SUSPENSION BRIDGE MODELING

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A dissertation submitted in partial fulfillment of the
requirements for the award of the degree of
Master of Science (Engineering Mathematics)

Faculty of Science
University of Technology Malaysia

DEC, 2010

Especially for my lovely family. My dad, Haa Chan Ping and my mom, Su Ah fong.
Also for all my brothers and sisters.

With gratitude for all the love support that all of you had given to me. You all are the best in my life. Thank you!

ACKNOWLEDGEMENTS

With this opportunity, I would like to express a billion thanks to my project supervisor, Assoc. Prof. Dr. Shamsudin bin Ahmad and panel Assoc. Prof. Dr. Khairil Anuar Arshad for their guidance, invaluable advice and encouragement throughout the process to complete this project.

I also like to appreciate to my family and friends for the moral support and encouragement throughout the process in this thesis and finally making it success.

Last but not least, thank you to all those involved directly or indirectly in helping me to complete the project which I would not state out every one of them. Thank you for everyone for their generosity and tolerance in doing all the things

ABSTRACT

The purpose of this study is to model the suspension bridge that oscillated by the external forces and investigate the phenomenon of resonance that would induce the destructive of the suspension bridge. Theoretically, the resonance will occur when the external frequency of the forces are tend to or equal to the natural frequency of the bridge. Resonance is a phenomenon of wave oscillation that can produce large amplitude even due to small periodic driving forces. A big building can collapse easily by the resonance due to the vibration of earthquake. A high frequency of sound can cause resonance to occur and break the glass or mirror. The mathematical model involves a suspension bridge that suspended at both end and it is vibrating under external forces (marching soldiers). In this model, the oscillation of the suspension bridge will be in linear wave equation form and will be solved by using the methods in Ordinary Differential Equation (ODE's) and Partial Differential Equation (PDE's). Different types of graph will be plotted by using MAPLE. Simulation results demonstrated that the bridge will collapse during the first two modes of the vibration when resonance occurred. Different lengths and angles of the suspension bridge also influence the period of the vibration when resonance occurred.

ABSTRAK

Kajian ini dilakukan bertujuan untuk model jambatan gantung yang terumbang-ambing oleh pengaruh luaran dan menyiasat fenomena resonansi yang akan mendorong kemusnahan jambatan gantung. Secara teori, resonan akan terjadi ketika frekuensi luaran daripada pengaruh luaran cenderung atau sama dengan frekuensi alam dari jambatan. Resonan adalah fenomena ayunan gelombang yang dapat menghasilkan amplitud besar walaupun kekuatan pendorong berkala kecil. Sebuah bangunan besar dapat diruntuhkan dengan mudah oleh resonan akibat getaran gempa. Frekuensi yang tinggi boleh menyebabkan resonan suara berlaku dan memecahkan kaca atau cermin. Model matematik ini melibatkan jambatan gantung yang ditangguhkan pada kedua-dua hujung jambatan dan bergetar di bawah kuasa pengaruh luaran (tentera berbaris). Dalam model ini, ayunan jambatan gantung ini adalah dalam bentuk persamaan gelombang linier dan akan diselesaikan dengan menggunakan kaedah Persamaan Pembezaan Biasa (ODE’s) dan Persamaan Pembezaan Separa (PDE's). Berbagai jenis graf akan diplotkan dengan menggunakan MAPLE. Keputusan simulasi ini menunjukkan bahawa jambatan akan runtuh pada dua mode pertama semasa resonan berlaku. Panjang dan sudut jambatan gantung yang berbeza juga mempengaruhi tempoh getaran resonan.

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LIST OF ABBREVIATIONS

DE	-	Differential Equation
ODE	-	Ordinary Differential Equation
PDE	-	Partial Differential Equation
IC	-	Initial Condition
BC	-	Boundary Condition

LIST OF SYMBOLS

f	-	Frequency of oscillation
t	-	Time
P	-	Period in one complete cycle of oscillation
ω	-	Angular frequency
F	-	Force
T	-	Tension
m	-	Mass
a	-	Acceleration
L	-	Length
ρ	-	density of material
A	-	Cross sectional area of an object
Δx	-	Distance between two points
θ , α	-	Angles

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CHAPTER 1

BACKGROUND OF THE STUDY

1.1 Introduction

This research involves the mathematical modeling of suspension bridges that suspended at both end. Mathematical modeling now a day becomes one of the most important parts of our daily life. Theoretical work in science and design work in engineering are often done by using mathematical modeling. Scientists and engineers are usually using the mathematical models to discover scientific principles or to predict the behavior of a real-world system. This mathematical tool is always deal with differential equations (DE's).

Differential equation is an equation that contains a derivative (or derivatives) of an unknown function [1]. Differential equation included Ordinary differential equation (ODE's) and partial differential equation (PDE's). Further detail and discussion about differential equation will be continued in Chapter 3. Since this research involved the suspension bridge, therefore, Chapter 2 will briefly discuss and introduce some basic knowledge of the suspension bridges.

Suppose that, there is a force (by wind or mankind) that disturbing the suspension bridge from its equilibrium position. The suspension bridge will oscillate up and down by the external forcing. Hence, we wish to model the oscillations of suspension bridges under the external forcing to find out the general function of the wave equation of the suspension bridge. The wave equation is obtained by using the mathematical and physical theory on a suspension bridge due to the force. Then, the solution will be obtained by solving partial differential equations and ordinary differential equations of the wave equation.

