space (3D), the area of a parallelogram and triangle can be expressed as the magnitude of the cross-product of two edge vectors, since $|v \times w| = |v||w||\sin\theta|$ where θ is the angle between the two vectors v and w. Thus for a 3D triangle with vertices $V_0V_1V_2$ putting $v = V_1 - V_0$ and $w = V_2 - V_0$, one gets:

$$\begin{aligned} A(\Delta) &= \tfrac{1}{2}|v \times w| \\ &= \tfrac{1}{2}\left|(V_1 - V_0) \times (V_2 - V_0)\right| \end{aligned}$$



In two dimensional space (2D), a vector can be viewed as embedded in 3D by adding a third component which is set = 0. This lets one take the cross-product of 2D vectors, and use it to compute area. Given a triangle with vertices $V_i = (x_i, y_i) = (x_i, y_i, 0)$ for $i = 0, 2$, we can compute that:

$$(v_1 - v_0) \times (v_2 - v_0) = \left(0, 0, \begin{vmatrix} (x_1 - x_0) & (x_2 - x_0) \\ (y_1 - y_0) & (y_2 - y_0) \end{vmatrix}\right)$$

And the absolute value of the third z-component is twice the absolute area of the triangle. This formula for area is a very efficient computation with no roots or trigonometric functions involved - just 2 multiplications and 5 additions, and possibly 1 division by 2, which sometimes can be avoided.

$$\begin{aligned} 2A(\Delta) &= \begin{vmatrix} (x_1 - x_0) & (x_2 - x_0) \\ (y_1 - y_0) & (y_2 - y_0) \end{vmatrix} = \begin{vmatrix} x_0 & y_0 & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} \\ &= (x_1 - x_0)(y_2 - y_0) - (x_2 - x_0)(y_1 - y_0) \end{aligned}$$

where $V_i = (x_i, y_i)$

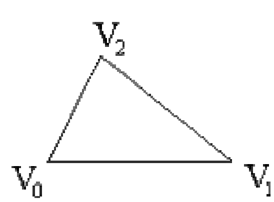


Figure 2.3 shown the flow diagram for the computation of surface area measurement.

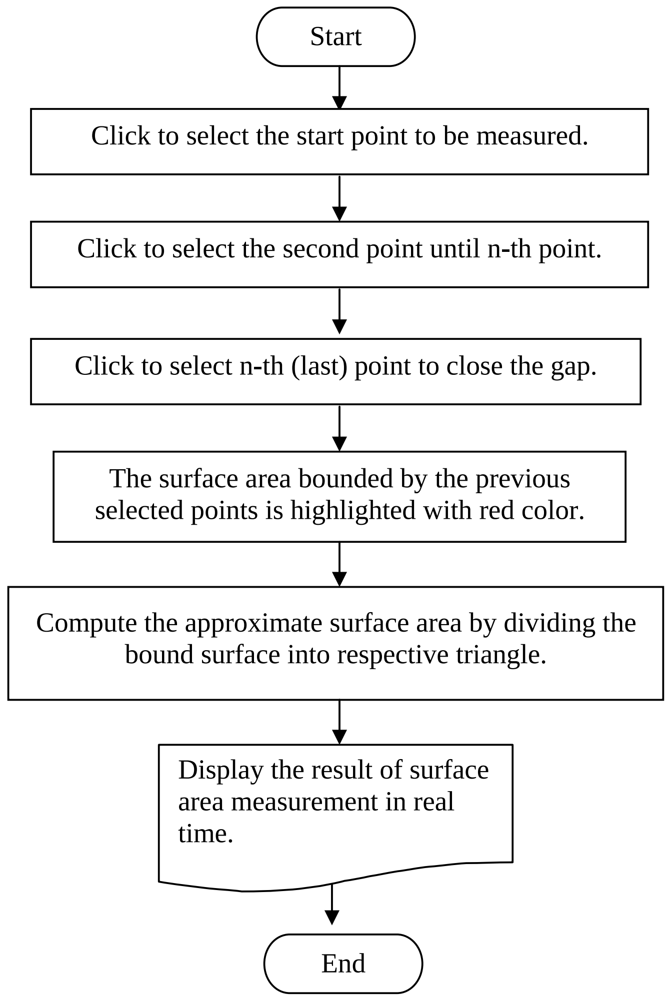


Figure 2.3: Flow chart of surface area computation using algebraic variation derived from Heron's formula.

