MULTI-OBJECTIVE SLIDING MODE CONTROL OF ACTIVE MAGNETIC BEARING SYSTEM

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ABSTRACT

Active Magnetic Bearing (AMB) system is known to inherit many nonlinearity effects due to its rotor dynamic motion and the electromagnetic actuators which make the system highly nonlinear, coupled and open-loop unstable. The major nonlinearities that are associated with AMB system are gyroscopic effect, rotor mass imbalance and nonlinear electromagnetics in which the gyroscopics and imbalance are dependent to the rotational speed of the rotor. In order to provide satisfactory system performance for a wide range of system condition, active control is thus essential. The main concern of the thesis is the modeling of the nonlinear AMB system and synthesizing a robust control method based on Sliding Mode Control (SMC) technique such that the system can achieve robust performance under various system nonlinearities. The model of the AMB system is developed based on the integration of the rotor and electromagnetic dynamics which forms nonlinear time varying state equations that represent a reasonably close description of the actual system. Based on the known bound of the system parameters and state variables, the model is restructured to become a class of uncertain system by using a deterministic approach. In formulating the control algorithm to control the system, SMC theory is adapted which involves the formulation of the sliding surface and the control law such that the state trajectories are driven to the stable sliding manifold. The surface design involves the transformation of the system into a special canonical representation such that the sliding motion can be characterized by a convex representation of the desired system performances. Optimal Linear Quadratic (LQ) characteristics and regional pole-clustering of the closed-loop poles are designed to be the objectives to be fulfilled in the surface design where the formulation is represented as a set of Linear Matrix Inequality optimization problem. For the control law design, a new continuous SMC controller is proposed in which asymptotic convergence of the system's state trajectories in finite time is guaranteed. This is achieved by adapting the equivalent control approach with the exponential decaying boundary layer technique. The newly designed sliding surface and control law form the complete Multi-objective SMC (MO-SMC) and the proposed algorithm is applied into the nonlinear AMB in which the results show that robust system performance is achieved for various system conditions. The findings also demonstrate that the MO-SMC gives better system response than the reported ideal SMC (I-SMC) and continuous SMC (C-SMC).

ABSTRAK

Sistem bearing magnet aktif (AMB) diketahui mempunyai pelbagai pengaruh kesan ketaklinearan disebabkan oleh pergerakan dinamik rotor dan penggerak sistem elektromagnet yang telah menyebabkan sistem ini mengalami ketaklinearan yang tinggi, terganding dan tidak stabil dalam kawalan gelung terbuka. Faktor penyumbang utama kepada ketaklinearan ini dikaitkan dengan kesan giroskopik, ketidakseimbangan berat rotor dan ketaklinearan elektromagnet di mana kesan giroskopik dan ketakseimbangan berat rotor adalah berkadar terus dengan kelajuan putaran rotor. Untuk mendapatkan sambutan sistem yang memuaskan dalam julat operasi sistem yang luas, kawalan aktif adalah diperlukan. Tesis ini membincangkan permodelan sistem AMB yang tak linear dan pembangunan pengawal tegap berasaskan kawalan ragam gelincir (SMC) di mana sistem yang dikawal akan mencapai prestasi tegap dalam pelbagai ketaklinearan sistem. Model AMB yang dibangunkan ini adalah berdasarkan integrasi antara dinamik rotor dan elektromagnet. Persamaan tak linear tersebut adalah berubah dengan masa dan persamaan ini mewakili penghampiran kepada ciri sistem yang sebenar. Berdasarkan kepada batasan parameter sistem yang diketahui, model ini distrukturkan semula menjadi satu kelas sistem tak pasti menggunakan pendekatan secara deterministik. Dalam membangunkan algoritma kawalan untuk mengawal sistem tersebut, teori kawalan ragam gelincir telah digunakan di mana kaedah ini melibatkan rekabentuk permukaan gelincir dan juga pembangunan hukum kawalan yang boleh memastikan trajektori sistem terpacu ke arah permukaan gelincir yang stabil. Rekabentuk permukaan gelincir melibatkan penukaran sistem kepada satu bentuk berkanun khas di mana pergerakan gelincir boleh diwakilkan oleh perwakilan cembung yang merangkumi prestasi sistem yang dikehendaki. Kuadratik Linear (LQ) optimum dan kawasan gugusan kutub yang dihasilkan dari kawalan gelung tertutup adalah objektif-objektif yang perlu dipenuhi dalam rekabentuk permukaan gelincir di mana ianya boleh diwakili sebagai satu set permasalahan pengoptimuman Ketaksamaan Matrik Linear. Untuk rekabentuk hukum kawalan, satu pengawal ragam gelincir berterusan yang baru telah dicadangkan. Hukum kawalan ini dapat menjamin sistem trajektori sampai ke kawasan kestabilan asimptot dalam satu masa yang terhingga. Ini dapat dicapai dengan menggunakan teknik kawalan setara yang digabungkan dengan lapisan sempadan yang menurun secara eksponen. Permukaan gelincir dan hukum kawalan yang baru dibangunkan ini membentuk pengawal kawalan ragam gelincir berbilang objektif (MO-SMC) lengkap. Pengawal ini kemudian diaplikasikan kepada sistem AMB tak linear di dalam pelbagai keadaan dan prestasi sistem secara tegap telah terbukti tercapai. Penemuan ini juga menunjukkan bahawa MO-SMC menghasilkan sambutan sistem yang lebih baik berbanding dengan teknik kawalan lain yang sedia ada iaitu kawalan ragam gelincir unggul (I-SMC) dan kawalan ragam gelincir berterusan (C-SMC).

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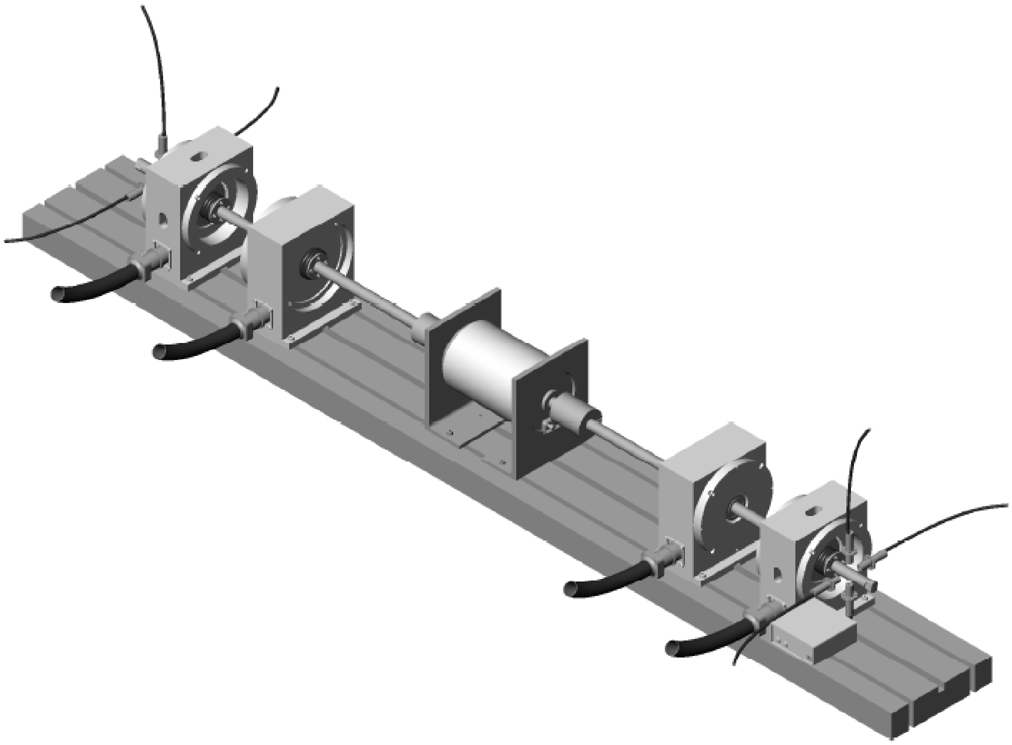
CHAPTER 1

INTRODUCTION

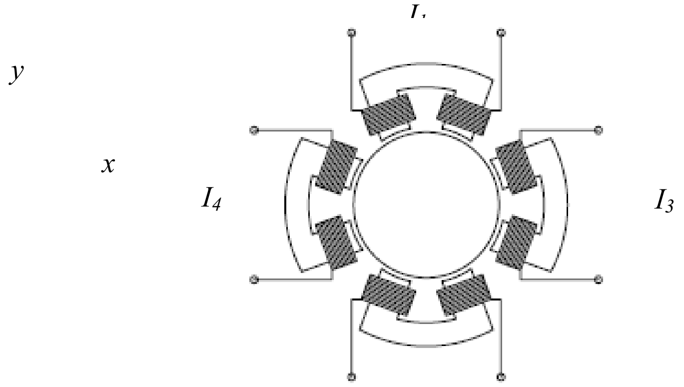
1.1 Introduction to Active Magnetic Bearing System

Bearings are one of the most essential components in all rotating machinery and the study on its mechanism and development is becoming more indispensable as the technology need pushes for more high-precision high-speed devices. By standard definition, bearing is the static part of machine (stator) that supports the moving part (rotor) of a system. While air and fluid bearings may be found in multi-degree-of-freedom ball and socket joint of machines, ball bearings, which allow for pure rotation, are by far the most popular and widely used in many industrial application mainly due to its low production cost and ubiquitariness (Wilson, 2004). Magnetic bearings are alternative to this traditional types or bearings, in which the bearings are constructed from permanents magnets, electromagnets or both in which the bearing in this combination is called hybrid magnetic bearing. An active magnetic bearing (AMB) system is then defined as a collection of electromagnets used to suspend an object via feedback control. For one degree-of freedom (DOF) system, usually AMB is synonymously called magnetic suspension system as used in ground transportation system where the vehicle is floated by the combination of controlled electromagnetic and permanent magnetic forces i.e. Maglev Train (Trumper *et al.*, 1997; Namerikawa and Fujita, 2004; Fujita *et al.*, 1998; Bleuler, 1992). For system with higher DOF, AMB system contains a suspended cylindrical rotor that rotates in varying speed depending on the applications. Thus, the obvious feature of AMB system is its non-contact suspension mechanism, which offers many advantages compared to conventional bearings such as lower rotating losses, higher operating speed, elimination of high-cost lubrication system and lubrication contaminations, suitability to operate at temperature extremes and in vacuum and having longer life span (Okada and Nonami, 2002; Knospe and Collins, 1996; Bleuler, 1992). Due to these significant reasons, AMB has been applied in a wide range of applications such as industrial machineries and medical equipment, power and vacuum technologies, and artificial heart, to quote a few applications (Knospe, 2007; Mohamed *et al.*, 1997b; Shen *et al.,* 2000; Maslen *et al.*, 1999, Tsiotras and Wilson, 2003; Kasarda, 2000; Lee *et al.* 2003).

