# THEORETICAL FOUNDATION FOR DIGITAL SPACE OF FLAT ELECTROENCEPHALOGRAM

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A thesis submitted in fulfilment of the
requirements for the award of the degree of
Doctor of Philosophy (Mathematics)

Faculty of Science
Universiti Teknologi Malaysia

DECEMBER 2009

*To my family,*

*for all they have done for me in the past,*

*all they mean to me in the present,*

*and all I hope will yet be in the future.*

*My beloved parents,*

*Puan Fadzilah Ismail*

*Encik Ahmad Che Nik (in memory)*

*My understanding husband,*

*Mohamad Rizam Ramli*

*My beautiful daughter,*

*Nur Rusydina Mohamad Rizam*

*My handsome son,*

*Ahmad Rifqi Mohamad Rizam*

*I am nobody without all of you.*

# ACKNOWLEDGEMENT

All praises are due to Allah the Almighty, the lord of all that exits. May Allah peace and blessing be upon His final Prophet and Messenger, Muhammad, his family and his Companions.

First of all, I would like to express my deep gratitude to my supervisor, Assoc. Prof. Dr. Tahir Ahmad for his invaluable guidance and constant assistance during this research. I wish to thanks my external supervisor Dr Hussain Imam Bin Hj Muhammad Ismail for his suggestions and encouragement in this research.

I want to thank Universiti Utara Malaysia for giving me the opportunity to further my study. Thank you also to the Minister of Higher Education (MOHE) for financial support through the SLAB award.

I wish to express to my dearest thanks to my husband for his understanding, endless patience and encouragement and to my children for bringing so much happiness and joy into my life. Special thanks to my mother, my mother in-law and my family members for their prayers and loving support.

Last but not least, I also would like to thank members of Fuzzy Research Group and all my friends for their help, friendship and encouragement.

# ABSTRACT

Epilepsy is one of the most common disorders of the brain characterized by recurrent seizures. Epileptic seizure, which is caused by abnormal electrical activity in brain can be measured by using Magnetoencephalogram (MEG) and Electroencephalogram (EEG). MEG measures magnetic field whereas EEG measures electrical potential during seizure. Fuzzy Topographic Topological Mapping (FTTM) is a mathematical model for solving neuromagnetic inverse problem. The model was developed to accommodate static simulated and experimental MEG signals and their transformed image. However, in this thesis, digital topology is adopted for FTTM, in particular with Khalimsky topology where the actual structure of digital objects can be visualized. The new construction is called FTTM digital and is denoted as $FTTM_{dig}$. All four components of $FTTM_{dig}$ are shown to be homeomorphic as in the older versions of FTTM. In addition, real time recorded EEG signal during epileptic seizure is constructed to be topological space by inducing on metric space. Finally, this topological space of real time recorded EEG signal is incorporated with $FTTM_{dig}$ via relational topology. The integration of these two topologies is the key to the foundation of the novel Flat EEG.

# ABSTRAK

Sawan merupakan salah satu daripada gangguan yang biasa berlaku di dalam otak yang dikategorikan mengikut serangan yang berulang-ulang. Serangan sawan ini disebabkan oleh ketidaknormalan aktiviti elektrik pada otak yang dapat diukur dengan menggunakan *Magnetoencephalogram* (MEG) dan *Electroencephalogram* (EEG). MEG mengukur medan magnet, sementara EEG pula mengukur keupayaan elektrik. *Fuzzy Topographic Topological Mapping* (FTTM) merupakan satu model matematik yang direka untuk menyelesaikan masalah songsangan neuromagnetik. Model ini dibangunkan untuk menempatkan isyarat pegun MEG yang diperolehi melalui simulasi dan eksperimen serta menempatkan imej terubah. Walaubagaimanapun, dalam tesis ini, topologi digital dibina pada FTTM, khususnya topologi Khalimsky bagi menggambarkan struktur sebenar objek digital. Pembinaan ini dinamakan FTTM digital dan diwakilkan sebagai $FTTM_{dig}$. Semua komponen $FTTM_{dig}$ ditunjukkan sebagai homeomorfisma seperti dalam versi FTTM yang lama. Sebagai tambahan, masa rakaman sebenar isyarat EEG semasa serangan sawan berlaku dibina sebagai ruang topologi dengan menggunakan ruang metrik. Akhir sekali, ruang topologi bagi masa rakaman sebenar isyarat EEG ini digabungkan dengan $FTTM_{dig}$ melalui topologi hubungan. Penggabungan kedua-dua topologi ini merupakan kunci kepada asas pembinaan EEG sekata.