2.2.3 Angular Measurement

Purpose:
Angular measurements are carried out to define angles between anatomical or pathological structures. The angle at branching of vascular structures might be essential for vascular analysis, angles which describe orientation of bones are often important for the diagnosis and treatment planning in orthopaedics. In craniofacial surgery planning, angular measurements play an inevitable role. It is used by surgeons and doctors in determination of craniofacial appearance abnormalities and malfunctions. The measured angles indicated to what degree a lower jaw of a patient should be pushed back or pulled out or to how much a patient nasion part should be elongated. Figure 2.4, 2.5 and 2.6 shows four different measurements conducted on craniofacial surface.

i) Nasofronatal angle: It is formed by drawing a line tangent glabella through the nasion that will intersect a line drawn tangent to nasal dorsum.

ii) Nasofacial angle: It is formed by drawing a vertical line tangent to forehead at the glabella and tangent to the chin at the pogonion so that a line drawn along the nasal dorsum intersect it.

iii) Nasomental angle: It is formed by a line drawn through the nasal dorsum intersecting a line drawn from the nasal tip to soft tissue chin at the pogonion.

iv) Mentocervical angle: A vertical line tangent to forehead passing at glabella and second line intersecting tangent to the chin at Pogonion.

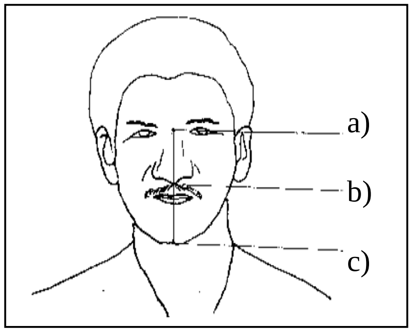


Figure 2.4: The land marks on the face a) Nasion (N), b) Subnasal (SN) and c) Menton (MN)

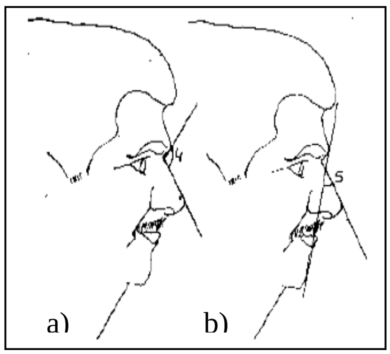


Figure 2.5: a) Nasofrontal angle and b) Nasofacial angle.

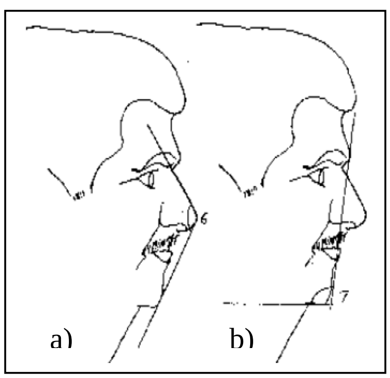


Figure 2.6: a) Nasoemental angle and b) Mentocervical angle.

Angular Measurements:
In angular measurements, three coordinates are required representing the apex of the angle and terminating the legs. In order to provide consistency across the measurement tools, the linear distance is used again for this purpose. So, the angular measurement tool thus consists of two linear distance lines. The apex of the leg is emphasized with a sphere which can be easily selected. It turned out that without orientation aids it is often difficult to access the size of an angle. Therefore, semitransparent polygons are used as orientation aids when the angle is transformed.

Two semitransparent rectangles are created perpendicular to the legs of the angle. Two shapes have been designed to emphasize the plane in which the angle is located. First, the triangle formed by the three vertices of the angular measurement tool is displayed transparently. The use of a triangle as orientation aid is restricted to angles of less than 180 degrees (or would be ambiguous for angles larger than 180 degrees). Therefore, [3] have improved the initial design. With the new design, a portion of a circle is employed instead of the triangle to communicate the extent of the angle. The portion of the circle is smaller than the triangle. The portion is scaled such that the radius corresponds to the half length of the smaller leg.