In this research, we assumed the suspension bridge is passing through by a union of marching soldiers. The external force that exerted to the suspension bridge is come from the marching soldiers. When the external force frequency get close to or equal to the natural frequency, then the resonance of the oscillation will occur [1][2]. The suspension bridge may collapse due to the resonance of the oscillation by the marching soldiers. The way of construction of the suspension bridge's model will be shown in Chapter 4.

After constructing the model, we need to determine which mode of the maximum amplitude of the resonance would bring to the bridge collapse and the other factors that will affect the bridge to resonant by the external force (soldiers) such as the lengths and the angles of the suspension bridge.

The graphical representation will be carried out by using MAPLE. Different graphs will be presented such as amplitude versus period, amplitude versus frequency and etc. The analysis about the phenomenon and behavior of the graphs will be discussed in Chapter 5. The conclusions and recommendations will be made in Chapter 6.

1.2 Background of the Problem

In the summer of 1940, Tacoma Narrow's Bridge was completed. Almost immediately, observer noted that sometimes the wind appeared to set up large vertical oscillations of the roadbed. The bridge became a tourist attraction as people came to watch, and perhaps ride the undulating bridge. Finally, on 7th November, 1940, during a powerful storm, the oscillations increased beyond any previously observed. Soon the vertical oscillations became rotational, as observed by looking down the roadway. The entire span was eventually shaken apart by the large oscillation, and the bridge collapsed. Another case was the collapsed of the Broughton Bridge near Manchester, England by a column of soldiers marching in union over the bridge [3].

These disasters have often been cited in textbooks on ordinary differential equation as examples of resonance, which happens when the frequency of forcing matches the natural frequency of oscillation of the bridge, with no discussion given on how the natural frequency is determined, or even where the ordinary differential equation used to model this phenomenon comes from. The modeling of bridge vibration by partial differential equation, although still simple minded, is a big step forward in connecting to reality.

The mathematicians, Lazer and Mckenna, one of the researcher state that the main cause of the collapsing bridge is due to the nonlinear effect, but not to the resonance. They state that the main cause leading to the destruction of suspension bridge was the large oscillations of the bridge which amplitude increases over time every cycle and proportional to the wind velocity. McKenna has defined a different viewpoint of the torsional oscillations in the bridge [4][5].

In the other hand, Professor Farquharson of University of Washington stated that in the Tacoma Narrow's Bridge, the wind speed at the time was 42 mph, giving a frequency by the vortex shedding mechanism of about 1 Hz. He observed that the frequency of the oscillation of the bridge of the bridge just prior to its destruction was about 0.2 Hz. So he concluded that the bridge collapsed due to the torsional (twisting) vibration by the wind [6]. Besides, others were arguing the bridge collapsed was due to the structure of the bridge itself. So, there was no agreement of the researchers about the main cause that can induce the collapsing of the Tacoma Narrow's suspension bridge.

Hence, this study was interested in how if there were another cause which can induce the collapsed of the bridge? We will try to construct a simple mathematical model based on the mathematic and physic theory of the suspension bridge. The model will be in linear wave form. So what could be happen to the suspension bridge? How did it collapse? So, we made a hypothesis that the bridge was collapsed due to the resonance of the external forces.

1.3 Statement of the Problem

The research was conducted in order to model the linear wave motion of suspension bridges under forcing by a column of soldiers that marching over the suspension bridge. The wave equation obtained will include the ordinary differential equation (ODE's) and partial differential equation (PDE's). Then we will solve the differential equations by using suitable method such as method of separation of variables. Then, the graph will be plotted and the value of natural frequency, period of the oscillation, amplitude, and etc will be calculated. The phenomenon for non-resonance and resonance by the external force will be discussed. Also, we wish to find out which mode of the oscillation will induce the resonance to give the real impact to the suspension bridge due to the external force (soldiers). We also interested in what others effect will influence the resonance of the suspension bridge such as the length and angle of the bridge.

1.4 Research Objectives

The research objectives in this study will be:

i. To derive the mathematical model of a suspension bridge that oscillates under external force.
ii. To analyze and discuss the behavior of the vibration due to different external frequencies for non-resonance mode and resonance mode.

iii. To analyze and discuss the period of the vibration in different vibration mode when the resonance occurred.

iv. To analyze and discuss the effect of different lengths and angles to the period of vibration when resonance occurred.

v. To identify which mode of vibration will give the real impact to the suspension bridge.

1.5 Scope of the Study

This research was only considering that the suspension bridge was collapsed by the resonance due to the external force (soldiers). Other consideration such as mechanical structure failure of the bridge was out of the scope in this study. The model was focused on partial differential equations and ordinary differential equations in linear wave equation. The graphical representation of the model will be constructed by using MAPLE.

1.6 Significant of the Study

Since our aims were to find out the period of different vibration mode and identify which mode will give the real impact to the suspension bridge when the resonance occurred. We also investigate the effect of different length and angle for the bridge safety. Hence, the results will help the engineers to take for consideration in the construction of the suspension bridge. Today, wind tunnel testing of bridge designs is mandatory. Therefore, they will design a more stable bridge instead of only focus on the material use for the bridges. Finally, the bridges will be more safety for all users.

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