

Investigation of integer-coefficient partial response coding in STBC-OFDM system

Sharifah K. Syed-Yusof, Norsheila Faisal and Muladi

Abstract—The carrier-to-interference power ratio in a space-time block coded (STBC) Orthogonal Frequency Division multiplexing (OFDM) system using the partial response coding approach is investigated. In this paper, the integer polynomial partial response coefficients are implemented. At frequency offset of 0.2, this method enhances the carrier-to-interference power ratio (CIR) by between 0.8 dB to 7.3 dB when the length of polynomial is 3 and 2 respectively. It also effectively reduces the error floor caused by Doppler frequency offset.

Index Terms—Carrier-to-interference ratio (CIR), frequency offset, inter-carrier Interference (ICI), Orthogonal Frequency Division multiplexing (OFDM), partial response coding (PRC), space-time block coded (STBC)

I. INTRODUCTION

IN time variant mobile radio environment, the relative movement between transmitter and receiver resulted in frequency offset due to Doppler frequency shifts, hence the carriers cannot be perfectly synchronised. This imperfection destroys orthogonality among subcarriers in OFDM-based systems and causes intercarrier interference (ICI) to occur in addition to signal rotation and attenuation. The degradation of BER performance increases rapidly with increasing frequency offset occurrence in OFDM system [1].

In single-input single output (SISO) OFDM system, several methods have been proposed to reduce the effect of the ICI. One of the methods is frequency-domain equalization [2]. Time-domain windowing is another way to reduce the effect of frequency offset [3]. A self-ICI-cancellation approach has been proposed, which transmits each symbol over a pair of adjacent or non-adjacent subcarriers with a certain phase shift [5, 6, 7]. This method can suppress the ICI significantly with a reduction in bandwidth efficiency. In single-carrier systems, partial response signaling has been studied to reduce the sensitivity to time offset without sacrificing the bandwidth [8]. In the frequency domain, the partial response with correlation polynomial $F(D) = 1 - D$ was used to mitigate the ICI caused by carrier frequency offset [9]. ICI is actually deliberately introduced in a controlled manner through the polynomial functions. The optimum weights for partial

response coding that minimize the ICI power were derived in [10]. However, by using polynomial coefficients with integer values reduces the complexity of the receiver. The ICI suppression in multiple-input multiple-output (MIMO) OFDM is studied in [4] by using time-domain filtering based. In this paper, we investigate the performance of partial response coding with integer polynomial coefficients in a special case of MIMO OFDM, known as space-time block-coded (STBC) OFDM system. A simple symbol-by-symbol suboptimum detection technique is used in this study.

This paper is organised as follows. In Section II we describe the partial response coding (PRC) OFDM system. The ICI expressions and analysis is included in Section III. Then, in Section IV, the simulation results are presented to demonstrate the performance of PRC in STBC-OFDM systems with integer polynomial coefficients.

II. PRC OFDM SYSTEM

Let X_k be the symbols to be transmitted and c_i be the coefficients for partial response polynomial, the transmitted signal at the k -th subcarrier can be expressed as

$$S_k = \sum_{i=0}^{K-1} c_i X_{k-i} \quad (1)$$

where K is the number of coefficients or length of the polynomial. Without loss of generality, $E|X_k|^2 = 1$ and $E(X_k X_j^*) = 0$ for $k \neq j$ is assumed.

Different polynomial length (up to $K=3$) and coefficients are investigated. The coefficients are limited to the value of ± 1 and 0. The transmitted SISO-OFDM signal in time domain is

$$y(t) = \sum_k S_k e^{j2\pi f_k t}, \quad 0 \leq t \leq T_s \quad (2)$$

where $f_k = f_0 + k\Delta f$ is the frequency of the k -th subchannel, $\Delta f = 1/T_s$ is the subchannel spacing, and T_s is the symbol duration.

After passing through a time-varying channel with the impulse response $h(t, \tau)$, the received signal is

$$\tilde{y}(t) = \int h(t, \tau) y(t - \tau) d\tau \quad (3)$$

The channel impulse response for the frequency-selective fading channel can be described by

$$h(t, \tau) = \sum_{l=0}^{v-1} h_l(t) \delta(\tau - \tau_l(t)) \quad (4)$$

where v is the total number of non-zeros taps in the channel response, $h_l(t)$ represents the time variant attenuation factor of the l -th path and $\tau_l(t)$ is time varying

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delay of l -th path. The channel impulse response can also be represented in terms of Doppler frequency shifts, $f_{D_l}(t)t$ caused by movement of mobile receiver as

$$h(t, \tau) = \sum_{l=0}^{v-1} h_l \exp(j 2\pi f_{D_l}(t)t) \delta(\tau - \tau_l(t)) \quad (5)$$

where h_l is the amplitude of the l -th path.

III. ICI ANALYSIS OF PRC-OFDM

The received signal can be written as

$$\tilde{y}(t) = h(t, \tau) * y(t) + z(t) \quad (6)$$

where $z(t)$ is the additive white Gaussian noise (AWGN).