Figure 1.1 illustrates an example of the standard structure of six-DOF AMB system and the schematic arrangement of the rotor and magnetic coil (stator) of the system. The system is composed of a cylindrical rotor or shaft made of laminated or solid ferromagnetic material, sets of electromagnetic coils, power amplifiers, position sensors and digital controller. The shaft is coupled to an external driving mechanism such as pumps, electric motors or piezo actuators by a flexible coupling which provides the rotational motion that forms the sixth DOF of the system. The electromagnetic coils generate the magnetic forces by the current I_i and the position sensors monitor the gap between the rotor and stator in which the captured information is used by the digital controller to determine the control signal necessary to suspend the rotating rotor to the centre of the actuating bearings. The control signal is sent to the power amplifiers for necessary amplification of the current I_i such that forces produced are able to withstand the dynamic requirement of the rotor as well as the external mechanical load. In addition, with some changes in the configurations of the AMB, the electromagnetic coils are not only able to supply the radial forces, but also generate the forces for rotational motion consequently eliminating the need of external driving mechanism. This so-called self-bearing motor appears rather appealing for space-constraint application, however the design construction and formulation of the control system is considerably much more complex (Kasarda, 2000; Kanekabo and Okada, 2003; Bleuler, 1992).



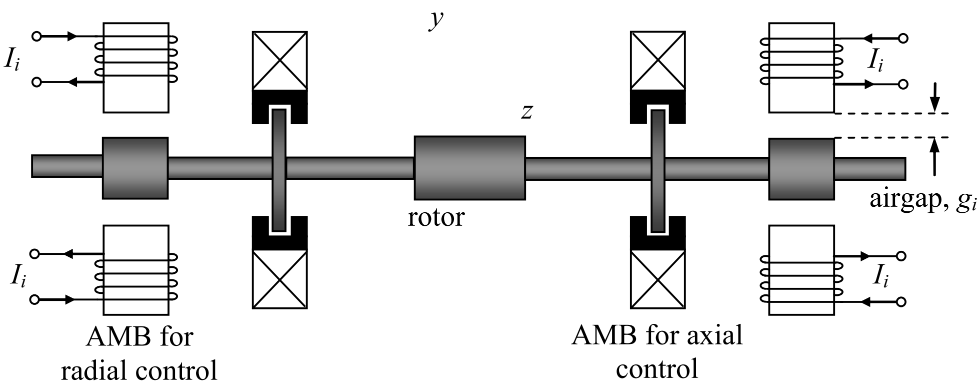
(a) Typical AMB system set-up



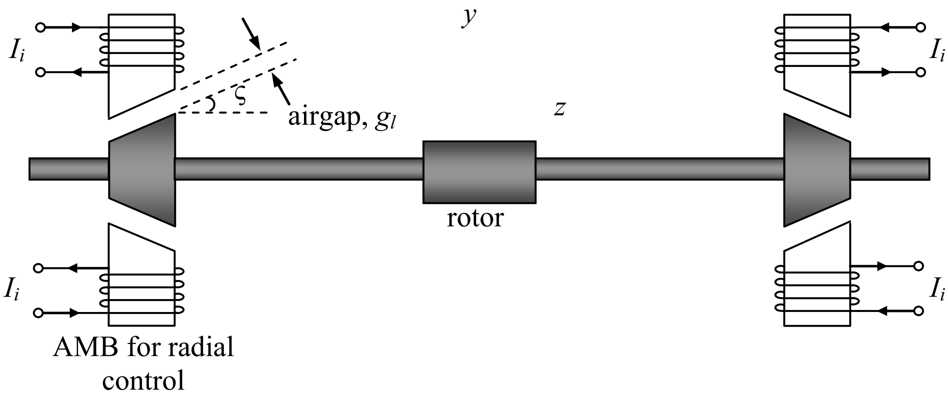
(b) Rotor and electromagnetic coils (stator) with respective coil currents, I_1, I_2, I_3 and I_4.

Figure 1.1 Active Magnetic Bearing System

In most of AMB system, there exist separate sets of electromagnetic coils that control the radial (*x*- and *y*- axes) and axial (*z*-axis) movement of the rotor due to negligible dynamic coupling between these axes of motions. Advantageously, separate control schemes are feasible to regulate the motions in radial and axial of the system. As illustrated in Figure 1.1 (a), at each end of the rotor, a set of electromagnetic coils is used for radial control where in each set, it contains two pairs of coil as shown in Figure 1.1 (b). Based on this figure, at this end of the system, the coil currents I_1 and I_2 supply the forces in y-direction while I_3 and I_4 supply the forces in x-direction. For the axial motion, one magnetic coil is located on each side of the rotor end. As an alternative to electromagnetic coil, in some AMB system where the rotor movement is very minimal, permanent magnets are sufficient to supply the regulating axial force and thus more favored to be used.



(a) System with cylindrical AMB



(b) System with conical AMB

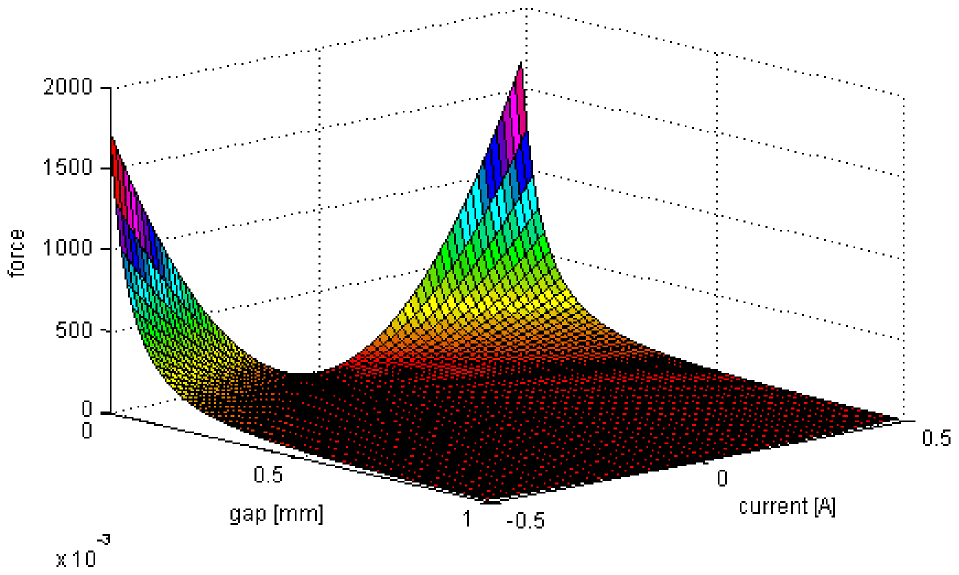
Figure 1.2 Configurations of Active Magnetic Bearing System

Figure 1.2 further illustrates two configurations of AMB system where in Figure 1.2 (a), the cylindrical rotor is used. This is similar to the aforementioned description of system in Figure 1.1 in which the axial motion is separately controlled by a pair of electromagnetic coil. In contrast to this configuration, conical magnetic bearing (Figure 1.2 (b)) where the rotor surface at the bearing end has small angle, ς, which makes the airgap between the rotor and bearing to be in slanted position. With this set-up, the electromagnetic coils supply both the axial and radial forces to the system and the most obvious advantage obtained is the elimination of a pair of axially-control electromagnetic coils. Nevertheless, this system experiences high coupling effect between the axes and formulation of reliable controller under wide operating is a very challenging task (Mohamed and Emad, 1992; Huang and Lin, 2004; Cole *et al.*, 2004).

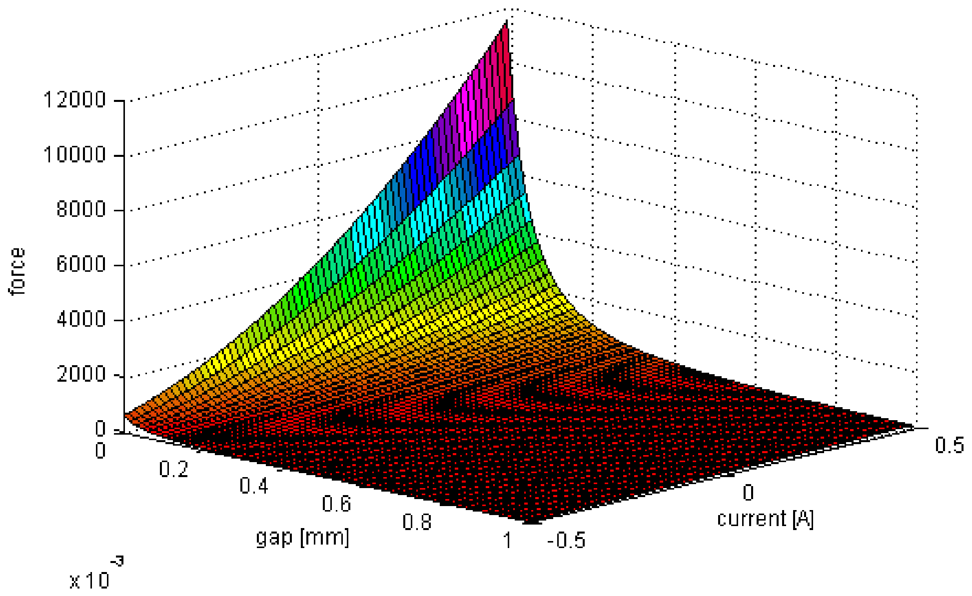
The various structural designs of AMB system are constructed to meet different kind of requirements of the real-world application in order to exploit the advantage of this non-contact lubrication-free technology. However, there are also numerous nonlinearities inherited in AMB system that cause the system instability. One of the most prominent nonlinearities is the relationship between the force-to-current and force-to-airgap displacement. The general equation that governs the magnetic force in AMB system is given as:

$$\begin{aligned} f_m &= \mu_o A_g N^2 \left(\frac{I_i}{g_i}\right)^2 \\ &= K \left(\frac{I_i}{g_i}\right)^2 \end{aligned} \tag{1.1}$$

where μ_o is the permeability of free space, A_g is the cross-section area of the airgap, N is the number of turn of the coil, and I_i and g_i is the current and airgap at i-th coil, respectively. By using the parameters given in (Mohamed and Emad, 1992; Lin and Gau, 1997), this relationship can be plotted and shown in Figure 1.3 (a). Noticeably, the relationship of the magnetic force in which the magnitude is proportional to the square of the input current and inversely proportional to the square of the rotor position causes sudden surge of the force magnitude as the airgap approaches zero. Theoretically, this so-called negative stiffness imperatively causes singularity error in many controller designs which practically translated to the saturation of magnetic actuator. As one of the techniques to overcome this difficulty, a small bias current, I_b, is usually introduced to the coil such that the linearity of the force-to-current about



(a) Nonlinear magnetic force



(b) Nonlinear magnetic force with biased current $I_b = 0.8\text{A}$

Figure 1.3 Nonlinear relationship between magnetic force and current/airgap the centre of the system can be established to some degree which provides higher system bandwidth and easier controller design. Figure 1.3 (b) shows the effect when $I_b = 0.8$ A is added to the equation (1.1) where an almost linear relationship between force and current and no singularity point is observed when the gap is zero.

