NUMERICAL MODELLING AND SIMULATION OF BLOOD FLOW THROUGH A MULTI-IRREGULAR STENOSED ARTERY

NORZIEHA BT MUSTAPHA

UNIVERSITI TEKNOLOGI MALAYSIA

NUMERICAL MODELLING AND SIMULATION OF BLOOD FLOW THROUGH A MULTI-IRREGULAR STENOSED ARTERY

NORZIEHA BINTI MUSTAPHA

A thesis submitted in fulfilment
of the requirements for the award of the degree of
Doctor of Philosophy (Mathematics)

Faculty of Science
Universiti Teknologi Malaysia

JUNE 2009

To my husband, parents and family

– thank you for everything

ACKNOWLEDGEMENT

Alhamdulillah, thanks to Allah s.w.t. for graciously bestowing me the perseverance to undertake this research. I owe a great debt of gratitude to many people who supported and assisted me in the preparation of this thesis. A special thanks and appreciation to my supervisor, Prof. Dr. Norsarahaida S. Amin for providing invaluable guidance and advice.

I am also indebted to Assoc. Prof. Dr. Prashanta K. Mandal from the Department of Mathematics, Visva-Bharati, West Bengal, India and Assoc. Prof. Dr. Peter Johnston, from Griffith University, Brisbane, Australia, for their dedicated time and effort to ensure the success of this research. I would also like to extend my gratefully acknowledging to my co-supervisor, Dr Norma Alias.

Financial support provided by the Public Services Department of Malaysia and Universiti Teknologi Malaysia is greatly appreciated. Finally, my sincere thanks to all my colleagues, staffs and friends at UTM who have kindly provided assistance, helpful suggestions and comments to ensure the success of this thesis.

ABSTRACT

The blood flow problem in a multi-irregular stenosed artery is important from the physiological considerations in view of many clinical situations. For instance, the patient is found to have multiple stenoses in the same arterial segment and the geometry of the stenosis is irregular. In this study, numerical modelling of blood flow through a multi-irregular stenosed artery is developed. Experimental investigations have also revealed that blood exhibits non-Newtonian properties at low shear rate. This research deals with both Newtonian and non-Newtonian models of blood flow. Such effects have been studied in the form of generalized Newtonian fluid, where apparent viscosity decreases by increasing shear rate. Two external factors namely periodic body acceleration and magnetic field have been considered. Numerical solutions are established under the assumptions that the flow is axisymmetric, unsteady, laminar, fully developed and two-dimensional. Numerical computation by finite difference Marker and Cell (MAC) method has been used to discretize the governing equations of motion for unsteady flow in the cylindrical polar co-ordinate system. The obtained pressure-Poisson equation was then solved through the successive-over-relaxation (S.O.R.) method. The obtained numerical results show good agreement with the experiments. It is found that under the influence of body acceleration, the velocity and flow rate are increased. The pressure drop gives higher values and the separation region is found to be larger in the case of blood flowing through a flexible artery having multiple irregular stenoses when compared to blood flowing through a single irregular stenosed artery. Furthermore, when magnetic field is increased, the velocity gradient near the wall and the wall shear stress will also be increased. With a sufficiently large magnetic field, the flow separations completely disappeared. The generalized Newtonian model results in a higher pressure drop with a smaller separation region in comparison to the Newtonian fluid.