The demodulated signal can be written as

$$Y(n) = \sum_{k=0}^{N-1} \tilde{y}(k) \exp\left(j \frac{2\pi}{N} n k\right) \quad (7)$$

The output of the DFT at the receiver for a time-block $\{-(v-1), \dots, N-1\}$ where N is the number of carriers can be written as

$$Y(p) = G(p, p)S(p) + \sum_{\substack{q=0 \\ q \neq p}}^{N-1} G(q, p)S(q) + Z(p) \quad (8)$$

for $p=0, \dots, N-1$. The $G(p, p)S(p)$ gives the desired signal value for subcarrier p with an average carrier power of $E[|G(p, p)S(p)|^2]$. While $G(q, p)$ is defined as the subcarrier frequency offset response for the p -th subcarrier [5]. It is also the ICI effect of the q -th subcarrier to the p -th subcarrier with the occurrence of normalised frequency offset, ε . In the case of time-variant the equation $G(q, p)$ becomes

$$G(q, p) = \frac{1}{N} \sum_{n=0}^{N-1} \sum_{l=0}^{v-1} h_{l,n} \exp(j\theta_{l,n}) \exp\left(j \frac{2\pi n}{N} (p-q)\right) \exp j \frac{2\pi}{N} n \varepsilon_{l,n} \quad (9)$$

By letting c_i be the coefficients for partial response and K is the number of coefficients or length of partial response, the transmitted signal can be expressed as in (1). Therefore, the ICI power on the m -th subcarrier can be expressed into

$$P_{ICI}(p) = E \left| \sum_{\substack{q=0 \\ q \neq p}}^{N-1} G(q, p)S(q) \right|^2 \quad (10)$$

In terms of the partial response coding coefficients, the total intercarrier interference power it can be expressed as

$$P_{ICI} = \sum_{i=0}^{K-1} \sum_{l=1}^{N-1} c_i^2 |G(l)|^2 + \sum_{k=1}^{K-1} \sum_{i=0}^{K-1-k} c_i c_{i+k} \left[\sum_{l=0}^{N-1} G(l)G^*(l-k+i) + G(l+k-i)G^*(l) \right] \quad (11)$$

In the case when we have transmit and receive diversities with M_r receive and M_t transmit antennas, we can modify (8) to

$$\mathbf{Y}(p) = \mathbf{G}(p, p)\mathbf{S}(p) + \sum_{\substack{q=0 \\ q \neq p}}^{N-1} \mathbf{G}(q, p)\mathbf{S}(q) + \mathbf{Z}(p) \quad (12)$$

where $[\mathbf{G}(q, p)]_{i,j}$ is the subcarrier frequency offset for the p -

th subcarrier between the i -th receiver and j -th transmitter for $(1 \leq i \leq M_r, 1 \leq j \leq M_t, 0 \leq p \leq N-1, 0 \leq q \leq N-1)$.

Also the received vector for the p -th subcarrier frequency is $\mathbf{Y}(p) = [Y^{(1)}(p), \dots, Y^{(M_r)}(p)]^T$, while the transmitted vector is $\mathbf{S}(p) = [S^{(1)}(p), \dots, S^{(M_t)}(p)]^T$ and the noise vector is $\mathbf{Z}(p) = [Z^{(1)}(p), \dots, Z^{(M_r)}(p)]^T$.

The ICI power on the n -th subcarrier can then be evaluated as

$$P_{ici}(n) = E \left[\sum_{\substack{p=0 \\ n \neq p}}^{N-1} \sum_{\substack{n=0 \\ n \neq m}}^{N-1} \mathbf{G}(m, p)\mathbf{S}(p)\mathbf{S}^H(n)\mathbf{G}^H(m, n) \right] \quad (13)$$

In the MIMO case, the intercarrier interference is defined to include interference from all other subcarrier frequency for the same antenna and interference from all other subcarriers (including n -th) from other antennas.

IV. RESULTS

In this section, we study the STBC-OFDM system based on Alamouti code [11] using the mechanism as in [12]. The preliminary simulation results for the performance of STBC-OFDM system with integer polynomials PRC is presented here. The block diagram of STBC-OFDM using PRC is illustrated in Fig.1. The number of subcarriers used in this simulation is 128 subcarriers. L denotes the number of symbol samples per carrier. Different polynomial length (up to $K=3$) and coefficients are investigated. The coefficients are limited to the value of $\pm 1, \pm 2$ and 0. This restriction is necessary as the amplitude level of partial response signaling is proportional to the values of coefficients. At each polynomial length, the polynomial that gives the lowest carrier to interference ratio (CIR) is chosen. Our simulation is conducted with the assumption of flat fading channel and the channel is perfectly known at the receiver. Symbol-by-symbol suboptimum maximum likelihood (ML) detection is used at the receiver.