Another major nonlinearity existed in AMB system is vibration due to the mass unbalance of the rotor, or called imbalance. Imbalance is a common problem in all machineries with rotational shaft when the principle axis of inertia of the rotor does not coincide with its axis of geometry due to mechanical imperfections occurred in fabricating machine parts, as shown in Figure 1.4 (Herzog *et al.* 1996; Shafai *et al.* 1994; Huang and Lin, 2004). When the rotor is 'forced' to rotate around its center of inertia, G_m, instead of its centre of geometry, G, a centrifugal force caused by the acceleration of the inertia centre creates a synchronous transmitted force and furthermore manifested into synchronous rotor displacement. In the worst case scenario, since the imbalance effect is proportional to the rotor rotational speed, at high-speed operation the rotor whirls exceeding the allowable airgap and causes the rotor to partially or worse yet annularly rub the stator which result in permanent damage to the bearing system (Choi, 2002). Among the commonly considered design solution to prevent this to occur is to have a mechanical retainer bearing

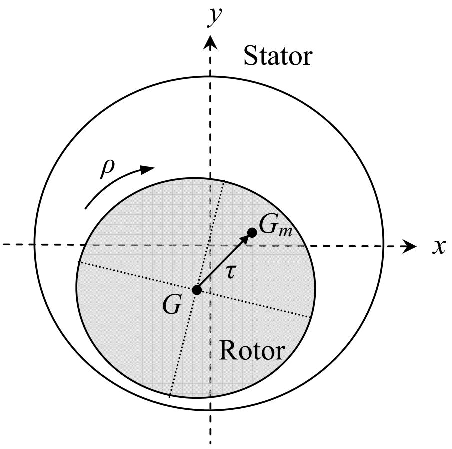


Figure 1.4 Illustration of unbalance rotor

installed as of the safety measures, however, the contact further exaggerates the nonlinear dynamic motion to cause a more chaotic motion (Knospe, 2007; Grochmal and Lynch, 2007; Li *et al*., 2006; Sahinkaya *et al*. 2004).

Other significant nonlinearities associated with the rotor dynamics are gyroscopic effect and bending modes for flexible rotor. Gyroscopic effect present in AMB system results in the coupling between the pitch (rotation around x-axis) and yaw (rotation around y-axis) motion and the magnitude is proportional to the rotor rotational speed. This imposes a more challenging task for stabilization of the system for high-speed application (Li *et al*., 2006; Hassan, 2002). In addition, in some applications where a long rotor is required, the excitation of flexible mode of the rotor becomes crucial which may result in an inherently unstable system (Li *et al*., 2006; Jang *et al*., 2005; Nonami and Ito, 1996).

In all AMB-related applications, the main objective is either asymptotically regulating the rotor to center position (zero airgap deviation) of the system or tracking a predefined rotor positions. However, with the presents of these nonlinearities, the AMB system is liable to exhibit unpredictable and irregular dynamic motions which complicate the design of effective system controller (Jang *et al*. 2005, Kasarda, 2000). Conventional feedback controller methods developed by assuming that the motions on each system axis are dynamically decoupled rarely meet the stringent system requirements which result in limited operational range of the system. Furthermore, nominal parameter values are commonly used in the system where in real application, the exact values are poorly known and subjected to variation which consequently result in deterioration of some controller performances on the system. The need for more advanced control strategies is thus becoming indispensable in order to achieve the desired system performance. In the following section, the various control methods that have been designed for AMB system is discussed.

1.2 AMB System Configuration and Control Strategies

The idea of active control magnetic bearing system has sparked interest as early as 1842 after Earnshaw (1842) proved that the levitation of ferromagnetic body and maintaining a stable hovering in six-DOF position is impossible to achieve by solely using permanent magnet (Matsumura and Yoshimoto, 1986). Ever since then, numerous control methods have been proposed by many research groups not only to stabilize the system, but also to improve the performance of the system under wide operational condition. Figure 1.5 illustrates the hardware set-up for the closed-loop control of AMB system. The measurement of the four gap deviations forms as the feedback information used by control algorithm executed in a fast Digital Signal Processor (DSP) based processor. The calculated control signal is further amplified to perform the required vibration control, positioning or alignment of rotor of the system.

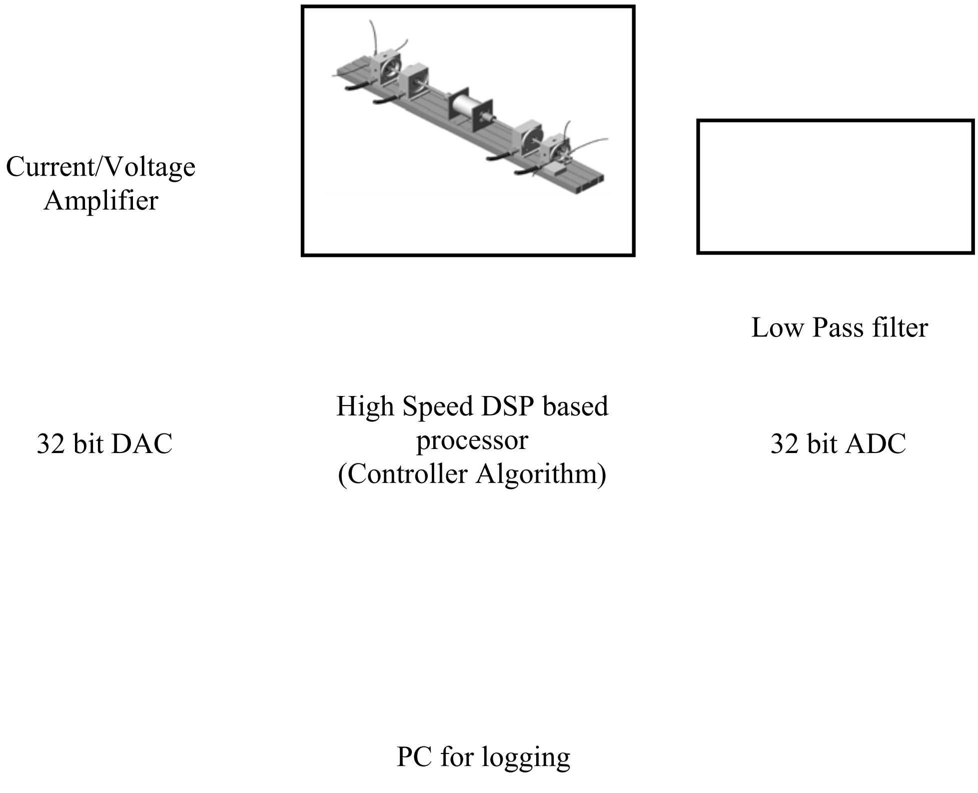


Figure 1.5 Hardware configuration for closed-loop control of AMB system

The electromagnets can be controlled by either the coil current (current-based control) or the voltage (voltage-based control). In voltage-based control approach, two design steps are usually adapted in which in the first step, a low-order current controller is designed such that desired electromagnetic force is produced. Then, tracking this current trajectory signal is used as the control objective for the design of input voltage controller. The common assumption in this approach is the combination of the processor and voltage amplifiers is able to fulfill the timing of the two-stage nature of the controller which usually is very difficult to meet (Bleuler *et al.*, 1994b). Another important drawback is due to the inclusion of dynamic of the power amplifier and circuit constraints, the linearization of amplifier and system dynamics usually involved in the controller formulation which further limit the system performance (Hassan, 2002; Charara *et al.* 1996). Some nonlinear control methods such as differential flatness (Levine, *et al.*, 1996), backstepping-type control (DeQueiroz, *et al.* 1996a; DeQueiroz, *et al.* 1996b) and feedback linearization and passivity-based control (Tsiotras and Arcak, 2005) are proposed but the difficulty of overcoming singularity problem results in more complicated controller structures. In the current-based control method, since there is a direct relationship between the coil input current and the magnetic force shown by equation (1.1), the abovementioned challenges in voltage-based control design can be relaxed and becomes more advantages to AMB control system (Bleuler *et al.*, 1994a).

The current-based control scheme can be classified into three modes of operations of power amplifiers as shown in Table 1.1 (Sahinkaya and Hartavi, 2007; Hu *et al.*, 2004). The configuration of the tabulated coil currents is based on a single pair of electromagnet in which one of the coils produces the opposite force of the other coil. For Class-A control, a bias current, I_b, is applied to both coil and a differential control current, I_c, is added to the bias current in one coil and subtracted from the opposite coil depending on the net force required. The bias current is set to half of the maximum allowable current, I_{max}. This mode of operation, also named as Constant Current Sum (CCS) control, is the most widely used method in controlling AMB system due to the fact that high bearing stiffness and good dynamic range can be achieved (Grochmal and Lynch, 2007; Sahinkaya and Hartavi, 2007). In Class-B mode of operation, or also known as Current Almost Complementary (CAC) condition, a small bias current is supplied to both magnetic coils and at one instant of

Table 1.1 Mode of operation for a pair of electromagnet

Mode of Operations	**Input Current**
Class A	$I_1 = I_b + I_c,$ $I_2 = I_b - I_c,$ where $\lvert I_c \rvert < I_b,$ $I_b = 0.5 I_{max}$
Class B	$I_1 = I_b + I_c$ and $I_2 = I_b,$ or $I_1 = I_b$ and $I_2 = I_b + I_c.$
Class C	$I_1 = I_{c1}, I_2 = 0,$ or $I_1 = 0, I_2 = I_{c2},$

time, the control current is added to only one of the coils to produce the desired control force. Although a possible lower power losses can be attained due to smaller I_b, the bearing stiffness is reduced quite significantly which make the system to be suitable for low vibration application. In this control mode, a possibly large feedback gain is required to achieve the required bearing stiffness and likely will result in current saturation. Tsiotras and Wilson (2003) and Tsiotras and Arcak (2005) have shown that the control of AMB system with saturated input and low bias current is nontrivial and a challenging nonlinear control problem. Another mode of operations is the Class-C control where the bias current is totally eliminated and the two coils are alternatively activated at an instant of time. This is equivalently called Current Complementary Condition (CCC) where only one coil is energized depending on the direction of the required force needed. Under this mode of operation, the nonlinearity effects are severe and controller singularity problem occurred when the gap deviation approaching zero is one of the most crucial design problems which result in controller complexity. Apart from this design issue, the lacks of robustness against changes in operating condition as well as poor dynamic performance are also major shortcomings of this approach (Sahinkaya and Hartavi, 2007; Charara *et al.* 1996; Levine *et al.* 1996).