ABSTRAK

Masalah aliran darah dalam arteri yang mempunyai berbilang stenosis dan berpermukaan tak rata adalah penting dari sudut fisiologi dalam kebanyakan situasi klinikal. Misalnya, pesakit didapati mempunyai lebih daripada satu stenosis dalam satu segmen arteri yang sama dan permukaan stenosis adalah tak rata. Dalam kajian ini, permodelan berangka bagi aliran darah melalui arteri yang berbilang stenosis dan berpermukaan tak rata dipertimbangkan. Kajian-kajian yang dijalankan secara bereksperimen telah mendedahkan bahawa darah mempunyai sifat-sifat tak Newtonan pada kadar ricih yang rendah. Kajian ini mempertimbangkan kedua-dua model Newtonan dan tak Newtonan. Kesan-kesan ini dikaji bercirikan model Newtonan teritlak, yang mana kelikatan ketara berkurang apabila kadar ricih meningkat. Dua faktor luaran iaitu pergerakan jasad berkala dan medan magnet dipertimbangkan. Penyelesaian berangka dilaksanakan dengan andaian bahawa aliran darah adalah berpaksi simetri, tak mantap, berlamina, terbentuk sepenuhnya dan dalam dua matra. Kaedah berangka iaitu kaedah beza terhingga Marker dan Cell (MAC) digunakan untuk menyelesaikan persamaan pembezaan separa yang mewakili aliran tak mampat dalam sistem koordinat silinder. Persamaan tekanan - Poisson yang diperoleh diselesaikan menggunakan kaedah pengenduran berlebihan berturutan (S.O.R). Keputusan berangka yang dicapai menunjukkan persamaan yang baik dengan keputusan eksperimen. Keputusan berangka menunjukkan bahawa di bawah pengaruh pergerakan jasad, halaju dan kadar alir meningkat. Sementara itu, susutan tekanan dan rantau pemisahan meningkat apabila aliran darah melalui arteri fleksibel yang mempunyai dua stenosis berbanding dengan satu stenosis. Selain itu, dengan peningkatan medan magnet, halaju berhampiran dinding arteri dan tekanan ricih pada dinding arteri meningkat. Dengan kekuatan medan magnet yang cukup besar, pemisahan aliran susut sepenuhnya. Di samping itu, model tak Newtonan menunjukkan peningkatan susutan tekanan dengan rantau pemisahan lebih kecil berbanding model Newtonan.

TABLE OF CONTENTS

CHAPTER	TITLE		PAGE
	TITLE PAGE		i
	DECLARATION		ii
	DEDICATION		iii
	ACKNOWLEDGEMENT		iv
	ABSTRACT		v
	ABSTRAK		vi
	TABLE OF CONTENTS		vii
	LIST OF FIGURES		xi
	LIST OF TABLES		xv
	LIST OF SYMBOLS / NOTATIONS		xvi
	LIST OF APPENDICES		xix
1	**INTRODUCTION**		
	1.1	Research Background	1
	1.2	Objectives and Scope of the Study	4
	1.3	Research Methodology	4
	1.4	Significance of the Study	5
	1.5	Outline of Thesis	5
2	**LITERATURE REVIEW**		
	2.1	Introduction	8
	2.2	Studies on the Effect of Body Acceleration on Blood Flow	8
	2.3	Studies on the Geometry of Stenoses	10

	2.4	Studies on Magnetohydrodynamics (MHD) Effects on Blood Flow	13
	2.5	Studies on the Non-Newtonian Rheology of Blood Flow	15
	2.6	Method of Solution	17
	2.7	The Governing Equations	19
		2.7.1 The equation of Newtonian model	20
		2.7.2 The equation of body acceleration	22
		2.7.3 Magnetohydrodynamics (MHD) flow	22
		2.7.4 The equation of generalized Newtonian Model	24
3	**UNSTEADY RESPONSE OF BLOOD FLOW TO BODY ACCELERATION THROUGH MULTI-IRREGULAR ARTERIAL CONSTRICTIONS**		
	3.1	Introduction	26
	3.2	The Stenoses Geometry (The Wall Model)	27
	3.3	Governing Equations	30
		3.3.1 Boundary and Initial Conditions	31
	3.4	Solution Procedure	32
		3.4.1 Radial Coordinate Transformation	32
		3.4.2 Finite Difference Approximation	35
	3.5	Determination of Blood Flow Characteristics	37
	3.6	Numerical Algorithm	39
	3.7	Stability and Accuracy	39
	3.8	Numerical Results and Discussions	42
4	**A TWO DIMENSIONAL PULSATILE BLOOD FLOW THROUGH A MULTI-IRREGULAR STENOSED ARTERY**		
	4.1	Introduction	60
	4.2	Governing Equations	61
		4.2.1 Boundary and Initial Conditions	61

