ABSTRACT

The method of Painleve analysis is introduced and used to explore the integrability of certain physically significant nonlinear partial differential equations (nPDEs). These nPDEs include the Burgers' equation, Korteweg-de Vries equation (KdV) and nonlinear Klein-Gordon equation. Furthermore, it is shown primarily that this Painleve test is intended to determine necessary conditions for such class of nPDEs to have the Painleve property. However, the nonlinear Klein-Gordon equation is shown instead to be integrable with respect to the Weak Painlevé test.

ABSTRAK

Kaedah analisis Painlevé diperkenalkan dan digunakan untuk meneroka kebolehkamiran sesuatu persamaan pembezaan separa yang berkepentingan fizikal. Persamaan pembezaan separa ini termasuklah persamaan Burgers, persamaan Korteweg-de Vries (KdV) dan persamaan Klein-Gordon tak linear. Tambahan pula, telah dibuktikan bahawa ujian Painlevé ini digunakan untuk menentukan syarat yang perlu ada untuk sesuatu kelas persamaan permboleh ubah separa untuk mempunyai sifat Painlevé. Walaubagaimanapun, persamaan Klein-Gordon tak linear ditunjukkan mempunyai sifat kebolehkamiran dengan melalui ujian Painlevé lemah.

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CHAPTER 1

INTRODUCTION

In this chapter, we state briefly the background of the problem of finding exact solution for some nonlinear evolution equations. We also outline the objectives and the scope of the study. Finally, the organization of this dissertation is presented.

1.1   Background of the problem

Painlevé analysis is named after a French mathematician Paul Painlevé (1863-1933). It is about the study of the singularity structures of differential equations which involves the searching of solutions of differential equations that are compatible with integrability. It is a widely applied and quite successful technique to investigate the integrability of nonlinear ordinary differential equations (ODE) and partial differential equations (PDE) by analyzing the singularity structure of the solutions. In particular, any differential equation is believed to be integrable if it possesses the Painlevé property.

In the study of integrability, in the sense of series expansions of the solutions, some local properties of the system of equations have been extended to answer the global question of integrability. In recent year, much attention has been focused on higher order nonlinear PDEs, known as evolution equations.

This project begins with an overview of the history of Painlevé analysis and a specialization on it. In this project, Painlevé property for PDE will be defined and will be shown as to how it determines the integrability of the equations. Several soliton equations will be presented and used.

1.2   Statement of the Problem

This project wishes to study in detail the Painlevé analysis for a class of nPDE equations. The examples of nPDEs such as Burgers' equation, Korteweg-de Vries (KdV) equation and nonlinear Klein-Gordon equation will be used. This project intends to find whether these equations satisfy Painlevé property. If it satisfies Painlevé equation, then its integrability is determined.

1.3   Objectives of the Study

The main objectives of this research are:
 a) To study in general the Painlevé analysis with its properties and conditions.
 b) To apply Painlevé test on a class of nPDE.
 c) To determine the integrability of respective nPDEs.

1.4   Scope of the Study

This project applies Painlevé analysis on certain class of physically significant nonlinear partial differential equations such as Burgers' equation, KdV equation and nonlinear Klein-Gordon equation. Our keywords are: nonlinear partial differential equations, Painlevé analysis and integrability.

1.5   Significance of the Study

It is important to identify and study nPDEs whose solutions are completely integrable. Integrability demands that the solutions are consistent, or match on the overlapping pieces of the sets on which they are defined. Painlevé analysis is the most successful approach in the understanding of the integrability and nonintegrability of the nPDEs.

1.6   Research Methodology

Firstly, literatures on the subject matters are searched for the literature review from books, journals and the internet. Informations related on this research are gathered. Then, mathematical equations for nPDEs are introduced. In this project, Painlevé test is used, seeking the integrability of the equations. Then, the result are compared. Lastly, report writing is done.