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## General Symbols

| | | |
|---|---|---|
| $X$ | - | Arbitrary set |
| $R$ | - | Relation |
| $\mathbb{R}$ | - | Real numbers |
| $\mathbb{Z}$ | - | Integers |
| $\in$ | - | Element of |
| $\forall$ | - | For all |
| $\exists$ | - | There exist (at least one) |
| $\bigcup$ | - | Set union |
| $\bigcap$ | - | Set intersection |
| $X/Y$ | - | Set difference |
| $\subset$ | - | Proper set inclusion |
| $\subseteq$ | - | Set inclusion |
| $=$ | - | Equal |
| $\neq$ | - | Not equal |
| $\tau$ | - | Topology |
| $d$ | - | Metric |
| $(X,\tau)$ | - | Topological space |
| $(X,d)$ ,$M$ | - | Metric space |
| $xR$ | - | Afterset |
| $Rx$ | - | Forset |
| $S_1$, $S_2$ | - | Subbase of topology |
| $\cong$ | - | Homeomorphism |
| $\varnothing$ | - | Empty set |
| $\theta$, $U$, $V$ | - | Open set |
| $\mu_{B_z}$ | - | Fuzzified magnetic field |
| $B_z$ | - | Absolute values of magnetic field |
| min $B$ | - | Minimum absolute values of magnetic field |
| max $B$ | - | Maximum absolute values of magnetic field |
| $f$ | - | Function |
| $f^{-1}$ | - | Inverse function |
| [ ] | - | Closed interval |

| | | |
|---|---|---|
| $(\ )$ | - | Open interval |
| $card$ | - | Cardinality of a set |
| $t$ | - | Time |
| $P$ | - | Potential difference |
| $P_i$ | - | Potential difference for each channels |
| $P_{it}$ | - | Potential difference for each channels at any particular time |
| $s(t)$ | - | A value of signal at time |
| $N_{EEG}$ | - | Space-time of EEG signal |
| $X$, $Y$, $Z$ | - | Cartesan coordinates |

**Abbreviations**

| | | |
|---|---|---|
| FTTM | - | Fuzzy Topographic Topological Mapping |
| MC | - | Magnetic Contour Plane |
| BM | - | Base Magnetic Plane |
| FM | - | Fuzzy Magnetic Field |
| TM | - | Topographic Magnetic Field |
| MI | - | Magnetic Image Plane |
| BMI | - | Base Magnetic Image Plane |
| FMI | - | Fuzzy Magnetic Image Field |
| TMI | - | Topographic Magnetic Image Field |
| $\mathrm{FTTM}_{dig}$ | - | Fuzzy Topographic Topological Mapping digital |
| $\mathrm{MC}_{dig}$ | - | Magnetic Contour Plane digital |
| $\mathrm{BM}_{dig}$ | - | Base Magnetic Plane digital |
| $\mathrm{FM}_{dig}$ | - | Fuzzy Magnetic Field digital |
| $\mathrm{TM}_{dig}$ | - | Topographic Magnetic Field digital |
| EEG | - | Electroencephalography |
| MEG | - | Magnetoencephalography |
| MRI | - | Magnetic Resonance Imaging |
| CT | - | Computerized Tomography |
| PSP | - | Postsynaptic Potential |
| EPSP | - | Excitatory Postsynaptic Potential |
| IPSP | - | Inhibitory Postsynaptic Potential |
| FCM | - | Fuzzy c-Means |
| SQUID | - | Superconducting Quantum Interference Device |
| $fT$ | - | femto Tesla |
| $\mu V$ | - | micro Volt |

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# CHAPTER 1

# INTRODUCTION

## 1.1 Introduction

Epilepsy is a common neurobiological disorder of recurrent seizure (Shorvon, 2005) that affects approximately 1% of the world's population which is more than 50 million individuals worldwide. Epilepsy can affect anyone at any stage of life but has the greatest incidence from infancy to adolescence and in persons over the age of 65 (Engel, 1989).