	4.3	Solution Procedure	61
		4.3.1 Non-dimensionalization of the Equations	62
		4.3.2 Radial Coordinate Transformation	63
		4.3.3 Marker and Cell Method	64
		4.3.4 The Poisson Equation for Pressure	70
		4.3.5 Successive Over Relaxation (SOR) Method	73
		4.3.5 Pressure and Velocity Corrections	74
	4.4	Stability Restriction	77
	4.5	Determination of Blood Flow Characteristics	78
	4.6	Numerical Algorithm	78
	4.7	Numerical Results and Discussions	80
5	**UNSTEADY MAGNETOHYDRODYNAMICS BLOOD FLOW THROUGH IRREGULAR MULTI-STENOSED ARTERIES**		
	5.1	Introduction	97
	5.2	Mathematical Formulation	98
		5.2.1 Boundary and Initial Conditions	99
		5.2.2 Radial Coordinate Transformation	102
		5.2.3 Solution Procedure	103
	5.3	Numerical Results and Discussions	104
6	**A NUMERICAL SIMULATION OF GENERALIZED NEWTONIAN BLOOD FLOW THROUGH MULTI-IRREGULAR ARTERIAL STENOSES**		
	6.1	Introduction	113
	6.2	Mathematical Formulation	114
		6.2.1 Boundary and Initial Conditions	116
		6.2.2 Radial Coordinate Transformation	116
		6.2.3 Solution Procedure	117
	6.3	Numerical Results and Discussions	118

7	**CONCLUSION**		
	7.1	Summary of Research	127
	7.2	Suggestions for Future Research	130
REFERENCES			132
Appendices A-D			141-159

LIST OF FIGURES

FIGURE NO.	TITLE	PAGE
2.1	A sketch of the physical problem	24
3.1	A published data of Back *et al* (1984)	28
3.2	Profile of multiple stenoses with irregular and cosine curves	29
3.3	Finite difference non-uniform grids	35
3.4	Numerical algorithm	40
3.5	Axial velocity profile for three different grid sizes at the critical height of stenosis when t = 0.5 s	41
3.6	Axial velocity profile for a single stenosis at the critical height of stenosis when t = 0.5 s	50
3.7a	Distribution of the axial velocity over the entire arterial segment at t = 0.75 s ($a_0 = 100$ mm/s^2, $\phi = 0$, $x = 0.8$)	50
3.7b	Distribution of the axial velocity at different times ($a_0 = 100$ mm/s^2, $\phi = 0$, $x = 0.8$)	51
3.8	Profiles of the radial velocity for different axial positions at t = 0.75 s ($a_0 = 100$ mm/s^2, $\phi = 0$, $x = 0.8$)	51
3.9	Profiles of the radial velocity at z = 57.1 mm for different times ($a_0 = 100$ mm/s^2, $\phi = 0$, $x = 0.8$)	52
3.10a	Distribution of the rate of flow at t = 0.75 s ($a_0 = 100$ mm/s^2, $\phi = 0$, $x = 0.8$)	52
3.10b	Distribution of the rate of flow at different times ($a_0 = 100$ mm/s^2, $\phi = 0$, $x = 0.8$)	53
3.11a	Distribution of the wall shear stress over the entire arterial	53