Due to many possible combinations of design configurations and actuating schemes exists in the control of AMB system, there exists abundance of control design techniques that have been proposed to meet the control objectives which are stabilization of the system and fulfilling specific application-related system performances. The control strategies can be essentially divided into three main groups: the linear control, nonlinear control and the control approach based on mimicking human's decision making process and reasoning or known as Intelligent Control (IC) method. The linear and nonlinear control strategies are model-based approaches where a mathematical model representing the AMB system as a class of a dynamical system is a required for the development of the control. As an alternative, due to the complexity in formulating the control law especially for the nonlinear control techniques, the adaptation of the IC methods in AMB control has found growing interest especially Fuzzy Logic (FL), Genetic Algorithm (GA) and Neural Network (NN), or the fusion of any of the method with existing mathematical-based methods.

The conventional Proportional-Derivative (PD), Proportional-Integral (PI) and Proportional-Integral-Derivative (PID) control for AMB system are among the earliest controllers considered for the control of AMB system due to its simplicity in the design as well as hardware implementation (Bleuler *et al.*, 1994b) and until today, the controller still receives considerable attention in some specialized application. In the work done by Allaire *et al*. (1989) and William *et al.* (1990), discretized PD controller is designed based on linearized model at a nominal operating point. The main emphasis of the work by Allaire *et al.* (1989), however, is the design construction of AMB system to accommodate the variation of the load capacity in thrust motion and the PD controller is used to achieve closed-loop stability. Due to apparatus limitation, mechanical shims are used to gauge the airgap and the controller is manually adjusted. William *et al.* (1990) has continued the study where the relationship between the characteristic of the developed PD controller to the stiffness and damping properties of AMB system is established. Other than stiffness and damping curves, the rotor vibratory response is also used to show the effectiveness of the control algorithm where from the experimental result, due to time delay in feedback response and hardware limitation, the high frequency response does not agree with the theoretical result. To overcome the difference, the so-called Proportional-Derivative-Derivative (PDD) and Proportional-Integral-Derivative-Derivative (PIDD) are proposed and applied into the system which yields quite a satisfactory result.

In more recent year, Hartavi *et al.* (2001) has studied the application of PD controller on 1 DOF AMB system where the electromagnetic model is developed based on Finite Element Method (FEM), initially proposed by Antilla *et al.* (1998). Good system stability is achieved, however, only under limited range of operating condition. Polajzer *et al.* (2006) has further proposed a cascaded decentralized PI-PD for control of the airgap and independent PI current controller to achieve high bearing stiffness and damping effect of a four DOF AMB system. The controller is designed based on simplified linearized single-axis model where the effect of magnetic nonlinearities and cross-coupling effect are ignored. A considerable improvement has been achieved in term of its static and dynamic response in comparison to PID control developed in previous work. In an AMB system where the rotor is flexible, the control of vibration due to bending mode of the rotor is crucial. For the AMB system developed by Okada and Nonami (2002), a hybrid-type magnetic bearing is used and PD controller is proposed to perform the inclination control such that the system with flexible rotor is able to step through the bending modes occurred at five critical rotational speeds. The five bending modes are analyzed from the finite element model of the rotor that is transformed into a linear state equation and the controller parameters are designed based on the linearized model. With the central rotor position is controlled separately to provide sufficient stiffness, the system with the proposed PD controller for inclination control is able to run up to 6300 rpm rotational speed.

Due to limited performance of PD, PI or PID controller and design procedure to incorporate various design requirements, other linear controller methods have been proposed to fully exploit the possible active potentials of the AMB system in permitting to a much higher degree of rotor vibration and position control (Bleuler *et al.*, 1994a; Huang and Lin, 2003). Another most popular linear control method used by researchers is the Linear Quadratic Regulator (LQR) control which is based on optimal control theory (Anderson and Moore, 1990). LQR design method is designed by selecting the so-called weighting matrices that minimizes a pre-defined linear quadratic cost function. Matsumura and Yoshimoto (1986) are considered as among the earliest researchers that have applied the LQR-type controller in AMB system. In their study, an LQR controller is designed and cascaded with and integral term such that the steady-state error of the airgap deviation is eliminated. This optimum servo-type control is formulated based on a linearized 5-DOF AMB system at a constant biased current, where the deviations of rotor position from this equilibrium are treated as system states to be regulated and the input to the system is the electromagnetic voltages. The digital simulation results show that the system achieve stability condition at zero speed and 90000 rpm, however, at this high rotational speed, the coupling effect influence the control performance significantly. The method is further applied into a system where the integral servo-type control is to perform both the radial and thrust control for a cylindrical AMB system (Matsumura *et al.*, 1987). Through this study it is verified that multi-axial control of AMB system is difficult to achieve with the proposed type of controller. Since both of these works are based on a linearized model at one operating point, Matsumura *et al.* (1999) has used a different linearization technique called exact linearization approach such that the linear model can represent a wider range of the nonlinear model. The design LQR controller for this newly linearized model confirms to achieve wider range of stabilization area. The control method of this highly-cited work (Matsumura and Yoshimoto, 1986) is also further adapted in a new type of horizontal hybrid-type magnetic bearing (Mukhopadhyay *et al.* 2000). In this work, the new type AMB system is developed by using a rotor made from strontium-ferrite magnet and both the top and bottom stators are made from Nd-Fe-B material where the combination of this permanent magnet configuration is proven to provide high bearing stiffness to produce repulsive force for rotor levitation. The force-to-airgap relationship is established by using finite element analysis (FEA) where the relationship is integrated with the dynamic model of the AMB system. The optimum integral servo-type control is designed to stabilize the system and tested on the system up to the 800 rpm rotor speed.

In a quite similar scope of work, Lee and Jeong (1996) has designed centralized and decentralized LQR controller with integrator to perform a control on a vertical conical AMB system. For the centralized control, the coupling effect between the axial and thrust motions is considered and this effect is ignored on the decentralized controller design. The relationship between the current and voltage is emphasized where the mathematical model of the electromagnetic coil dimension and its dynamics are included in the design procedure where it is illustrated that the coupling effect between the axial and radial axes of motion is quite insignificant for the particular AMB system which result both the centralized and decentralized controller produce comparatively similar performances. In a rather different approach, Zhuravlyov (2000) has explored the design of LQR controller for not only regulating the rotor position but also to reduce the copper losses in the coils. Two-stage LQR based controller is developed such that the first controller is meant to stabilize the rotor to the reference position with magnetic force is the system input. For the second stage, another LQR controller is developed to produce the coil current and voltage which produces the optimized bearing force while at the same time, the copper losses in the coil is also minimized. Instead of taking the real value of the system matrix, this approach has used the complex state-space system such that the frequency content of the system can be incorporated. The study also shows that the real implementation of the controller is difficult especially when the second stage controller requires a switching term to achieve the desired objective, and controller simplification is needed for practical purposes.

The works in the development of controller based on μ-synthesis have also been reported by many researchers. Fujita *et al.* (1995) has proposed the μ-synthesis controller that is designed based on a few set of active electromagnetic suspension model. The combination of the nominal model, four set of model structures and possible model parameter values are used to determine uncertainty weighting function which form a sufficient representation of the range where the real system is assumed to reside. A special so-called D-K iteration is then used to tune the controller parameter to achieve robust stability as well as robust performance. Nonami and Ito (1996) have used μ-synthesis method for stabilization of five-axis control of AMB system with flexible rotor. The modeling of the system is performed by using FEM technique and the resulted high order system is truncated by removing the flexible mode for the purpose of controller design. It is shown that the controller can achieve robust performance for this system and the it is noted that by value of the structured singular value, μ, in the D-K iteration contribute to achieving good robust performance.

Namerikawa and Fujita (1999) have further included more nonlinearities in the AMB model by specifically classifying linearization errors, unmodelled dynamics, parametric variations and gyroscopic effect as the uncertainties in the system. These uncertainties are represented structurally in matrices and Linear Fractional Transformation (LFT) technique is used to uniformly represent the AMB as a class of uncertain system for controller development. Instead of using standard μ test, a so-called mixed μ test is adapted to reduce the design conservatism. Losch *et al.* (1999) have designed and implemented the μ-synthesis controller in a feed pump boiler equipped with active magnetic bearings. They have proposed a systematic and formalized way for deriving the controller design parameters based on model uncertainties, control requirements and known system limitations. A new method for determining suitable uncertainty weighting function has been proposed in which the effectiveness of the designed controller is demonstrated by the robust performance of the pump.

In different scope of research, Fittro and Knospe (2002) has designed the μ-synthesis controller for specifically solve the rotor compliance minimization problem – to reduce the maximum displacement that may occur at a particular rotor location collocated at the region the disturbance frequency is not specified. Although the controller produces a significant improvement compared to PD controller, the results obtained however has suggested that a more accurate plant mode is required to yield a more accurate result in minimizing the rotor compliance.