The CIR performance of PRC in STBC-OFDM is shown in Fig.2. We can observe clearly from this figure that depending on the polynomial length, the PRC has around 0.8 to 7.3 dB improvement over without PRC realization when $\varepsilon=0.2$. At this particular frequency offset, $K=2$ has about 6.5 dB improvement gain over $K=3$. However, there is slight CIR improvement which is about 0.8 dB when $K=3$ compared to without PRC implementation.

From the observation from Fig.3, the PRC improves the BER of ordinary STBC-OFDM system with the presence of frequency offset, due to Doppler shift in the channel, As expected, the BER performance when $\varepsilon=0.1$ is better compared to when $\varepsilon=0.2$. As shown from Fig.3, the partial response coding has a much lower BER floor compared to the system without PRC. This can be seen from Fig.3 that at $\varepsilon=0.2$, the shorter length polynomial, $K=2$ is able to reduce the error floor of $K=3$ from 10^{-2} to 10^{-5} .

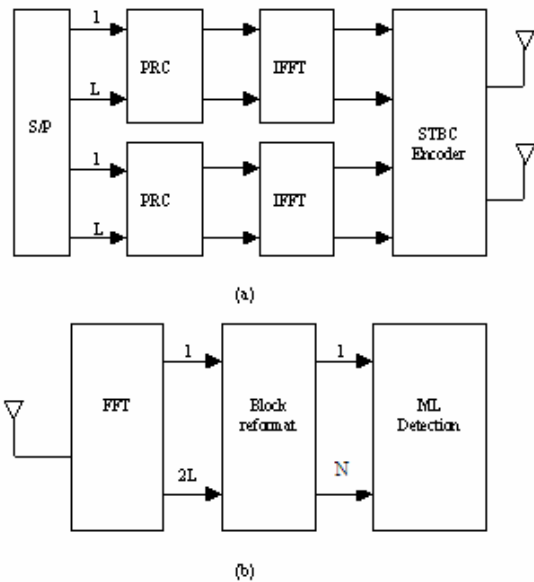


Fig.1. (a) Transmitter structure (b) Receiver structure

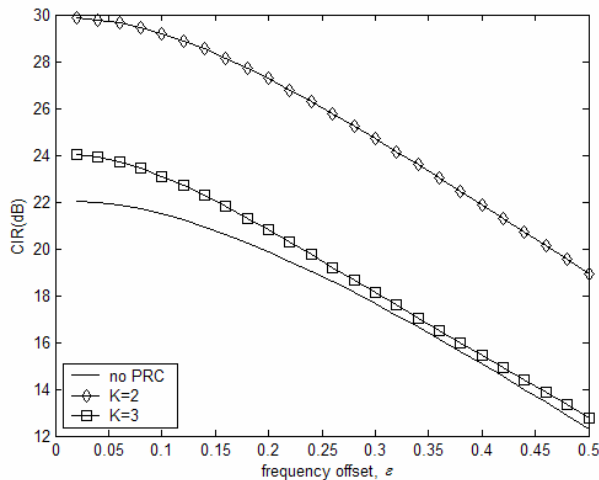


Fig.2. CIR comparison of PRC in STBC-OFDM with respect to constant frequency offset

V. CONCLUSION

In this paper, STBC-OFDM system with integer polynomial coefficients PRC is being studied. ICI is deliberately introduced in a controlled manner through the polynomial functions. The effectiveness of PRC (with symbol-by-symbol suboptimum detection) in improving the CIR of STBC-OFDM system is investigated.

From the results obtained, at $\epsilon=0.2$, about 0.8 to 7.3 dB CIR improvement is shown and the PRC effectively reduces the error floor caused by frequency offset due to Doppler frequency when the length of polynomial is shorter (2 in this case). Finally, we can conclude that PRC with integer polynomial coefficients gives a potential solution to adverse the effect of ICI in STBC-OFDM systems in a simple manner and further studies should be conducted in the future.

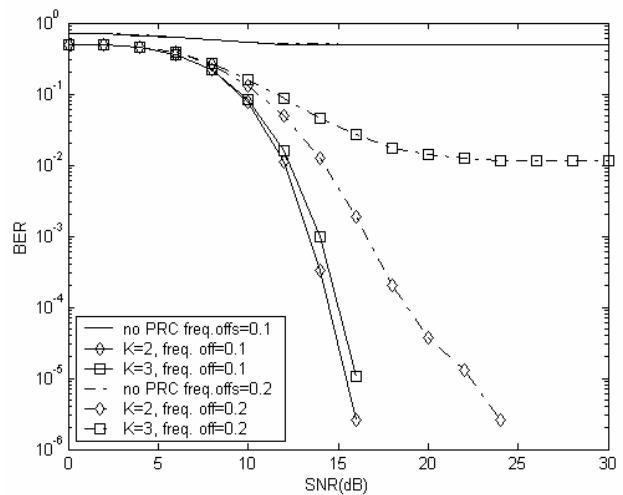


Fig.3. BER comparison of PRC in STBC-OFDM at frequency offset, ϵ of 0.1 and 0.2

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