	segment at $t = 0.75$ s ($a_0 = 100$ mm/s^2, $\phi = 0,\ x = 0.8$)	
3.11b	Distribution of the wall shear stress over the entire arterial segment at different times ($a_0 = 100$ mm/s^2, $\phi = 0,\ x = 0.8$)	54
3.12a	Axial variation of the fluid acceleration at $t = 0.75$ s ($a_0 = 100$ mm/s^2 $\phi = 0, x = 0.8$) for irregular and cosine curves	54
3.12b	Axial variation of the fluid acceleration at $t = 0.75$ s ($a_0 = 100$ mm/s^2 $\phi = 0, x = 0.8$)	55
3.13a	Variation of the rate of flow with time for different f_b at $z =$ 57.1mm ($f_p = 1.2$Hz, $a_0 = 100$ mm/s^2, $\phi = 0,\ x = 0.8$)	55
3.13b	Variation of the rate of flow with time for different f_p at $z =$ 57.1mm ($f_b = 1.2$Hz, $a_0 = 100$ mm/s^2, $\phi = 0,\ x = 0.8$)	56
3.13c	Variation of the rate of flow with time for different z ($f_b = 1.2$Hz, $f_p = 1.2$Hz, $a_0 = 100$ mm/s^2, $\phi = 0,\ x = 0.8$)	56
3.14a	Variation of wall shear stress with time for different f_b at $z = 57.1$ mm ($f_p = 1.2$Hz, $a_0 = 100$ mm/s^2, $\phi = 0,\ x = 0.8$)	57
3.14b	Variation of wall shear stress with time for different f_p at $z = 57.1$ mm ($f_b = 1.2$Hz, $a_0 = 100$ mm/s^2, $\phi = 0,\ x = 0.8$)	57
3.14c	Variation of wall shear stress with time for different z ($f_b = 1.2$Hz, $a_0 = 100$ mm/s^2, $\phi = 0,\ x = 0.8$)	58
3.15	Instantaneous patterns of streamlines for different a_0 ($f_b = 1.2$Hz, $f_p = 1.2$ Hz, $\phi = 0, t = 20$ s)	59
4.1a	Typical MAC cell	64
4.1b	Control volume of MAC cell	65
4.2	Numerical algorithm	79
4.3	Profile of multi-irregular stenoses and a long single irregular stenosis	87
4.4	Comparison of dimensionless pressure drop across a single irregular stenosis	88

4.5	Dimensionless pressure drop across various stenoses of the same total length	88
4.6	Cross-sectional profiles of the axial velocity component corresponding to different axial positions in the multi-irregular stenoses at Re =1000	89
4.7	Variation of the axial velocity component along the axis of each stenosis at Re = 1000	89
4.8	Variations of normalized wall shear stress through various stenoses at Re = 20	90
4.9	Variations of normalized wall shear stress through various stenoses at Re = 1000	90
4.10	Variations of the combinations of the stenoses severity	91
4.11	The normalized wall shear stress for multi-irregular stenoses with various severity combinations at Re = 500	91
4.12	Instanteneous patterns of streamlines for different combinations of stenoses severity at Re = 500	92
4.13	Instanteneous patterns of streamlines for multi-cosine severe stenoses and a long single severe stenoses at Re = 500	92
4.14	Vorticity contour for the model of different combination of stenoses severity at Re = 500	93
4.15a	Vorticity contour for multi-irregular stenoses at Re = 500	93
4.15b	Vorticity contour for multi-cosine stenoses at Re = 500	94
4.15c	Vorticity contour for a long single stenosis at Re = 500	94
4.16	The normalized wall shear stress for different times at Re = 1000	95
4.17	Dimensionless pressure drop across the stenosis of irregular geometry at Re = 700	96
4.18	Patterns of streamlines for multiple modified irregular (74% severity) at t = 60.0089 and t = 68.0001 at Re = 500	96
5.1	Dimensionless pressure drop across multi-irregular stenosed artery	108
5.2	Variations of normalized wall shear stress for multi-irregular (M = 6, 13) and multi-cosine (M = 6) stenoses at Re = 1000	108