Another robust linear control design that has received considerable attention in the control of AMB system is H_∞ technique. Since the linear model does not always express the exact representation of the system due to various uncertainties present in the system, H_∞ control technique offer a nice procedure to construct the uncertainties into a proper structure for control design process. Fujita *et al.* (1990) has worked on verifying the well-established H_∞ controller on an experimental set-up of a one DOF magnetic suspension system. The main objective is to achieve robust system stabilization when the system is subjected to external disturbance. Various model uncertainties are also considered by formulating frequency weighting function which is included in the design procedure. Fujita *et al.* (1993) further develop H_∞ controller for five DOF AMB system by using the Loop Shaping Design Procedure

(LSDP). The so-called unstructured multiplicative perturbation which describes the plant uncertainties with the frequency weighting function is established which reflects the magnitude of uncertainties present. After specifying the uncertainty and performance weightings, by using the LSDP the shaping function is designed where the H_∞ controller is developed and tested experimentally which shows that some minor online adjustment on the shaping functions is still required to achieve a more favourable system response in term of regulating the airgap at various frequencies.

A simplified H_∞ controller has been designed by Mukhopahyay *et al.* (1997) for repulsive type magnetic bearing where an AMB configuration with permanent magnet in the radial axis is used to increase the bearing stiffness. The result from the study shows with the combination of the proper placement of the permanent magnet and controller design the radial disturbance is able to be attenuated for an 8 kg non-rotating rotor.

A continuous and discrete time H_∞ controller have been proposed by Font *et al*. (1994) to regulate the rotor to the center position of an electrical drive system by using AMB. The first six bending modes of the rotor is included in the system model such that the design controller can achieve robust stability towards the frequency excitation occurred at these modes. For the continuous controller, instead of using the truncated method, an aggregation method to reduce the order of the system is adapted where this technique offers the advantage of retaining the most important poles in the reduced order system. Satisfying closed-loop behaviors have been obtained, however, the power amplifier introduces severe constraint on the control capability.

Namerikawa and Fujita (2004) and Namerikawa and Shinozuka (2004) have used the H_∞ controller design technique for disturbance and initial-state attenuation (DIA) on magnetic bearing and magnetic suspension system, respectively. In the design procedure of the proposed H_∞ DIA controller, the selections of the frequency weighting related to the disturbance input, system robustness and the regulated variables are performed iteratively for the construction of linearized generalized system plant. A so-called weight matrix N obtained from this procedure is found to indicate the relative importance between attenuation of disturbance and intial-state uncertainty which further affects the calculated controller gains. Four H_∞ DIA controllers have been designed under different values of frequency weighting to assess the variation of matrix N on the system performance where it is shown the system overshoot is inversely proportional to the magnitude of N. In (Namerikawa and Fujita, 2004), the non-rotational AMB is used which implies that no gyro-scopic effect and imbalance present.

In a more recent work, Tsai *et al.* (2007) has proposed H_∞ control design for four-DOF vertical AMB system with gyroscopic effect. The well-known Kharitonov polynomial and Nyquist Stability Criterion are employed for the design of the feedback loop and it is confirmed experimentally that the controlled current produced is much less compared to the current produced by LQR or PID control methods. The performance of the system is verified in the range 6500 rpm to 13000 rpm rotor rotational speed.

Linear controller based on Q-parameterization theory has also been widely tested and applied in AMB system starting with the work from Mohamed and Emad (1992). In this work, a Q-parameter controller based on linearized conical AMB model is proposed which can meet various system requirements such as disturbance rejection, rotor stability and tolerances towards plant parameter variations. In the design procedure, these requirements are treated as constraints and can be classified by the doubly co-prime factorization matrices and the sets of stabilizing controllers which include the free design parameter Q. The search of the desired Q-parameter that produces the desired controller gain becomes an optimization problem where Q's are chosen through a customized optimization program. In this work, the controller is designed for imbalance-free rotor at speed $p = 0$, and good transient and force response is achieved until $p = 15000$ rpm. Since the order of the controller equal to the order of the plant and the order of the weighting function describing the constraint, the works are further extended by Mohamed *et al.* (1997a) where the linear system is transformed into three single-input-single-output (SISO) systems with the inclusion of the rotor imbalance. This simplification results in solving a set of linear equation rather that finding the solution from the complex optimization problem, where good rotor stabilization is achieved at three pre-defined rotor speed.

The Q-parameterization controller in discrete form is proposed by Mohamed *et al.* (1999) to specifically overcome the imbalance at various speed. The rotational speeds are scheduled in a table and appropriate gain adjustment according to the selected speed will be selected as the Q-parameter for the controller. This gain-scheduling method shows the elimination of imbalance at three rotor rotational speed is achieved with simpler design technique, however, a large look-up table is required to accommodate the operation at wider range of rotor rotational speeds.

With the linearization of force-to-current and force-to-airgap displacement relationship, AMB model belongs to a class of linear parameter varying (LPV) system which is suitable for LPV controller design. Zhang *et al.* (2002) has proposed a class of LPV controller that can maintain robust stability and performance at wide range of rotor speed. The augmented AMB system model is characterized as many sets of convex representation of system where the system matrix is considered as affine function of the rotor speed and treated as a set of structured uncertainty range. Due to the convexity property, H_∞ control rules is applied to each vertex yield stable closed-loop system and the LPV controller gain can be computed based on the convex representation of the system. The simulation result confirms that the robustness of the controller is obtained in which with the 3% uncertainty present, the nominal performance index, $\bar{\gamma}$, is well below 1 where the desired $\bar{\gamma}$ is only 1. However, in the experimental verification, due to the high computational time, some simplification is introduced in the controller algorithm to achieve acceptable system performance.

The synthesis of the LPV controller involves finding the solution of a single Lyapunov function that produces a stabilizing controller over a specified parameter range. When finding the solution is not possible, the normal approach is to formulate a few LPV controllers at many smaller parameter sub-regions which form a so-called switched LPV system. Lu and Wu (2004) have worked on this type of controller for AMB system and proposed hysteresis and average-dwell-time-dependent switching methods to maintain the system stability when the system switches from one sub-region to another. Both of the switching techniques lead to non-convex optimization problem that is difficult to be solved, however, the convexification of the hysteresis switching method is possible by using Linear Matrix Inequality (LMI) technique.

The simulation of the five-DOF vertical AMB system shows the effective of the switching methods but imposes extra calculation overhead.

Unlike the linear control methods where the controller synthesis is based on an approximate linear model, nonlinear control can be more suitable for a wider range of system operation and conditions with the possible inclusion of system uncertainties and nonlinearities. Among the prominently covered nonlinear control techniques for AMB system are back-stepping method, feedback linearization, adaptive control and sliding mode control or the fusion between any of the methods. For back-stepping method, DeQueroz *et al.* (1996a) have proposed a class of back-stepping type controller for a planar two DOF AMB system such that the tracking error of the rotor position can be globally exponentially eliminated. In the proposed method, the desired force trajectory signal is designed such that the rotor position tracks the predefined position trajectory. Based on this force trajectory, a special structure of a so-called static equation is established in which a desired current trajectory is constructed to satisfy the static equation. In the final design step, the produced current trajectory is set as the control objective for the design of voltage input. In order to ensure global exponential rotor position tracking, composite Lyapunov function is used. The simulation of the tracking of non-rotating rotor confirms the validity of the method, however, it is observed that the selection of the controller parameters is crucial when there exists some variations in the system parameter.

When the airgap between the rotor and the stator is large, the nonlinear magnetic effect becomes more critical due to the variation of the values of coil inductance, resistance and back electromotive force (e.m.f) against currents and rotor position. This effect is studied by DeQuiroz *et al.* (1998) where it is shown that the relationship between the produced electromagnetic force and the current is highly coupled and complex. By extending the method previously proposed by DeQueroz *et al.* (1996b), due to the nonlinear electromagnetic force, the design of the current trajectory is shown to be extensive yet an achievable task. The tracking of the rotor position is achieved quite satisfactorily as shown by the simulation result and as suggested by the research group, extending to a higher DOF AMB system requires the adaptation with other control techniques to reduce the design complexity.

As highligthed by many works including (Montee *et al.*, 2002; Tsiotras and Wilson, 2003), application of standard back-stepping method may cause singularity problem when the electromagnetic flux approaches zero. To overcome this problem, Montee *et al.* (2002) proposes to introduce an exponentially decaying bias flux and a new back-stepping control algorithm is designed in such a way that the system is stabilized at a faster rate than the decaying bias flux. The main advantage of this method is twofold: 1) singularity problem can be avoided, 2) zero ohmic loss at steady state. The controller is designed in both the Class B and Class C control mode and the study concludes that the Class C mode with the exponentially decaying flux produces the least power dissipation due to ohmic loss while retaining satisfactory rotor positioning to the center, however stability of the system is more prominent in Class B mode.

Tsiotras and Wilson (2003) has proposed a novel integral back-stepping type control law to alleviate the singularity problem in Class C voltage-input AMB system when the produced control flux is zero. In this work, a new flux-based one DOF AMB system is derived based on the so-called generalized complementary flux condition in which the model produced is suitable for both zero and low bias flux control type (Class B and Class C). By adapting other control tools such as control Lyapunov function, homogeneity and passivity technique, the integral backstepping controller constructed is able to overcome the singularity problem or in some system condition, the region of singularity is reduced significantly. The simulation works confirm the finding of the study and as a by-product of the control method and it is shown that robustness against the system parameter variation is also achieved.

Back-stepping control is a full-state feedback approach where for AMB system, measuring the velocity of rotor is often difficult. In a different scope of study, Sivrioglu and Nonami (2003) have investigated the design of adaptive back-stepping control based on output feedback. A nonlinear observer is constructed to estimate the unmeasured state (rotor velocity) and based on back-stepping method, a dynamic controller is formulated with the objective to eliminate the rotor tracking error. The inclusion of the adaptive-type observer in the design is shown to achieve global stability by using Lyapunov function. To verify the result, a flywheel AMB system modeled and experimentally used where the gyroscopic and imbalance are excluded in the dynamic model. At low rotor speed, the result give satisfactory tracking performance of the rotor while the current used is also minimized as suggested in Class C control mode.

In most application where the AMB system is used, the construction of the AMB-embedded system usually remains in static position (fix base). However, for some application such as flywheel battery for energy storage system, the body of the system is subjected to movement and undesired disturbance that causes the operating point of the rotating rotor to be disrupted. This situation is always true for flywheel battery installed in space craft (Wilson, 2004), large energy storage system in the earth-quake prone area (Sivrioglu, 2007) and single-gimbal gyro for satellite application (Liang and Yiqing, 2007). This shaking-like movement of the AMB system will introduce disturbance to the planned motion of the rotor and might cause possible system instability which is very undesirable for this high-energy capacity system. Sivrioglu (2007) has proposed a nonlinear adaptive back-stepping method to overcome this so-called vibrating base effect where the formulation of the controller is based on an imbalance-free vertical AMB model. In this study, the AMB system is coupled to a ‘shaker’ that introduces a bounded acceleration disturbance to the system and the controller in similar type of structured designed in (Sivrioglu and Nonami, 2003). Accessing the controller at low speed where the gyroscopic coupling is minimal, the system is able to achieve stability where the rotor whirls around the allowable airgap, however, the finding shows that a comparable performance can be achieved with PID controller for the flywheel system.

