PARALLEL CALCULATION OF DIFFERENTIAL QUADRATURE METHOD FOR THE BURGERS-HUXLEY EQUATION

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To my beloved parents and sisters,

Ng Peng Long, Tan Gek Sim, Ng Su Yee, Ng Su Rou,

MoMo

and all of my dear friends who support me.

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ABSTRACT

In this study, the Burgers-Huxley equation with Dirichlet's boundary conditions is treated by the Differential Quadrature method (DQM). The different types of grid points spacing used in the DQM, which is the equally-spaced and the Chebychev-Gauss-Lobatto grid points have been investigated on their effect on the accuracy of results generated. The finite difference method (FDM) will be used to solve some examples of Burgers-Huxley equation in order to compare with the solutions obtained by DQM to show the stability and accuracy of the numerical method. Then, the set of ordinary differential equations obtained is solved by using different types of implicit Runge-Kutta (RK) methods. C language programmes have been developed based on the examples discussed. Also, shared memory architecture of parallel computing is done by using OpenMP in order to reduce the time taken in simulating the numerical results. Consequently, the results showed that the Differential Quadrature method is a good alternative in approximating the Burgers-Huxley equation with excellent accuracy and stability.

ABSTRAK

Dalam projek ini, persamaan Burgers-Huxley dengan syarat sempadan Dirichlet akan diselesaikan dengan menggunakan kaedah Pembezaan Kuadrature (DQM). Pelbagai jenis cara pengiraan titik-titik yang digunakan dalam DQM seperti titik sama jarak, dan titik Chebychev-Gauss-Lobatto telah dikaji mengenai kesan cara pengiraan titik-titik terhadap ketepatan jawapan yang diperolehi. Kaedah beza terhingga (FDM) akan digunakan untuk menyelesaikan beberapa contoh persamaan Burgers-Huxley untuk dibandingan dengan penyelesaian yang diperolehi dengan DQM untuk menunjukkan kestabilan dan ketepatan kaedah berangka ini. Kemudian, set persamaan pembezaan biasa yang diperolehi akan diselesaikan dengan menggunakan pelbagai jenis kaedah Runge-Kutta (RK) secara implisit. Program bahasa C telah diaturcarakan berdasarkan contoh-contoh yang dibincangkan dalam projek ini. Selain itu, perkongsian memori secara OpenMP dalam pengiraan berkomputer secara selari dijalankan untuk mengurangkan masa penghitungan oleh komputer. Akhirnya, hasil kajian menunjukkan bahawa kaedah Differential Quadrature adalah alternatif yang lebih baik dalam menganggar persamaan Burgers-Huxley.

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LIST OF ABBREVIATIONS

FDM	-	Finite Difference Method
PDE	-	Partial Differential Equation
DQM	-	Differential Quadrature Method
RK	-	Runge-Kutta
OpenMP	-	Open Multi-processing
ODE	-	Ordinary Differential Equation
C-G-L	-	Chebyshev-Gauss-Lobatto

LIST OF SYMBOLS

E_p	-	Efficiency of parallel algorithm
t_p	-	Execution time of parallel algorithm
t_s	-	Execution time of sequential algorithm
x_1	-	Start point of domain
x_N	-	End point of domain
h	-	step size for time interval
n_steps	-	number of step size
p	-	Number of processors
t	-	Time
x	-	Grid point
N	-	Total number of grid points
$S(p)$	-	Speedup of parallel algorithm
f	-	Parallelizable fraction in source code

CHAPTER 1

INTRODUCTION

1.1 Background of the Problem

In most of the science and engineering fields, we might encounter with single or a system of partial differential equations (PDEs). As stated by Chang Shu (2000), the examples are: Newtonian fluid flows which are modeled by the Navier-Stokes equations; the vibration of thin plates is governed by a fourth order partial differential equation; whereas acoustic waves and microwaves can be simulated by the Helmholtz equation. In this study, the partial differential equation considered is the Burgers-Huxley equation which can effectively models the interaction between reaction mechanisms, convection effects and diffusion transports (Murat Sari and Gurhan Gurarslan, 2009).