5.3	Variations of normalized wall shear stress at Re = 1000	109
5.4	Axial velocity profiles at the downstream of stenoses (z = 57.1) at Re = 1000	110
5.5	Variations of normalized wall shear stress for more severe stenoses (74% areal occlusion) at Re = 500	110
5.6	Instantaneous patterns of streamlines for 74% areal occlusion at Re = 500 for M = 0, 4, 10 and 17	111
5.7	Vorticity contour for the M = 0, 4, 10 and 17 of more severe stenoses at Re = 500	112
6.1	Comparison of dimensionless pressure drop across multiple stenoses (GN- generalized Newtonian)	122
6.2	Variations of normalized wall shear stress at Re = 300 (GN-generalized Newtonian)	122
6.3	Variations of normalized wall shear stress at Re = 1000 (GN-generalized Newtonian)	123
6.4	Variations of normalized wall shear stress through each stenosis geometry for 74% areal occlusion at Re = 300 (GN- generalized Newtonian)	123
6.5	Radial variation of viscosity in the multi-irregular stenoses at different positions for Re = 500	124
6.6	Instantaneous patterns of streamlines for 74% areal occlusion (proximal) & 48% areal occlusion (distal) at Re = 500	124
6.7	Instantaneous patterns of streamlines for 74% areal occlusion (proximal & distal) at Re = 500	125
6.8a	Vorticity contour for generalized Newtonian fluid through severe stenoses at Re = 500	125
6.8b	Vorticity contour for Newtonian fluid through severe stenoses at Re = 500	126

LIST OF TABLES

TABLE NO.	TITLE	PAGE
3.1	Variation of stenosis radius along the axis (mm) (See Back *et al.* (1984))	27
3.2	The results of velocity at certain z for different step size of t	40
4.1	Results of velocity for grid size of 979 X 40 and different Reynolds numbers in a straight tube	81
4.2	Dimensionless pressure drop across a single and multiple irregular stenoses	82
4.3	Values of critical Reynolds number	84

LIST OF SYMBOLS / NOTATIONS

Roman Letters

a_0	-	amplitude
$A_\circ$	-	constant amplitude of the pressure gradient
A_1	-	amplitude of the pulsatile component
$\tilde{\mathbf{A}}_1$	-	rate of deformation tensor
B_0	-	uniform magnetic field
$\tilde{\mathbf{B}}$	-	total magnetic field
$\tilde{\mathbf{B}}_0$	-	uniform magnetic-field
$\tilde{\mathbf{B}}_1$	-	induced magnetic field
$\tilde{\mathbf{E}}$	-	electric field
f_p	-	pulse frequency
f_b	-	body acceleration frequency
$\tilde{F}$	-	fluid acceleration
$\tilde{\mathbf{g}}$	-	gravitational force
$\tilde{G}(\tilde{t})$	-	periodic body acceleration
$\tilde{h}(\tilde{z})$	-	unstressed wall thickness
I_0	-	Bessel function
$\tilde{\mathbf{J}}$	-	current density
k_R	-	wall oscillation
$l_\circ$	-	length of the stenosis
L	-	the finite difference arterial segment

M	-	Hartmann number
p	-	pressure
P_A	-	pressure at the beginning of the throat
P_B	-	pressure at the end of the throat
Q	-	volumetric flow rate
r_0	-	constant radius of the non-stenotic artery
$\tilde{r}_m$	-	unstressed mid-wall radius
$\tilde{R}_m$	-	mid-wall radius of the vessel in deformed state
$R(z,t)$	-	radius of the arterial segment in the stenotic region
Re	-	Reynolds number
$\tilde{\mathbf{S}}$	-	extra stress
$\tilde{\mathbf{T}}$	-	stress tensor
$u(r,z,t)$	-	the radial velocity component
U_0	-	cross-sectional average velocity
V	-	volume
$\tilde{\mathbf{V}}$	-	fluid velocity field
$w(r,z,t)$	-	the axial velocity component

Greek Letters

β	-	a combination factor
$\frac{\partial p}{\partial z}$	-	pressure gradient
$\dot{\gamma}$	-	shear rate
δ	-	radial extension ratio
Δx	-	increment in the radial directions
Δz_i	-	increment in the axial directions
Δt	-	small time increment
Δp_d	-	dimensionless pressure drop
$\Delta\tilde{p}$	-	pressure drop