Epilepsy is diagnosed when two or more seizures of an unknown cause occurs (Freeman, 2002). A seizure occurs when a volume of nerve cells, or neurons within the cerebral cortex experience a sudden surge of synchronized electrical activity which is temporarily disrupting the cells functionally. The electrical disruption can irritate surrounding cells causing the seizure to propagate to other parts of the brain.

Seizures are classified into two major categories as either partial or generalized (Marks, 2003). A partial seizure occurs when the initial discharges occur at a localized focus while a generalized seizure has multiple foci at various locations throughout the whole brain, in both hemispheres. Not all seizures can be easily defined as either partial or generalized. Some people have seizures that begin as partial seizures but then spread to the entire brain. Others may have both types of seizures but with no clear pattern.

There is no single test that can prove or disprove whether someone has epilepsy. Among the primary diagnostics and evaluation tools for diagnosing epilepsy seizure are Magnetoencephalography (MEG), Electroencephalography (EEG), brain scans such as Magnetic Resonance Imaging (MRI) and Computerized Tomography (CT) scans. However, only the MEG and EEG can directly measure the electrical activity of the brain. MEG measures the magnetic field outside the head produced by electrical activity of the neurons in the brain (Hamalainen et al., 1993). On the other hand, EEG measures the potential differences on the scalp induced by electrical activity of the brain (Richey and Namon, 1976).

Once epilepsy is diagnosed, the extensive drug treatments are recommended to control it. The treatments are designed to restore the chemical imbalance in brain cells that results in excessive electrical activity which leads to epileptic seizures. If epilepsy fails to respond to drug treatment, the patient may indicate a need for surgical therapy. However, if surgery is needed, the surgeons have to find out where the seizure originates. In general, the epilepsy surgery is a procedure that either removes or isolates the region of the brain where the seizure begins (Vossler and Wyler, 2004).

The main problem in epilepsy surgery is to resolve the size and location of the epileptic foci. Ensuring only a minimal amount of gray cortex is being removed is needed in order to minimize the loss of normal functioning gray matter. According to Kalitzin *et al*. (2005) there is no single technique that has been validated to give the definite answer to predict the exact location of epileptic foci. Furthermore, there are six cortical zones that have been defined in presurgical evaluation of candidates for epilepsy surgery (Rosenow and Luder, 2001). Therefore, it is crucial to determine an accurate technique which is capable to localize the epileptogenic focus from the MEG and EEG data in patient who is suffering from epilepsy.

Researches have been extensively studied to address this problem either in theoretical development or applied study. This study concentrates on developing mathematical theory of topological spaces for MEG and EEG data of epileptic suffer.

## 1.2 Research Background

Recovering electrical activity of the brain from MEG and EEG measurement is called bioelectromagnetic inverse problem (Figure 1.1). The bioelectromagnetic inverse problem consists of detecting pointwise dipolar current sources (modeling epileptic foci) located in the brain, from measures of the magnectic field or electric potential on the surface of the head (Hamalainen et al., 1993; Baillet *et al*., 2001, Darvas *et al*., 2004).