For the feedback linearization method, the main objective is to transform the nonlinear system dynamics into a fully or partially linear model and the established linear control methods can be employed (Slotine and Li, 1991). This is achieved by designing an input that cancels the nonlinearities and the resulted closed-loop system is linear and controllable. Li (1999) has investigated the feedback linearization technique on CCS, constant flux sum (CFS) and constant voltage sum (CVS) mode of operation on one DOF AMB system. The CVS control is obtained by linearizing the model under CFS mode. The three constant-sum configurations are compared in term of closed-loop performance, nonlinearity and the effect of the current constraints where in the studies the CVS is proven to be the least difficult in the design procedure while CFS yield the most complex controller structure. The work is further continued to investigate the constraints imposed on feedback linearization design such that only single input actuates at one instantaneous time (Li and Mao, 1999). This is found crucial since there exist many constraints in feedback linearization controller that can produce linearized model in which some of the imposed constraints can result the linear plant tends to be nonlinear. In the study, minimum copper loss and constant upper bound of force slew rate have been derived to be the design constraint where it is proven that the feedback linearization with constant bound on force slew rate produces a more linearized plant.

In a rather different scope of work, Levine *et al.* (1996) has proposed a nonlinear control law based on feedback linearization procedure where the main objective is to be able to perform trajectory tracking while avoiding the use of premagnetization current. The differential flatness property is adapted in the design process since this method has simple parameterization curves for the system to track and the complicated integration of differential equations is replaced by solving simpler algebraic problem. The Class C mode of operation is used and current-based, voltage-based and cascaded controllers are proposed and investigated in the study. The design of these controllers is executed on one DOF system and then generalized to five-DOF AMB system without imbalance. The study has also shown the cases when the premagnetization current is necessary to achieve system stability and desired performance for the specific AMB system. Grochmal and Lynch (2007) continue this work by performing experimental work on five DOF AMB system to validate the design assumptions made in simulations and ensure robustness towards unmodelled dynamics. In this study, two nonlinear controllers based on CAC and CCS modes are proposed and compared with standard decentralized PID controllers in term of the system response during high-speed rotor rotation and the tracking performance for non-rotating rotor. The force parameter identification is done on both modes to establish the force-to-current and force-to displacement relationships. At p = 14000 rpm, it is shown that CCS controller can achieve far superior result compared to CAC and PID controller in which the radius of rotor movement is about 20μm while the other two controller almost twofold. In addition, when the system is sped up passing p = 5000 rpm, the CAC controller has forced the voltage source to reach the saturation limit, 12V, in order to maintain the rotor to an acceptable distance from the center. Under the tracking mode, it is shown that only CCS and CAC controllers can track the predefined sinusoidal curve while decentralized PID controller fails to give good tracking performance.

Hsu and Chen (2002), and Hsu and Chen (2003) work on feedback linearization method for a 3-pole AMB system. This type of AMB configuration is reported to cost much less than the normal 4-pole or 8-pole AMB system since less number of power amplifiers is used. On the other hand, due to the non-symmetric nature of 3-pole AMB, there exists a very strong nonlinearity resulted from magnetic flux coupling. With the voltage is treated as the input and magnetic flux as one of the system states, the imbalance-free system model can be established into an input-affine form that is feasible for feedback linearization control design. At rotor speed p = 2000, the performance of the controller is accessed at various initial positions where the rotor is able to be regulated to the center position, given that the initial positions reside in the designed admissible set constructed via Lyapunov analysis. As highlighted in the work, the inclusion of gyroscopic effect in the model needs further study for the development of the feedback linearization controller.

The design and application of feedback linearization method on magnetic suspension system is also considered by many researchers in which the system is actually analogous to one DOF AMB system. In magnetic suspension system, the main nonlinearity is due to the force-to-current and force-to-displacement (airgap) relationship whereby the analysis and design method is directly applicable to multi-DOF AMB system. Trumper *et al.* (1997) has used single-input and two-input suspension system for the feedback linearization controller design and the performance is compared to linear controller. Under the given range of operating condition, feedback linearization achieves better performance in term of regulating airgap deviation, however, due to possible modeling error, sustainable oscillations is still observable. In a similar line of study, Joo and Seo (1997) and Fabien (1996) work on the nonlinear controller design for magnetic suspension system with parametric uncertainties and observer-based feedback linearization, respectively. The emphasis of the work by Joo and Seo (1997) is the formulation of the controller for the system that is subjected to variation of the mass and bounded input disturbance, while for Fabien (1996), the controller design that is based only on the available output for stabilization of the system becomes the design objective. The results obtained from both studies verify that within the scope of the work the proposed feedback linearization controller can achieve good system stabilization by linearizing the system model, though, for the observer-based controller by Fabien (1996), more controller design parameters resulted since the addition of the observer increases the dynamic of designed controller.

As claimed in most of the above mentioned works related to the feedback linearization method, the biggest drawback of this control approach is the exact model of the system is required at design stage in which, in reality, obtaining the exact system representation with nonlinearities is next to impossible. The difference between the actual nonlinear model and the mathematical representation of the to-be-cancelled nonlinearity effect causes the design controller not to be able to linearize the system and worse yet, this residual effect possibly makes the system unstable. In order to overcome this limitation, the most prevalent approach is to integrate the feedback linearization controller with other type of robust controllers. Lindlau and Knospe (2002) have used the μ-synthesis based controller cascaded with the feedback linearization such that the robust performance can be achieved. For this work done on the single-DOF AMB system, the detailed nonlinear electromagnetic dynamic model is developed based on combination of both the analytic relationship and experimental data such that the nonlinearity is more accurately represented. Then, in order to accommodate the uncertainty due to the coil resistance variation, a special form of structured uncertainty is augmented to the established feedback-linearized model where the robust μ-synthesis technique is used. The system performance in term of disturbance rejection is confirmed to meet the μ performance specifications regardless the operating point and existence of the parameter uncertainty. The work is further continued by Chen and Knospe (2005) where the operation in current mode is used in order to overcome the difficulty in real implementation of voltage-mode controlled previously proposed in (Lindlau and Knospe, 2002). Under this current mode controller, it is found necessary to construct a corrective filter due to the fact that there always exist differences in 1) actual position and the position reading obtained from the position sensors and 2) actual coil current and the commanded current. Based on the structured residual uncertainty developed experimentally, the feedback-linearized μ controller is able to achieve robust system performance even subjected to large airgap variation.

The fusion of feedback linearization with back-stepping method is also considered by Hung *et al*. (2003). By first modeling the one DOF two-input AMB model into a fourth order nonlinear system, a nonlinear state-feedback control law is formulated to compensate the nonlinear magnetic effect which produces a linear controllable system. From this model, a so-called pseudo input is established with PD feedback law to stabilize the linearized dynamic. In order to construct the actual input current and voltage, a back-stepping type of controller combined with high-gain linear feedback control law is proposed. This multiple-loop control algorithm is run on experimental set-up with rotor speed up to 1800 rpm. The result shows the performance of the controller yields better performance than the PD controller in four areas which are: 1) better stabilization of rotor for large position variation, 2) smaller tracking error, 3) wider range of stable controller gain and 4) lower current consumption due to operation in Class C mode.

Another notable robust nonlinear control method that is frequently considered when robust stability and robust performance of nonlinear system is expected is Sliding Mode Control (SMC). SMC is known as a type of Variable Structure Control (VSC) where in this VSC control scheme, a discontinuous switching method is proposed to switch between two distinctively different systems structures in which it will produce a new class of system dynamics that slides on a so-called sliding surface. The main advantage of this control method is always associated with its invariance property towards so-called matched uncertainties and disturbance (DeCarlo *et al*., 1988; Hung *et al*., 1993; Edwards and Spurgeon, 1998). In the control of AMB system, SMC controller has been used in many forms established to achieve the required system performance or to tackle specific application-oriented problem. The fusion of SMC with feedback linearization method for AMB control has been proposed by numerous researchers. Smith and Weldon (1995) has worked on the nonlinear formulation of cascaded SMC and feedback linearization controller to achieve robust regulation of the rotor to the center while the system is subjected to external disturbance, parameter uncertainty and unmodelled dynamics. In this study, the voltage control is considered which required the system to be linearized at an equilibrium point set at a predefined bias current and the feedback linearized control law is designed to eliminate the second-order nonlinear coupling effect. Since the uncertainty is still present due to the parametric variations, the SMC control technique is developed such that the tracking of the rotor position can be performed. The simulation results verify that the proposed controller is effective in achieving the desired position tracking, however, no explicit method of choosing the surface parameters is proposed.

Charara *et al.* (1996) has taken a quite similar approach by developing feedback linearization and SMC control for a hybrid AMB system where permanent magnets are used for pitch, yaw and translation along *z*-axis motion control. In this work the dynamic model is derived based on Lagrange's equation where the resulted feedback-linearized model with pre-defined bounded unbalance effect is used for the SMC design. The work adapts the sliding surface based on (Slotine and Li, 1991) where it characterizes zero tracking error of rotor position displacement once the system in the sliding motion. The work on simulation verifies the superiority of the proposed control law, in the contrary, the limitation due to the need of high sampling frequency and ensuring the existence of sliding mode in all operating conditions result in degradation of system running on hardware set-up.

As highlighted by Hsu and Chen (2002), it is always deemed to have feedback linearization control law cascaded with other robust controller type so that the control law is more of practical use. Working still on the 3-pole AMB system with non rotating rotor, an integral SMC control law is designed based on the perturbed linearized plant where the difference between this perturbed model and the linearized model indicates the uncertain element of the system (Hsu and Chen, 2003). In contrast to normal approach where the sliding manifold is required to reach zero in finite time to ensure asymptotic stability, in this integral SMC design, it is only necessary to maintain the derivative of the sliding manifold to be zero since it will result the rotor positions to eventually approach zero. When comparing the controller performance to other linear controller and feedback linearization cascaded with linear controller, it is shown that integral SMC gives the best result in bringing the rotor to the center position with smallest overshoot while consuming the least amount of current. Results obtained from experimental works verify the effectiveness of the method, however, under large value of uncertainty, magnetic saturation may occur that degrades the performance. Chen *et al.* (2005b) continues this study by experimentally investigating the controller performance when a rotating rotor is used. The result shows that for this 3-pole AMB system, the maximum allowable rotor speed is 3000 rpm in which for any rotor speed that is higher than speed might cause the rotor to rub the retainer bearing.

