

## ABSTRACT

There are several optimization problems with number of feasible solution is polynomial bounded by the size of the given input instances. Graph Coloring is a classic NP-hard problem; hence, it is theoretically of great importance. Diverse applications of Graph Coloring have made the scientific community to be constantly searching for elegant solutions. Some of these applications are communication network, mobile radio frequency, computer register allocation, printed circuit board testing, time tabling and scheduling, pattern matching and Sudoku games. Many solutions have been proposed by the previous studies on solving Graph Coloring problems. But the most recent and efficient approach is commonly based on hybrid algorithms that use a particular kind of recombination operator. Hence, this study proposes a modified particle swarm optimization with fuzzy logic to obtain a high performance algorithm for solving the Planar Graph Coloring problem. Experimental results on several randomly generated graphs have illustrated the efficiency of the proposed method accordingly.

## ABSTRAK

Terdapat banyak masalah pengoptimuman dengan jumlah penyelesaian tersaur dibatasi secara polinomial oleh saiz input yang diberi. Pewarnaan Graf (PG) merupakan salah satu masalah polinomial keras konvensional yang secara teori masih memainkan peranan yang amat penting. Pelbagai aplikasi dalam PG telah membuatkan komuniti saintifik secara berterusan mencari penyelesaian yang tegar. Di antara aplikasi yang sering menggunakan kaedah PG adalah rangkaian komunikasi, frekuensi radio bergerak, umpukkan para komputer, pengujian papan litar bercetak, penskedul dan penjadualan, pola padanan dan permainan Sudoku. Banyak penyelesaian telah dicadangkan oleh kajian terdahulu bagi mengatasi masalah PG. Namun begitu, pendekatan terkini dan berkesan pada dasarnya bergantung kepada algoritma hibrid yang menggunakan gabungan operator tertakrif. Oleh yang demikian, kajian ini mencadangkan pembaikan pengoptimuman partikel berkelompok dengan mantik kabur bagi mendapatkan algoritma berprestasi tinggi dalam menyelesaikan masalah PG. Dapatan kajian yang dilaksanakan terhadap beberapa graf terjana telah menunjukkan bahawa kaedah pembaikan memberikan keputusan yang lebih baik dan amat berkesan.

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## CHAPTER 1

### Introduction

#### 1.1 Introduction

Optimization problem is a challenging field that searches for best solution among all possible solutions. There are several types of optimization problem such as NP-Optimization in which the number of feasible solution is polynomial bounded by the size of the given input instances. The optimization problems can be divided into two groups: Continuous and Discrete. The Continuous optimization problems are those that allow continuous variable to be the solution. In these cases, real number or real function to optimize a certain criteria will be found accordingly. The Discrete optimization problem allows only discrete variables and at the end, it finds a combinatorial object (Kubale n.d.). The importance of these techniques has been considered for many years for simulating research and development in different areas like network design (Woo et al. 2002), mathematical programming (Gamst 1986), sequencing and scheduling (Ambühl et al. 2006; Healy 2007; Z. Qin et al. 2010; Al-Anzi et al. 2006; Leighton 1979; de Werra 1985). Researchers have also shown that there are several combinatorial computations in user programs compared to the number of numerical computations. The graph coloring problem is one of the classical optimization problems.

The classical Graph coloring is a special case of graph labeling and the purpose of this problem is to assign colors to the vertices of a graph with a certain constraints which is called Vertex Coloring. The specific constraint in the classical graph coloring is that the assigned color to the vertices has to be unique among adjacent pair of vertices which connected through an edge. There are different types of graph coloring such as Edge Coloring where the colors will be assigned to the edges instead of vertices of graph and the mentioned constraint is no two connected edge shares the same color, List Coloring, Total Coloring, Harmonious Coloring, Exact coloring and others.

The application of graph coloring has been considered in different areas such as communication network (Woo et al. 2002), time tabling and scheduling (Ambühl et al. 2006; Healy 2007; Z. Qin et al. 2010; Al-Anzi et al. 2006; Leighton 1979; de Werra 1985), mobile radio frequency assignment (Riihijarvi et al. 2006; Gamst 1986), pattern matching, computer register allocation (K. D. Lee 2003; J. K. Lee et al. 2008; Werra et al. 2002; Chaitin et al. 1981; Chow & Hennessy 1990) and printed circuit board testing (Garey et al. 1976), Sudoku (logical based combinatorial number-placement puzzle).

## **1.2 Problem Background**

Graph Coloring is one of the most important problems among combinatorial and graph theory challenges. As mentioned in previous section there are several applications of graph coloring in different research areas that shows the importance of the graph coloring problem. These areas have been spread in different fields and most importantly it helps solving important issues.