$\tilde{\zeta}$	-	resistance
η_0, η_∞	-	asymptotic apparent viscosities
$\lambda, \Lambda, \wedge$	-	viscosity parameters
μ	-	constant viscosity
$\mu(\dot{\gamma})$	-	non-Newtonian viscosity (a function of shear rate)
ρ	-	density of blood
σ_0, β_0	-	stress-strain parameters
σ	-	electrical conductivity
τ_w	-	wall shear stress
ϕ	-	phase difference of the acceleration
ω_u	-	under relaxation parameter
ω_o	-	over relaxation parameter
ω_p	-	angular frequency
ω_b	-	angular frequency of body acceleration

LIST OF APPENDICES

APPENDIX	TITLE	PAGE
A	Derivation of the Governing Equation in Cylindrical Coordinates	143
B	The Matlab® Program to Obtain The Solutions of Problem of Unsteady Response of Blood Flow to Body Acceleration Through Multi-Irregular Arterial Constrictions (Chapter 3)	146
C	The Matlab® Program to Obtain The Solutions of Problem of A Two Dimensional Pulsatile Blood Flow Through A Multi-Irregular Stenosed Artery (Chapter 4)	149
D	List of Publications	160

CHAPTER 1

INTRODUCTION

1.1 Research Background

Cardiovascular diseases are the leading cause of death for both men and women all over the world (see Beaglehole *et al.* 2004). Cardiovascular disease is used most often to describe damage caused to the heart or blood vessels by atherosclerosis (arterial disease). Atherosclerosis often involves multiple narrowing of coronary vessel. This localized narrowing in a coronary artery commonly referred to as a stenosis (see Mazumdar (1989)). Multiple narrowing, which is more than one stenosis in the same arterial segment causes serious consequences of an arterial stenosis. One of the most serious consequences is a reduction of blood flow to the tissues which may develop across multiple stenoses. Once a single arterial stenosis is located in the artery, this will narrow or blocks the artery that provides oxygen and nutrients to the heart. Then, resulting blockage restricts blood flow to the tissues and heart. When the blood flow is completely cut off, it causes the cardiovascular diseases. Thus, when there are two or more stenoses in series, the effect becomes more critical.

The Doppler ultrasound spectral analysis technique can noninvasively detect the flow disturbances associated with moderate to severe degrees of constrictions in certain vessels. However, a method for the detection of minor (mild) arterial stenosis has not been found. This is partially due to the lack of detailed knowledge about the flow pattern in the presence of such stenosis. Thus the ability to understand completely the flow through blood vessels seems to provide the possibility of

diagnosing the disease at an earlier stage even before the stenosis appears to be clinically significant and hence an early treatment can be provided. Although the critical stenosis (more than 70% areal occlusion) is found to give more effects on the characteristics of blood flow, the mild and moderate stenosis should not be ignored.

In many cases of previous study, most of the assumptions made neglected the importance of physiological effects such as the rigid vessel wall, the flow is time independent and the geometry of the stenosis is smooth and symmetry (i.e. cosine curve). In reality, the arterial blood flow is unsteady and the effect of the unsteadiness on pressure drop is important. The cyclic nature of the heart pump creates pulsatile conditions in all arteries. The heart ejects and fills with blood in alternating cycles called systole and diastole. Besides, the movements of the arterial walls are limited by the surrounding tissue and should be described by various differential equations depending on thickness and the elasticity effects (see Liepsch *et al.* (1985)). Further, in general the arterial geometry has important impacts on the hemodynamics of arteries, and the changes of local blood flow would strengthen the influence on all kinds of mechanisms causing atherosclerosis. The stenosis geometry should be very complex, of which the shape is asymmetric and the surface may contain rough irregularities which resemble small valleys.

Human body is subjected to body accelerations or vibrations in many situations. Body vibration is capable of producing a wide variety of different effects on human body. Human responses to whole-body vibration are highly dependent on the frequency and direction of vibration, the posture and orientation of the body. The magnitude of the vibration displacement which is visible to the eye does not provide a good indication of the severity of a motion (see Griffin (1990)). Body accelerations or vibrations could be intentional or unintentional. Though human has remarkable adaptability to changes, a prolonged exposure of body to such vibrations lead to many health problems like headache, loss of vision, increase in pulse rate, abdominal pain, venous pooling blood in extremities and haemorrhage in the face, neck, eye sockets, lungs and brain.