As it has been manifested in many research works, SMC control approach is able to meet various control requirements for endless kind of applications such as power electronics, bioprocess, motion control and robotics, to name a few (Bartoszewics and Patton, 2007). In AMB application, among the earliest work that use the SMC technique is done by Rundell *et al.* (1996) where a static and dynamic SMC control law is developed for stabilization of a vertical AMB system. In this study the AMB model based on (Mohamed and Emad, 1993) is used where the model is linearized at an operating point and any system uncertainties and disturbance is classified as an external perturbation force. The design of the stable sliding surface in done in such a way that the external perturbation force mainly composed of the imbalance is included in structure of the surface. By having this type of surface, the imbalance effect is cancelled to produce stable sliding motion, but requires the rotor rotational speed as one of the feedback signal. Both the static and dynamic SMC control law is constructed with the inclusion of a discontinuous term to eliminate parameter uncertainties, where a constant gain is selected to be sufficiently large to bound the uncertainty effect. It has been shown by simulation result that both controllers are effective to stabilize the given plant until about 2800 rpm rotor rotational speed.

In the SMC control design, the system behavior is dictated by the dynamic of the designed sliding surface of sliding motion exists. Since AMB system can be treated as a LPV system at steady state, Sivrioglu and Nonami (1998) has proposed the design of time-varying sliding surface design based on H_∞ frequency-shaped technique where a new augmented system by using a prefilter is established. This is followed by specifying two frequency shaping filters to achieve robust stability and sensitivity reduction in which with the combination of the filter and the LPV AMB model, gain-scheduled controller is computed by using LMI technique. The performance of the proposed approach is confirmed to be able to stabilize the AMB system whereas at the critical rotor speed of p = 6000 rpm the orbiting movement of the rotor is noticed to have a large diameter of orbit.

Tian (1999) has considered the design of discrete-type SMC observer and SMC control law for AMB system with flexible rotor where the system is set to run on Class A mode with bias current 4A. In designing the controller, the flexible motor is modeled by using FEM technique and then incorporated with the electromagnetic dynamics. The resulted system in 26th order is reduced for the formulation of the controller where a state and disturbance observer is constructed in prior. The discrete controller is found to be smooth in which the excitation of unmodelled dynamic, that is often crucial for flexible rotor, is avoided. In comparison to linear controller, good system stability and tracking performance is also achieved at high rotor rotational speed.

In a slight different control approach, Lewis *et al*. (2001) has studied the design of continuous SMC controller for a flexible AMB system based on output feedback due to the fact that not all system states are practically measurable. The flexible rotor is modeled by using Hamilton’s principle that yields two high-order partial differential equations. The discretization of the equations by using the Galerkin’s method produces a form of system model in state-space representation that is further truncated for controller development. The SMC method based on (Slotine and Li, 1991) is adapted and the continuous function is used to replace the discontinuous term of the control law. Based on the open loop test, the critical rotor speed occurs at p = 6963 and the simulation result shows that the proposed control law is able to attenuate the effect of parametric variations and imbalance at this speed and up to 10000 rpm. It is also found that the gain constant and boundary layer thickness is crucial to ensure to achieve the desired system stability since it is noticed that the variation of this two controller parameters do have significant influence on the magnetic bearing stiffness and damping.

For some AMB system, the secondary electromagnetic effects such as flux leakage, fringing flux and finite core permeance are the contributing factors that degrade the system performance. Yeh *et al.* (2001a) have studied the influences of this nonlinearity effect in one DOF AMB system and proposed the used of SMC technique to stabilize the system. The bond graph model built based on Thevenin's theorem is developed to represent these secondary effect and FEM technique is used to obtain the possible range of the parameters. Based on deterministic method that is similar to the work by Osman and Roberts (1995), the bound of the uncertainties due to the parametric variation is defined and control law based on (Slotine and Li, 1991) is designed such that the system is robust within the specified range of operation. The performance of the controller in regulating to zero rotor position is confirmed by simulation and experimental work to be more superior than PID and feedback linearization method. In a relatively similar scope of work, Yeh *et al.* (2001b) has proposed a new SMC controller for both the current input and voltage input AMB system that is able to track the rotor position. In the voltage mode operation, the integrator back-stepping method is adapted to overcome the 'mismatched' between the control input and the rotor dynamics. For this non-rotating rotor AMB system, the tracking of a unit step input under both current and voltage input is found to be satisfactory where very small tracking error is produced.

In Lee *et al.* (2003), a continuous SMC control law based on special form of boundary layer technique is designed for a magnetic balance beam system and further the controller structure is generalized to multi-DOF AMB system. The design approach is similar to the conventional SMC design technique, however, the discontinuous term is replaced with a 'costumed' form of continuous term that is still able to bound the effect of uncertainties present due to the system parameter and external disturbance. The application of the controller on the magnetic balance beam shows that the external disturbance is able to be attenuated to a satisfactorily minimal level but the study on the multi-DOF AMB is not shown and remain as the future direction of the work.

For some application where flexible rotor is used, the possible contact with electromagnetic coil might occur and cause the system damage. As previously described, the use of a back-up mechanical bearing (retainer bearing) is often to be an acceptable solution, however, in the case where the rotor does have a contact with this retainer bearing, the dynamic of the system changes significantly which requires a stabilizing mechanism. Jang *et al.* (2005) has worked in this area where SMC technique is found to offer an excellent solution. In this study, a horizontal AMB system with flexible rotor and retainer bearing is modeled into a class of nonlinear system where the tracking error of the rotor position is treated as one of the system states. A PID-like sliding surface is constructed in which the pole assignment method is used to determine the surface parameter. The reachability of states to the sliding surface is guaranteed by using the approaching law method and the stability of the system is also theoretically ensured. The current-input Class-A AMB system is verified in simulations where stable system performance at high rotor speed is achieved with bias current of 1.8A.

In the study of AMB system, the control method specially designed to remove or attenuate the vibration effect due to rotor imbalance is considered by a handful of research groups. This is due to the fact that vibration caused by imbalance is proportional to the square of the rotor speed and undoubtedly becoming more significant for high-speed application. The imbalance effect is a synchronous-type disturbance where the magnitude and phase is dependent to the rotational speed which implies that which the exact identification of the disturbance signal amplitude and phase, the imbalance can be eliminated quite effectively. The adaptive vibration control naturally seems the most suitable control technique to meet this design objective. There exist two control techniques with regards to imbalance elimination which are autobalancing and unbalance compensation. For autobalancing, the rotor is forced to rotate around its center of inertia which eliminates the generation of the synchronous disturbance force. For the unbalance compensation method, the generation of the force that is opposite to the synchronous signal is performed to produce zero net force on the rotor that rotates on its center of geometry. Shafai *et al.* (1994) have used the adaptive force balancing compensator composed of a synchronous signal generator that is used to generate the imbalance-like disturbance signal and the Fourier Coefficient computer to filter the frequency of the input of the rotating rotor. This cascading controller works quite effectively on one DOF AMB system and it is noted that if there exists disturbance with higher harmonic content, another high frequency compensator is necessary.

This type of feedforward adaptive vibration technique is also considered by Betschon and Knospe (2001) since it is quite straightforward design process compared to feedback vibration control. The active vibration controller is inserted between the feedback controller and position sensors such that the synchronous disturbance is minimized. Then, the global stabilizing feedback controller is designed such that the quadratic function of rotor position is minimized which yields stable vibration-minimized AMB system performance that is confirmed by experimental work. Furthermore, in order to reduce the computational burden on the hardware, the adaptation algorithm is simplified by taking the diagonal element of the optimal adaptation gain matrix that is dependent on the rotor rotational speed which result minimal degradation in system performance operating at various operating speed.

Lum *et al.* (1996) has also considered adaptive autocentering technique such that autobalancing is achieved. In this method, an online identification of the coordinate of the imbalance mass and the rotor principle axis of inertia is constructed where the adaptation algorithm requires only the rotor displacement and velocity. Once the identification converges to the actual values of the imbalance coordinate and principle axis of inertia, with any stabilizing controller, the vibration due to imbalance is removed quite effectively and system stability is guaranteed regardless of the rotational speed of the rotor. The method is however limited to rigid rotor since for flexible rotor, the online identification algorithm of the principle axis of inertia and imbalance coordinate is challenging due to the existence of flexible modes of the rotor.

Shi *et al*. (2004) have also adapted the feedforward technique to attenuate the synchronous disturbance by proposing two adaptive compensators to achieve either autobalancing or unbalance compensation. The proposed adaptive methods are switched from one to the other depending on the bandwidth of the system where the autobalancing required lower bandwidth. To achieve acceptable disturbance attenuation, the performance measures are introduced called ‘direct’ and ‘indirect’ method in which for direct method, the performance measure of the adaptive algorithm is the direct function of the vibration signal to be minimized while for indirect method, the performance measure is based on the error of the position. Based on filtered-*x* least-means-square method adaptive algorithm, the direct and indirect unbalance compensation method is established and tested on AMB system experimental rig. The study shows that both methods perform quite effectively in minimizing the disturbance and pass the critical rotor speed at about 1300 rpm stably.

Multivariable generalized notch filter used for unbalance compensation is considered by Herzog *et al.* (1996). In many previously reported works related to notch filter design for vibration elimination, the filter is designed in open-loop and inserted to the closed-loop control system that may cause the system instability. In contrast to this approach, the filter is designed by cascading directly to the controller such that the filter is a part of the closed-loop system to be stabilized. The design parameter of the filter is found to be strongly dependent on the so-called inverse sensitivity matrix that is a function of rotational speeds. To cater the elimination of vibration at various speeds, a look-up table technique is used to store the matrix value for run-time use of the controller. The verification on a 500 HP turbo expander machine is performed at rotor speed close 30000 rpm where imbalance vibration is minimized quite effectively for this weakly gyroscopic coupled system.