Concerning the placement of the measurement number, two strategies have been used. First, the number has been integrated in one of the two distance lines which represent the legs. The second strategy is closer to the way angle are annotated in conventional technical drawings – the number is placed near the apex of the angle. The second strategy has the advantage that for larger angle the orientation is unambiguous. Figure 2.7 illustrated the flow chart of surface angle computation.

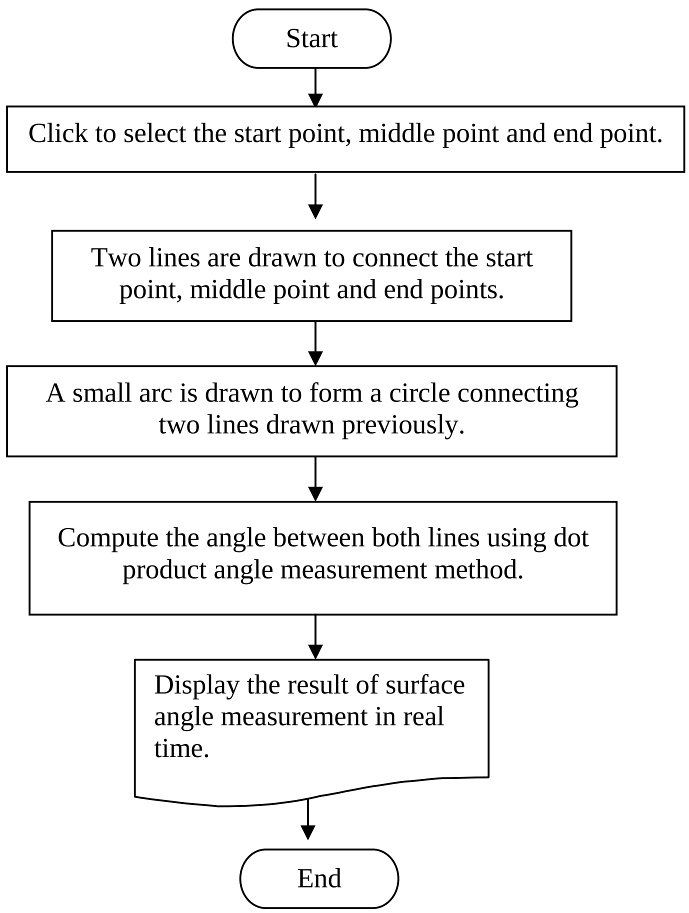


Figure 2.7: Flow chart of surface angle measurement using dot product angle measurement method.

Using a Dot Product, we can obtain the angle between two vectors *u* and *v* as follows. Figure 2.8 are the formula of surface angle computation with Dot Product formula. The cosine of theta is equal to the dot product of *u* and *v* divided by the product of *u* and *v*'s magnitudes. The magnitude of a vector *u* and *v* is denoted by $|u|$ and $|v|$ respectively.

$$\theta = arcCos\left(\frac{u \cdot v}{|u||v|}\right)$$

Figure 2.8: Surface angle computation using Dot Product formula.

3.0 IMPLEMENTATION AND RESULTS

3.1 Euclidean Distance Measurement

Linear distance measurement is conducted using Euclidean distance technique. Algorithm implemented in Delphi is illustrated in Figure 3.1 while Figure 3.2 shown the screen capture of the operation of Euclidean distance measurement.

```
function calculateEuclideanDistance(x1,y1,z1,x2,y2,z2 : single): single;
begin
  result := sqrt(Power((x2-x1),2)+ Power((y2-y1),2)+ Power((z2-z1),2));
end;
```

Figure 3.1: Euclidean distance measurement algorithm.