Nowadays, magnetic therapy is widely used for curing various diseases. The blood which is considered as a magnetohydrodynamics (MHD) fluid will help in controlling blood pressure and has potential therapeutic use in the diseases of heart and blood vessel (see Mekheimer (2004)). By using an appropriate magnetic field it can become effective to conditions such as poor circulation, travel sickness, pain, headaches, muscle sprains, strains and joint pain.

Blood is a complex mixture of cells, protein, lipoproteins, and ions by which nutrients and wastes are transported. Red blood cells typically comprise approximately 40% of blood by volume. Since red blood cells are small semisolid particles, they increase the viscosity of blood and affect the behaviour of the fluid. Blood is approximately four times more viscous than water. Moreover, blood does not exhibit a constant viscosity at all flow rates and is especially non-Newtonian in the microcirculatory system. The non-Newtonian behaviour is most evident in small vessels or at very low shear rates (less than $100s^{-1}$) when the red blood cells clump together into larger particles (see Ku (1997)). Since the shear rate is low in the downstream of the stenosis, a correct analysis of the flow pattern should include the non-Newtonian which is shear-thinning character of the blood.

The present study considers the problems of blood flow in a multi-irregular stenosed artery with Newtonian and non-Newtonian fluids. Blood rheology which includes the shear-thinning property known as generalized Newtonian fluid is considered in this study. The flow is assumed to be two-dimensional, laminar, unsteady and axisymmetric. Two cases of external factors discussed are the body acceleration and magnetic field. The analysis includes (1) formulation of the mathematical models to obtain the governing blood flow problems, (2) radial coordinate transformation and (3) numerical computation using a finite-difference scheme. The scheme employed is a finite difference based on the central difference approximation following Cavalcanti (1995) and Mandal (2005) and the finite difference method in staggered grid, known as Marker and Cell introduced by Harlow and Welch (1965). In the next section, we present the objectives and scopes of this present study. Then, follows by the research methodology in Section 1.3 and

the significance of the study is given in Section 1.4. Finally, the thesis outline, which describes the content of the entire thesis, is presented in the last section.

1.2 Objectives and Scope of The Study

The objectives of the present study are as follows:

Problem 1: To investigate the response of blood flow to body acceleration through a multiple of irregular arterial constrictions in Newtonian fluid using finite difference method.

Problem 2: To determine the effect of irregular surface on the characteristics of blood flow through a multiple arterial stenoses in Newtonian fluid using Marker and Cell (MAC) method

Problem 3: To investigate the effect of magnetohydrodynamics (MHD) blood flow in multi-irregular stenosed artery in Newtonian fluid using MAC method

Problem 4: To determine the effect of generalized Newtonian model on blood flow past a multiple of irregular arterial stenoses using MAC method

The scope of the study is a numerical investigation of blood flow through a multi-irregular stenosed artery. The geometry of the stenosis is a rough (irregular) stenosis profile, which is obtained from the published data by Back *et al.* (1984). The flow is assumed to be unsteady, incompressible, axisymmetric and laminar. The arterial wall is assumed to be deformable following Johnson *et al.* (1992). Only Newtonian and generalized Newtonian models of blood viscosity are considered. The external effects considered are body acceleration and magnetic field.

1.3 Research Methodology

The research methodology undertaken to solve the problems are problem formulation, non-dimensionalization of the equations, radial coordinate

transformation and numerical computation. The numerical computations employed in this thesis are the finite difference in both non-staggered and staggered (Marker and Cell Method) grids. The details about this research methodology will be described in Chapters 3 and 4.