In the case when the frequency of the disturbance is unknown, it is necessary to estimate the frequency of the disturbance in prior developing the adaptive algorithm. In addition during the estimation process, the susceptibility to noise deteriorates the algorithm convergences and causes error in the estimation process. Liu *et al.* (2002) has proposed a nonlinear adaptive unbalanced vibration control that features both the rotational synchronizing and asynchronizing harmonic disturbance to overcome this weakness. By first developing an adaptive single-frequency tracking algorithm, the method is expanded to adaptive multiple frequency tracking and a new modification law that guarantees output errors converge to zero asymptotically. The method is verified by simulation and experimental work with the range of rotor speed between 4000 rpm to 12000 rpm where the result shows that the attenuation of the disturbance at multiple frequencies considerably effective.

A decentralized automatic learning control method for unbalance compensation based on time domain is considered by Bi *et al.* (2005) in which the method adapts an intelligent-like updating law that reduce computational burden quite significantly. The four DOF AMB system is treated as four one DOF AMB system and individual learning law is constructed which includes learning gain and learning cycle parameters. The gain parameters are constructed in a look-up table format depending on speed requirements and the learning cycle is treated to be equal to the rotor rotational period such that operation at various rotor rotational speeds is met. By cascading the controller with PID controller experimental verification up to 3500 rpm rotational speed shows that imbalance vibration can be minimized quite effectively at steady state and some overshoot in the rotor displacement and coil current is noticed when there is an abrupt change in the rotor speed.

In the scope of utilizing IC methods and its associated tools such as FL, NN and GA for the design of AMB stabilizing controller, the growing interest can be noticed due to the fact that the control synthesis of this non-model-based control technique offers simpler solution in some AMB control application. Hung (1995) and Hong and Langari (2000) have used the fuzzy logic technique to represent the nonlinear AMB model that describes the input-output relationship for the entire input range. While Hung (1995) works on one DOF AMB system that produces simple fuzzy model, Hong and Langari (2000) have adapted the Takagi-Sugeno-Kang fuzzy model to represent five-DOF AMB system which includes the effect of harmonic disturbance and parametric uncertainties. In this modeling technique, many locally linearized models that valid for small region of operations are partially overlapped such that the nonlinear model can be sufficiently represented. The fuzzy control design technique thus is effectively used to accommodate the required system control performance. The result in Hung (1995) shows that the non-rotating rotor is able to be driven to central position at reasonably fast settling time and for Hong and Langari (2000), asymptotic rotor position is obtained at rotor speed up to 720 rpm. It is also observed that when there are variations in the weight of the rotor and the force constant, the rotor position remains in the bounded region of stability.

In a similar scope of work, Huang and Lin (2003) have also utilized the fuzzy technique to both model and controller design of six-DOF AMB system. Based on the nonlinear conical AMB model reported in the work by Mohamed and Emad (1992), Takagi-Sugeno fuzzy model of this system is developed which facilitates the fuzzy controller design. In this work, the controller objective is to attenuate the tracking error of the rotor to be below a predefined prescribed bound. This is achieved by using a Lyapunov-like function where the solution is found to be solvable through the use of LMI technique. The controller output of this controller is the magnetic force and it is assumed that the input current is able to supply the required controlled force and the simulation results verify the control effectiveness. This work is further continued in (Huang and Lin, 2005) by including the imbalance in AMB model. To overcome the imbalance vibration, a so-called imbalance compensator is integrated with the fuzzy controller in which robust system performance in tracking rotor position is achieved although with the present of the imbalance and bounded external disturbance.

NN method in the controller formulation of AMB system is found mostly in finding the bound of the uncertainties that present in the system and this bound further is used for the design of other model-based dynamic controller. The objective of using this intelligent uncertainties estimation technique is usually to produce less conservative controller output. Buckner (2002) has used the NN method, specifically 2-sigma network to identify the bound of the uncertainties by estimating the difference between a nominal system model and actual system (modeling error) or normally called as confidence interval. The estimation of the confidence interval represents the uncertainties model where the bound is used for finding the controller gain of SMC type controller. Similarly, in Lu *et al.* (2008), this method is used to find the LPV controller and in Gibson *et al*. (2003) for robust H_∞ controller.

In finding the best or so-called ‘optimum’ controller parameters values that meet various system requirements, many methods have been proposed. The heuristic method of tuning the parameters based on the output of the system usually gives satisfactory results but might be laborious if there are too many parameters to be considered. Besides mathematical-based optimization technique, GA seems to offer quite a nice solution in finding the controller parameters values. In Schroder *et al.*(2001), on-line GA method is used to tune a H_∞ controller and for fuzzy based controller, it is reported in the work done by Lin and Jou (2000). In both of these works, on-line GA tuning algorithm is utilized and the results confirm the AMB rotor can achieve robust rotor stabilization under predefined system operating range with a comparably slower settling time.

1.3 Summary of Existing Control Methods for AMB System

Based on this survey and discussion, the research work in controlling AMB system is driven into many directions involving modeling and designing control techniques for various kinds of system configurations that meet certain requirements of applications. Undoubtedly, the existence of many nonlinearity effects related to the rotor dynamics and electromagnetic have imposed great challenge in designing effective control algorithm that is able to produce a promising control performance and viable for practical use. The dynamic model of the system that mathematically represents the actual physical system is formed in numerous structures in which these equations serve an important design tool before any controller can be developed. Based on this review, there are many established models that have been developed and are found to be more than adequate for the development of the controller algorithm. In the realm of linear control, the most favored approach is to linearize these nonlinear models at an operating point and linear control method is synthesized such as PI, PID or LQR control. Unsatisfactory performances are usually observed when the system deviates from the operating point which indicates the need of more robust control algorithm. The H_∞, Q-parameterizations LPV and μ controllers have been proposed to overcome this weakness in which the variation of the system parameters, nonlinearity effects and disturbances are treated as uncertainties and structurally included in the design process. A significant improvements in term of system performance can be noticed where minimization of predefined performance indices is achieved and good system responses attained. As reported in many works, however, these robust control techniques based on nominal system model still shows degradation in performance when the uncertainties are ill-defined.

The use of nonlinear control methods seems a natural choice that can provide a more complete consideration of the parametric uncertainties, nonlinearities and disturbance present in the system while providing desirable system performances at wider range of operational speed. The model-based feedback linearization and backstepping control techniques have proven to give good system performances in many AMB system and the controllers are mostly cascaded with another linear of nonlinear robust controller to ensure the robust performance is attained in various system condition. This multi-loop control algorithm is resulted due to the fact that these control methods are rather sensitive to the error in the system modeling which produces residual effect that affects the closed-loop performance.

The intelligent based control also seems to offer a good solution in achieving good rotor stabilization and rejection of harmonic disturbance in AMB system. The adaptation of model-based control design techniques such as Lyapunov method and LMI has improved the design method where stability in certain operating range can be guaranteed. While this method is more on classifying the input and output relationship of the AMB system based on cognitive reasoning and scientific observations, establishing the input-output relationship based on experimental set-up seems to be more promising to design more effective intelligent-based controller that guarantees global stability of the system is achieved.

In the family of nonlinear robust control techniques for AMB system, SMC has shown to be capable of providing robust rotor positioning in wide range of system condition even with the present of parametric uncertainties, nonlinearities and disturbance. In recent years, the adaptations of many linear and nonlinear system design tools in the development of SMC control algorithm have enabled this controller type to accommodate various systems and design requirements systematically. This has offered a promising research contribution especially in the area of AMB control system. While the modeling techniques of AMB system is considered quite an established research field, the challenge remains in reconfiguring or rearranging the existing model in a certain structure in such a way that the major nonlinearity effects such as gyroscopic effect, nonlinear electromagnetic force and imbalance are appropriately represented. With the inclusion of this nonlinearity effects, the AMB model can be formed as a class of dynamic system that is suitable for the design of a dynamic robust controller.

1.4 Research Objectives

The objectives of this research are as follows:

I. To formulate a mathematical model of a nonlinear five DOF AMB system under Class A current input mode in state variable form. The complete model will be obtained by integrating the rotor dynamics and the nonlinear electromagnetic coils with the inclusion of gyroscopic effect, imbalance and nonlinear electromagnetic force.

II. To transform the nonlinear model of the AMB system into a class of nonlinear uncertain system comprising the nominal values and the calculated bounded uncertainties. These structured uncertainties exist due to the available limit of the airgap between the rotor and stator, its speed and variation in the rotor rotational speed.

III. To propose a new robust control algorithm technique based on deterministic approach for uncertain system. Particularly, SMC control technique will be utilized in the design where a new multi-objective sliding surface and robust continuous control law will be formulated.

IV. To implement the newly proposed controller into the mathematical model of AMB model so that the robustness of the new controller can be accessed. In particular, the effectiveness in minimizing the airgap deviation at various rotor speeds will be highlighted.

Verification on the stability and the reachability of the proposed controller will be accomplished by using the well-established Lyapunov’s second method. The performance of the AMB system will be accessed through extensive computer simulation performed on MATLAB platform and SIMULINK Toolbox as well as customly-developed LMI interface and solver which are YALMIP and SeDuMi.

1.5 Contributions of the Research Work

The following are the main contributions of the study:

I. A new representation of the nonlinear AMB model as a class of nonlinear uncertain system has been formulated which can accommodate the design of the controller.

II. New design algorithm for sliding surface that can accommodate many performance objectives in convex formulation. The solution can be systematically obtained by using LMI technique which produces the desired sliding surface parameter.

III. New design algorithm of a continuous SMC control law that is able to eliminate or attenuate the chattering while the reaching condition is guaranteed. Together with the sliding surface developed in I), a new complete SMC controller is established in which the control parameters can be parameterized systematically.

IV. Application and validation of this new robust controller on AMB system by extensive computer simulation.

1.6 Structure and Layout of Thesis

This thesis is organized into five chapters. In Chapter 2, the formulation of nonlinear models of 5-DOF AMB system is presented. Firstly, the dynamic of the rotor with force input in state space representation is illustrated. By defining the airgaps as the new state variable to be controlled, a geometric transformation is performed and a new state-space model of rotor is produced. Then, the nonlinear electromagnetic with current inputs is established and integrated with the rotor dynamic model to reach the complete AMB dynamic state-space model. Next, the AMB system is treated as a class of uncertain system. Based on the known allowable range of operation of the system and the maximum rotor rotational speed, the minimum and maximum uncertain bounds can be calculated to form a model with nominal and bounded uncertainties. This class of uncertain model representation serves the basis for the formation of the robust controller.

Chapter 3 presents the proposed new robust control strategy for AMB system based on SMC approach. The design method is composed of the sliding surface design and control law design. Since the inherited uncertainties satisfy the matching

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