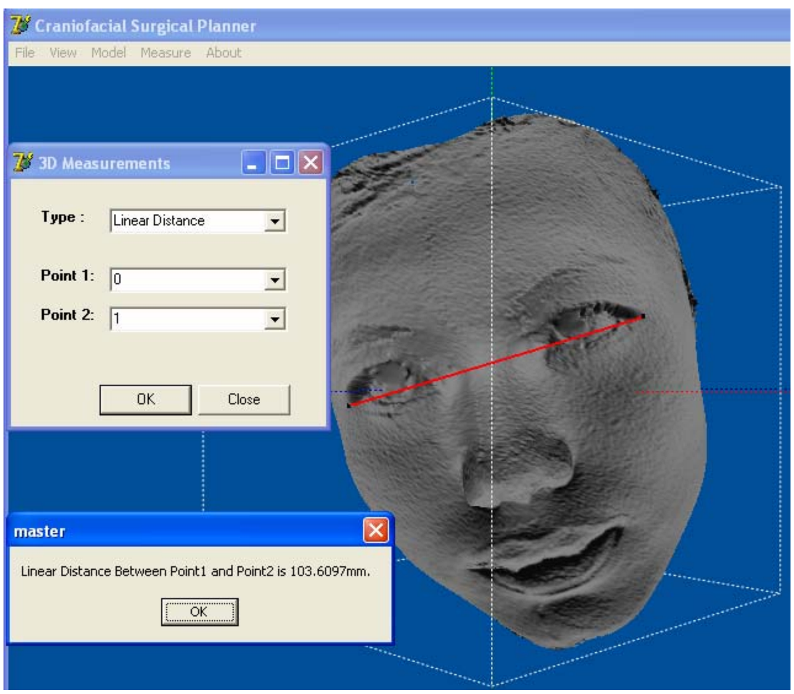


Figure 3.2: Euclidean distance measurement operation conducted using Delphi application developed.

3.2 Surface Angle Measurement

Figure 3.3 illustrated the source code of angle measurement operation conducted using the application developed. Figure 3.4 and 3.5 shown the sub-function for angle measurement operation. Figure 3.6 illustrated the screen capture of the operation of angle measurement.

```
function calculateAngle(v1,v2,v3 : single): single;
begin
    u := VectorSubtract(v1,v2);
    v := VectorSubtract(v3,v2);
    uv := VectorDotProduct(u,v);
    cosTheta := uv / (VectorLength(u) * VectorLength(v));
    theta := RadToDeg(ArcCos(cosTheta));
    result := theta;
end;
```

Figure 3.3: Angle measurement source code.

```
function VectorSubtract(const v1, v2 : TVector) : TVector;
begin
  Result[0]:=v1[0]-v2[0];
  Result[1]:=v1[1]-v2[1];
  Result[2]:=v1[2]-v2[2];
end;
```

Figure 3.4: Sub-function one for angle measurement operation.

```
function VectorDotProduct(const V1, V2 : TVector) : Single;
begin
  Result:=V1[0]*V2[0]+V1[1]*V2[1]+V1[2]*V2[2]+V1[3]*V2[3];
end;
```

Figure 3.5: Sub-function two for angle measurement operation.