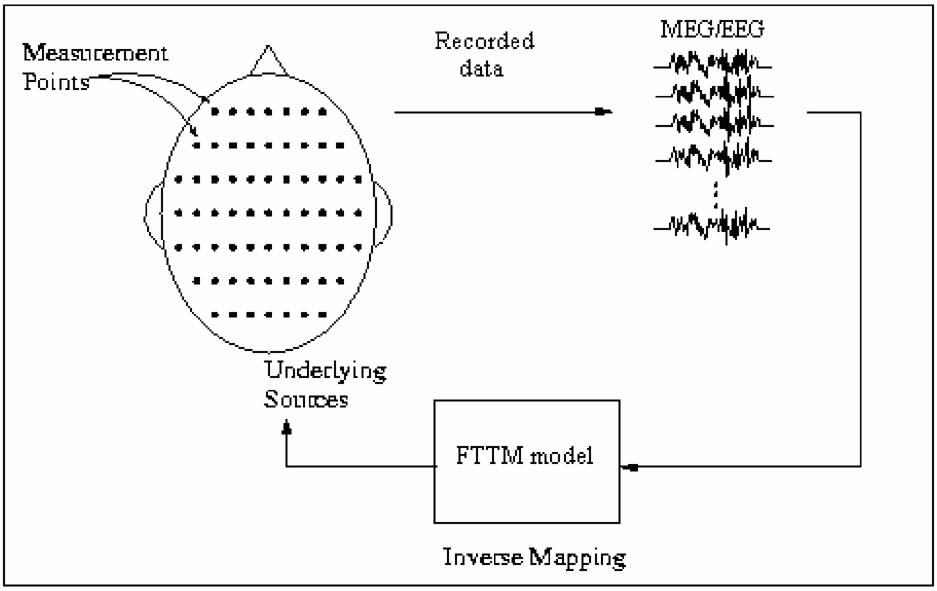


**Figure 1.1** An illustration of bioelectromagnetic inverse problem

Tahir *et al*. (2000) have introduced a novel method to solve this neuromagnetic inverse problem. The method is called Fuzzy Topographic Topological Mapping (FTTM) which is to determine the current source, i.e. epileptic foci. There are two versions of FTTM which is FTTM1 (Tahir et al, 2000; Liau, 2001; Fauziah, 2002; Tahir et al., 2004) and FTTM2 (Wan Eny Zarina et al., 2002; Wan Eny Zarina, 2006). In both versions of FTTM, topology has been used as fundamentals of development for the structure of FTTM components.

In this study, digital topology is adopted theoretically for FTTM to represent the current source. The question arises, why should the medium of FTTM need to be changed instead of the conventional topology that has been used by Tahir *et al.*(2000), Liau (2001) and Wan Eny Zarina (2006)? One of the main reasons is that Liau's work (2001) is no longer applicable to represent the digital objects in FTTM when computer is considered in analyzing the biomagnetic fields data of MEG signals.

Furthermore, the construction of digital topology of FTTM is supported from the result obtained in Wan Eny Zarina (2003) where the image is not really continuous. In fact, it is digital since the image is displayed in pixel. Generally, the objects on the computer screen are displayed as continuous image. These objects are actually discretized and are represented by using some sort of approximation. As a result, there is a need to change the fundamental of the standard topology in FTTM to the digital topology.

Besides that, FTTM model is not only able to work with biomagnetic fields data, it can also deal with any types of information which is related to electrical activity in human brain (Liau, 2006). Therefore, in FTTM model, bioelectric fields data of EEG signals can be used as one of the alternative information for source localization in the human brain. Thus, a topological space for EEG signals needs to be constructed by inducing metric space, in order to be equipped with a newly form of FTTM model, namely FTTM digital.

## 1.3 Problem Statement

This study attempts to develop theoretical foundation for digital space of Flat EEG. In order to describe digital space of Flat EEG, the theory of topology, digital topology and relational topology have been applied. The development of this theory enables any information on FTTM digital model be represented in digital objects which are highly relevant to the use of computer.

## 1.4 Research Objectives

The objectives of this research are:

1. to study the digital topology that can be suited to FTTM model;
2. to develop FTTM digital, a new FTTM model;
3. to construct topological space for EEG signals by inducing metric space;
4. to incorporate the constructed topological space of EEG signals with the FTTM digital via relational topology;
5. to support and generalize the experimental results against simulated MEG data and recorded real time EEG data.

## 1.5 Scope of the Study

This study develops mathematical theory of topological spaces on the bioelectromagnetic fields data of the epilepsy seizure which will be incorporated with FTTM model.

## 1.6 Significance of Findings

The findings of this study will contribute to:

1. the development of a new model of FTTM for solving bioelectromagnetic inverse problem; FTTM digital;
2. the theoretical development to the real time recorded EEG signal;
3. the application of determining the epileptic foci in pre-surgical evaluation made by the medical team.

## 1.7 Thesis Outline

This thesis has seven chapters. The development of this thesis is shown in Figure 1.2. The first chapter provides the introduction of the research. This chapter discusses the research background, the problem statement, the research objectives, the scope of study and the significance of the thesis.

Chapter 2 presents the selected reviews of previous studies on MEG and EEG. The origin of electrical brain activity, the application of MEG and EEG and the mathematical technique involved in MEG and EEG are also included.

Chapter 3 provides the mathematical backgrounds including topology, digital topology and FTTM that will be used throughout the thesis.

The main findings of this thesis are given in Chapter 4, 5 and 6. Chapter 4 is devoted to the construction of digital topology for FTTM model. The main objective of this chapter is to improve the topological structure of FTTM, particularly Khalimsky topology. All components of FTTM digital are shown to be homeomorphic as in the older versions of FTTM.