Generally, most of these problems may involve the nonlinear partial differential equations where the closed-form solution is not available or not easily obtained. This fact makes the scientists realize the importance of developing another alternative to approximate the solutions of these partial differential equations. After years of researches, scientists therefore approximate the solution of the system of partial differential equations by using numerical discretization techniques on some function values at certain discrete points, so-called grid points or mesh points. Among the three most widely used numerical methods in engineering and in computational fluid dynamics are: the finite difference, finite element, and the finite volume methods.

The Finite Difference method (FDM) is the simplest method where the functions are represented by their values at certain grid points and approximate the derivatives through differences in these values. The Finite Element method (FEM), represents the functions in terms of basis functions and solve the partial differential equation in its integral form. The Finite Volume method (FVM), divides space into volumes and the change within each volume, which represents the flow rate of flux across the surfaces of the volume is computed. However, according to Chang Shu (2000), these three methods fall into low order methods, whereas Spectral and Pseudospectral methods are considered as global methods. Spectral method is generally the most accurate because it has excellent error properties. It represents functions as a sum of particular basis functions, often involving the use of the Fast Fourier Transform. However, the Finite Difference method (FDM) will only be discussed in this study.

Another numerical discretization technique that will be discussed in this study is the Differential Quadrature method (DQM). DQM is an extension of FDM for the highest order of finite difference scheme (C. Shu, 2000). As stated by R.C. Mittal and Ram Jiwari (2009), this method linearly sum up all the derivatives of a function at any location of the function values at a finite number of grid points, then the equation can be transformed into a set of ordinary differential equations (ODEs) or a set of algebraic equations. The set of ordinary differential equations or algebraic equations is then treated by standard numerical methods such as the implicit Runge-Kutta (RK) method that will be discussed in this study in order to obtain the solutions.

Generally, when the grid points increase and simulation time become longer, the whole simulation become expensive especially when the system of ODE is solved by implicit RK method. It become the general practice that the whole problem can be parallelized in order to reduce the computation time. In this research, we are using OpenMP language which is attributed to share memory architecture.

1.2 Statement of the Problem

There are plenty of ways to approximate the solution of Burgers-Huxley equation. To solve this equation, a set of initial and boundary conditions are needed. In this study, the Differential Quadrature method (DQM) is applied in solving the examples of Burgers-Huxley equation with Dirichlet's boundary conditions. DQM is compared with the Finite Difference method (FDM) in terms of their accuracy and convergency in solving Burgers-Huxley equation. Finally, parallel programming with OpenMP architecture is implemented in order to reduce the execution time of the C language program developed in this study.

1.3 Objectives of the Study

The objectives of this study are:

I. To select the grid points spacing applied in DQM in order to solve some Burgers-Huxley equations.

II. To compare the DQM with FDM in terms of their accuracy and convergence study of numerical solutions in solving Burgers-Huxley equation with Dirichlet's boundary conditions.

III. To develop C language program codes for DQM and FDM with the implicit RK method in order to solve examples of the Burgers-Huxley Equation.

IV. To parallelize the C language program codes developed on implicit RK method using OpenMP language.

1.4 Scope of the Study

In this study, the main numerical discretization tehnique discussed is the Differential Quadrature method (DQM). The Finite Difference method (FDM) will be used for comparison with the DQM on their accuracy and convergence study. Also, the scope of the study will be focused on solving the Burgers-Huxley equation with Dirichlet's boundary conditions. The implicit RK method will be utilized to solve the set of ordinary differential equations obtained from the DQM and the FDM. Next, C language program codes will be developed for both of the numerical methods with the implicit RK method. Then, parallelization on the C language program of the implicit RK method will be done using OpenMP.

1.5 Significance of the Study

In this study, the DQM which is widely used in science and engineering fields in solving nonlinear differential equation will be discussed and applied to Burgers-Huxley equation with Dirichlet's boundary conditions. This study is important to show how the different types of grid points spacing used can affect the accuracy of the numerical solutions. Then, the accuracy and convergence study of this method will be compared with the FDM to show its outstanding characteristics in solving nonlinear differential equation. The implicit RK method that implemented in order to solve the ODE obtained after solved by DQM and FDM is effective in solving stiff equation. Next, the C language program codes for these methods will be developed in convenience of checking the performances of the numerical methods. OpenMP language that based on share memory architecture is used in parallelizing the algorithm of implicit RK method in C language in order to reduce the execution time.

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