

## **ABSTRACT**

Dynamic Optimization (optimal control) involves the determination of optimal control variable profiles and time invariant parameter values that optimize a performance criterion of an underlying differential-algebraic process model. In this project, Dynamic Optimization is applied to simplified mathematical model in fisheries. This is an attempt to produce a mathematically based solution on the globally perceived problem of decrease sea productivity as a result of overfishing and mismanagement of the fishing resources. Logistic Growth Model, Gordon Schaefer and Beverton Holt Model will be discussed to give understanding in achieving the optimal management in fisheries. To overcome the overfishing and mismanagement problem, Faustmann Formula / Model had been derived and utilize to obtain the optimal rotation period in fisheries. The application of Faustmann Formula will be then presented graphically using MATLAB programming tools to achieve the result. From these findings, we can conclude that the optimal rotation period that we obtained can be adapted in the tropical sea such as Malaysia to overcome the overfishing problem. As a result, Dynamic Optimization generally are very useful to applied in optimal fisheries management.

## **ABSTRAK**

Pengoptimuman Dinamik melibatkan penentuan pembolehubah kawalan dan pembolehubah masa dalam nilai-nilai parameter yang mengoptimumkan indeks prestasi berdasarkan model proses terbitan aljabar. Dalam projek ini, Pengoptimuman Dinamik digunakan untuk menyelesaikan masalah perikanan. Dengan ini, masalah kekurangan ikan dalam lautan akibat tangkapan berlebihan dan kecuaiian pengurusan dapat diselesaikan dengan menggunakan kaedah matematik. Logistic Growth Model, Gordon Schaefer Model dan Beverton Holt Model akan dibincangkan untuk memberi pemahaman dalam mencapai pengurusan optimal dalam perikanan. Selain itu, penggunaan Faustmann Formula juga diaplikasi bagi menentukan kitaran masa yang paling optimal dalam penangkapan ikan. Penggunaan Faustmann Formula akan dipersembahkan menerusi graf yang diplot menggunakan MATLAB untuk mendapatkan keputusan yang dikehendaki. Keputusan yang diperolehi boleh diaplikasikan ke atas spesis ikan lautan tropika khususnya di Malaysia bagi menyelesaikan masalah ketandusan ikan di lautan. Kesimpulannya pengoptimuman dinamik amat berguna untuk diaplikasikan dalam memperolehi pengurusan yang optimal dalam perikanan.

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## CHAPTER 1

### INTRODUCTION

#### 1.1 Introduction

Optimal control deals with the problem of finding a control law for a given system such that a certain optimality criterion is achieved. A control problem includes a [cost functional](#) that is a function of state and control variables (Lewis and Syrmos, 1995). An optimal control is a set of differential equations describing the paths of the control variables that minimize the cost functional (Bryson and Ho, 1969). **1.1**

#### **Background of the problem**

Interest in renewable-resources economics has increased greatly in recent years. In this research we will concentrate in the fishery management as the issue of fishery collapse is a common worldwide phenomenon. A new phrase sustainable development indicates a concern with conservation for the long term benefits of humanity.

The inherent problem of over fishing is exacerbated by uncertainty in fish stock

size and dynamics. Among other factors, they attribute fishery collapse to uncertainty in marine environments, and suggest that ignoring uncertainty can lead to excessive harvest.

The work of Beverton and Holt (1957) and Schaefer (1954) assumed a deterministic environment and provided analytically tractable models of renewable resource exploitation. In this research, the optimal policy is “bang-bang” in which the optimal escapement level occurs where the rate of return from harvesting the last fish and investing the money from doing so just equals the rate of return from letting that last fish grow to the next period.

To solve overfishing problem in fisheries, we apply the Faustman Formula (1849) to obtain the optimal rotation period in harvesting the fish. By obtaining this rotation period, fish will be harvested when it already reach the maturity period (recruitment) only. Besides considering the maturity of fish, this approach also considers selecting the lowest discount rate.

### **1.3 Statement of the problem**

Uncontrollable harvesting of fish stock will led to the reduction of fish supply in the immediate term. The long term effect could be the extinction of certain popular fish species.

In this research we seek to implement certain optimization technique to create a sustainable fishing policy. A few of mathematical model in fisheries such as Logistic Growth Model (1838), Gordon Schaefer Model (1954) and Beverton Holt Model (1957) had been discussed to obtain the optimal management in fisheries.

The main focus in this research is the application of Faustmann Formula in solving the overfishing of open access tropical sea fisheries problem where by this application the optimal rotation period for harvesting fish could be obtain.

#### **1.4 Objectives**

The objectives of this research are

- (i) To conduct a literature review on optimal management of open access fishery using dynamic optimization.
- (ii) To developed and adopt a suitable model for open access fisheries in warm tropical waters.
- (iii) To carry some simulation based on optimal control technique and apply the Faustmann Formula to determine the optimal rotation period in harvesting tropical sea fisheries.



## **1.5 Methodology**

From the literature review, we analyze three models that will be useful in optimal fisheries management which is Logistic Growth Model, Gordon Schaefer Model (1954), and Beverton Holt Model(1957). We will also discuss and apply the Faustman Formula to obtain the optimal rotation period in fisheries. In this study, we use MATLAB programming tools to obtain the optimal rotation period for harvesting. The graph of Faustmann formula and the proportional growth rate that we get from the data will be plotted, discussed and analyzed. Then the optimal rotation period for harvesting the fish will be determined.

## **1.6 Scope of the study**

In this study, our scope is in the optimal management of open access fisheries concerning warm tropical sea fisheries. There are three models that will illustrate this situation which are Logistic Growth Model, Gordon Schaefer Model and Beverton Holt Model. While our main focus is to determine the optimal rotation period for tropical sea fisheries where Chilean Jack Mackerel were use as an example to obtain it's rotation period. This is done by applying the Faustmann Formula.

## **1.7 Significance of the study**

The result of this research will be useful in solving the overfishing in open access fisheries problem where the optimal rotation period for harvesting fish in warm tropical sea will be obtained by applying the Faustmann Formula.

Furthermore, some mathematical model using dynamic optimization can be applied in obtaining the optimal management of open access fishery. This shows that the mathematical model is very useful in solving our real life problem. In this study, we can see that optimal control plays an important role in solving most of the mathematical fisheries problem.

The techniques of optimal control theory are outgrowths and extensions of the classical variational techniques of Euler, Lagrange, Legendre, Weierstrass, Hamilton, and Jacobi. Indeed the maximum principle, which is a necessary condition for optimality, encompasses most of the classical necessary conditions of the early mathematicians in a unified way that simplifies applications (Pontryagin, 1962). Optimal Control theory is a significantly improved theory in that covers both nonlinear and linear optimization problems inequality constraints (Clark,2005).