Chapter 5 depicts the construction of topological space of real time recorded EEG signal by inducing metric space. This chapter also illustrates how the space-time topology of EEG signal is developed to show the motion in that space.

Chapter 6 incorporates this topological space of real time recorded signal with FTTM digital via relational topology which provides the foundation of Flat EEG. Furthermore, this construction has highly linked with the idea of information granulation.

Finally, Chapter 7 concludes the whole thesis with a summary of the study and some insights of the possibilities for further research conducted in this area of study.

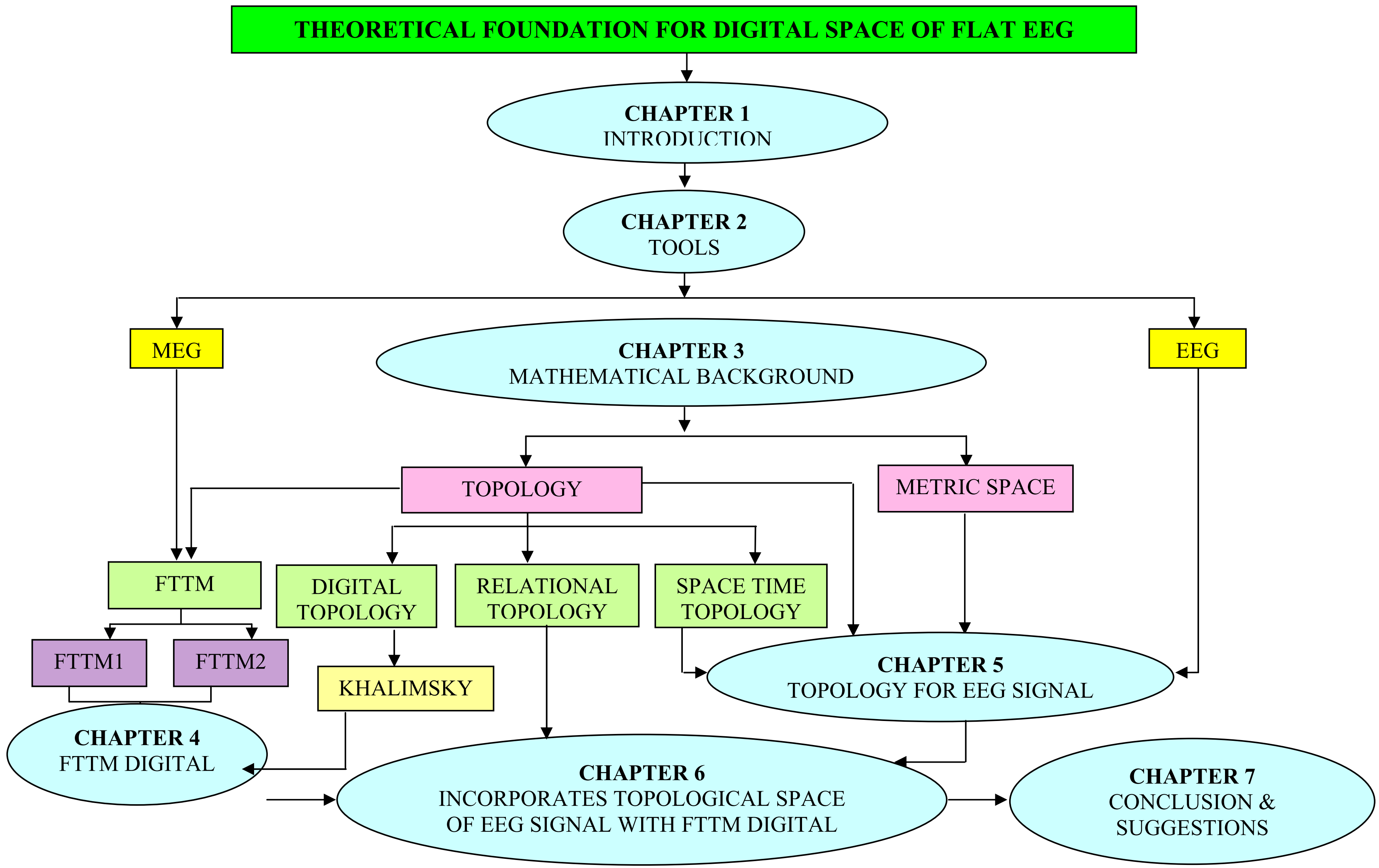


**Figure 1.2** Research Framework