1.4 Significance of the Study

The significance of the study is the behaviour of blood flow while flowing through multiple stenoses with the surface irregularities can be determined. Moreover, the complete understanding of the flow through blood vessels with the presence of mild and severe multi-irregular stenoses can provide the possibility of diagnosing the disease at an earlier stage even before the stenosis appears and hence an early treatment can be provided. In addition, a complete knowledge about the domain of biomechanics associated with the blood flow through stenosed artery is achieved. Besides, experimental results have shown that the existing classical Navier-Stokes theory which sufficiently describes Newtonian fluid is inadequate to describe blood rheology. Thus, more appropriate and realistic mathematical models to interpret and enhance understanding of blood flow behaviour to help elucidate and explain experimental and physical observation are developed. Meanwhile, the apparent positive effect that external forces such as body acceleration and magnetic fields have on blood flow would be explained through mathematical means. Further, the establishment of numerical techniques will pave the way for the solution of more difficult bio-fluid and mathematical problems, which would inevitably lead to the fundamental understanding of bio-fluid behaviour and contribute to the advancement of medical science.

1.5 Outline of Thesis

This thesis is divided into seven chapters including this introductory chapter. Chapter 1 presents some general introduction, research background, scope and

objectives, research methodology and significance of research. In this study, the problems of unsteady blood flow in a multi-irregular stenosed artery, the external factors such as body acceleration and MHD and the non-Newtonian rheology which is a generalized Newtonian model are investigated. These problems will be discussed in four chapters, namely Chapters 3 through 6. Chapter 2 contains the literature review. All of these problems are solved numerically using either the finite difference method or Marker and Cell (MAC) method.

In Chapter 3, the first problem which discusses the response of body acceleration on blood flow through a multi-irregular stenosed artery is presented. This chapter is divided into eight main sections including the introduction, the stenoses geometry, governing equations, solution procedure, determination of blood flow characteristics, numerical algorithm, stability and accuracy, numerical results and discussion. In this chapter, the discussion is related to the differences in the characteristics of blood flow with the presence of different values of body acceleration. The finite difference method based on central difference approximation used in this study is programmed in Matlab®. The complete program of the specific problem discussed in Chapter 3 is given in Appendix B.

Chapter 4 explaines the second problem which discusses a two dimensional pulsatile blood flow through a multi-irregular stenosed artery. This chapter will be divided into seven main sections where the first section is the introduction of the problem, second and third section, respectively describes the governing equations and solution procedure. In the next section, we describe a stability restriction and numerical scheme. In Section 4.5, we give the discussion about the determination of blood flow characteristics, followed by numerical algorithm. In Section 4.7, the numerical results are discussed. The discussion in this Chapter is about the difference between a cosine and irregular curve of multiple stenoses, a single stenosis, double stenoses and the long single stenosis, the severity of the stenosis and the critical Reynolds number. All related figures are presented, and some results are also given in the form of tables in all the chapters. In this chapter, a full discussions and details on the solution procedures which is MAC method is presented. This method has been found to be suitable and flexible to deal with the problems of blood flow. The MAC

method used in this study is programmed in Matlab®. The complete program of the specific problem will be discussed in Chapter 4 is given in Appendix C. The stability analysis and accuracy of the methods are presented. The model verification and validation by comparing with the experimental results and existing numerical results are given in this chapter.

In Chapter 5, the fourth problem related with the effect MHD blood flow in a multi-irregular stenosed will be discussed. This chapter contains three sections including introduction, governing equations, boundary and initial conditions, methods of solution and discussions of numerical results. In this chapter, the discussion concentrates on the effect of various magnetic field values on pressure drop, wall shear stress and separation region. With the sufficiently large magnetic field, the separation region of blood flow through a multi-irregular stenosed artery can be removed. Finally, the last problem is discussed in Chapter 6 which concentrates on shear thinning model of blood flow, known as generalized Newtonian model through a multi-irregular stenosed artery. This chapter contains three sections similar with the previous chapters. In this chapter, we present the discussion on the blood rheology effects on blood characteristics such as pressure drop, wall shear stress and streamlines pattern. The final and concluding chapter, Chapter 7, contains some concluding remarks, summary of research as well as some suggestions for future research.