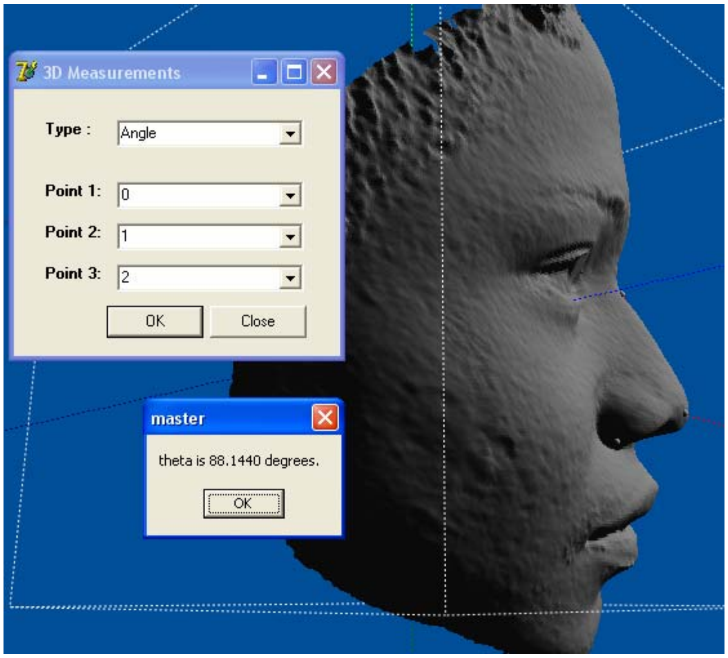


Figure 3.6: Angle measurement operation conducted using Delphi application developed.

3.3 Surface Area Measurement

Figure 3.6, 3.7 and 3.8 illustrated the sub-function of the source code of surface area measurement operation conducted using the application developed. Figure 3.9 shown the screen capture of the surface area measurement operation.

```
for n:=0 to (form1.p_indx-1-1) do
      begin
         dirEdgeDistance[n] := Form1.calcLinearDistance
                                    (form1.x_c[n+1],form1.y_c[n+1], form1.z_c[n+1],
                                     form1.x_c[0],form1.y_c[0],form1.z_c[0]);
      end;
```

Figure 3.6: Sub-function one for surface area measurement operation.

```
for n:=0 to (form1.p_indx-1-2) do
      begin
         indirEdgeDistance[n] := Form1.calcLinearDistance
                                    (form1.x_c[n+2],form1.y_c[n+2],form1.z_c[n+2],
                                     form1.x_c[n+1],form1.y_c[n+1],form1.z_c[n+1]);
      end;
```

Figure 3.7: Sub-function two for surface area measurement operation.

```
for n:=0 to (form1.p_indx-3) do
      begin
          area:= area +
          0.25 * sqrt(4*Power(dirEdgeDistance[n],2)*Power(indirEdgeDistance[n],2)-
          Power(Power(dirEdgeDistance[n],2)+Power(indirEdgeDistance[n],2)-
          Power(dirEdgeDistance[n+1],2),2));
      end;
```

Figure 3.8: Sub-function three for surface area measurement operation.

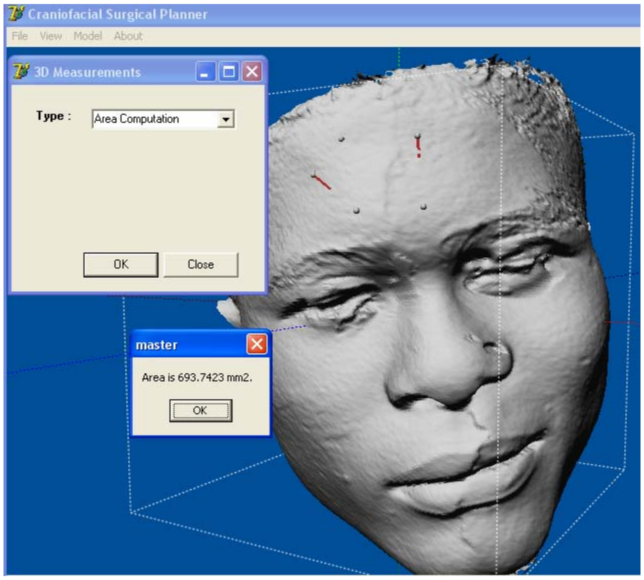


Figure 3.9: Surface area measurement operation conducted using Delphi application developed.

4.0 CONCLUSION

In this paper, several techniques about three-dimensional surface measurement have been reviewed in this paper. These included: linear distance measurement, area measurement and angular measurement. Linear distance measurement is the most simple and direct method in measuring linear distance between two points. Surface area measurement is applied by surgeons to determine the area involved when doing cross-sectional cutting or to measure the approximate area included. Angular measurement is conducted to determine the angle abnormality between craniofacial parts. All the above mentioned three-dimensional craniofacial surface measurement techniques are crucial and important in craniofacial surgery planning. This is due to that safety is the first practice in operation besides to restore a pleasing facial appearance.

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