In graph theory, graph coloring is a special case of graph labeling; it is an assignment of colors to elements of a graph subject to certain constraints. In its simplest

form, it is a way of coloring the vertices of a graph such that no two adjacent vertices share the same color. Graph Coloring problem is an NP-Hard problem and then there are no solution with polynomial time, but in several researches, the authors mentioned several methods for solving this classical problem. Among several types of graph coloring problem the four-coloring graph plays significant role. In this study the special case of vertex coloring that tries to color the planar graph with four number of color has been studied and this kind of graph coloring also has been studied in several papers.

Eventually, the Graph Coloring Problem gained notoriety as the prototype of a hard problem in the combinatorial optimization. Nowadays as the vast field of artificial intelligence has thrived, many algorithms are designed and used to solve this classic problem in the best possible way. However, it should be noted that many of these existing algorithms have serious flaws (Enami Eraghi et al. n.d.; Hin 1990; Galinier & Hertz 2006) such as 1) some only work efficiency for small problems (Galinier & Hertz 2006) 2) some fail noticeably when it comes down to accuracy (Akbari Torkestani & Meybodi n.d.; Enami Eraghi et al. n.d.), 3) some consume a long time in spite of managing to solve the problem (Enami Eraghi et al. n.d.; Glass & Prügel-Bennett 2003). Galinier P. et al. (Galinier & Hertz 2006) has shown that some graphs, especially large random graphs, cannot be colored efficiently by using pure local search algorithms, and several approaches have, therefore, been proposed to deal with these difficult instances.

In the past decades, several types of solution for graph coloring problem to find good coloring in a reasonable amount of time have been proposed such as exact algorithms and heuristic algorithms. A new approach of heuristic methods such as Simulated Annealing and Tabu search has been designed to avoid local traps and cycling. In Simulated Annealing, random factors are introduced to let the search jump out of the neighborhood of a local optimal point and to prevent cycling, while traditional Tabu Search uses its own mechanism for the same purpose, but both of these methods have their own weaknesses. Each of these methods has been used for solving the graph coloring problem (Cui et al. 2008; Mabrouk et al. 2009; Nakayama & Masuyama 1997; Talavn & Yez 2008; Hertz & Cosine 1991; Hertz & de Werra 1987; Klimowicz & Kubale 1993).



In addition of the above mentioned heuristic methods for solving graph coloring problem, there are several other methods has been considered. Genetic Algorithm was one of the solutions as it used in (Davis 1991; Fleurent & Ferland 1996) and also Genetic Local Search (Leighton 1979; Costa et al. 1995; Fleurent & Ferland 1996; Voudouris & Tsang 2003). Also new Genetic Local Search has been proposed by Dorne and Hao (Dorne & Hao 1999), and Galinier and Hao (Galinier & Hao 1999). Both algorithms use a simple way to manage the population while the local search operator is Tabucol. Also Variable Neighborhood Search (Mladenovic & Hansen 1997; Avanthay et al. 2003), Learning Automata (Enami Eraghi et al. n.d.; Akbari Torkestani & Meybodi n.d.), Amacol (Galinier et al. 2008), Neural Network (Gu & Yu 2004; Jagota 1996; Gassen & Carothers 1993), Particle Swarm Optimization (Cui et al. 2008) have been used for this purpose.

Graph coloring is a significant problem in computer science and operation research. Although several algorithm and heuristic have been proposed for solving graph coloring problem, only medium size problem can be solved optimally. It is also difficult to compute a provably good approximate solution. Solving the coloring problem for large graphs remains a computational challenge. The most recent and also most efficient approach is based upon hybrid algorithms that use a particular kind of recombination operator. In this study a hybrid method is proposed to solve the graph coloring problem with less complexity and better solutions.

### **1.3 Problem Statement**

Graph Theory is widely used in different areas of science for modeling various structures. One of the most significant challenges in graph theory is Graph Coloring which has been utilized for particular problems and most importantly the four-coloring theory for planar graph is mentioned as one aspect of this problem. As any other

optimization problems that has been considered, the four-coloring problem has been a domain of research for many years and researchers proposed many kinds of method for finding the best solution such as Genetic Algorithm, Neural Network, Tabu Search, Learning Automata and Particle Swarm Optimization (PSO).

Hence, in this study the PSO with modification and Fuzzy Inference System (FIS) are proposed to obtain best solution for graph coloring problem. Hence the hypothesis of the study can be stated as:

The modified particle swarm optimization algorithm enhanced with fuzzy inference system can be an efficient solution for graph coloring problem especially four-color problem.

#### **1.4 Dissertation Aim**

The aim of this research is to solve the graph coloring problem and specifically four-coloring problem in the proper time and storage complexity. For this purpose the Particle Swarm Optimization has been modified as one of the best heuristic method and for better computation, the Fuzzy Inference System has been utilized.

## 1.5 Objectives

The main goal of this study is to present a new hybrid method by which the Graph Coloring Problem can be solved relatively accurately and fast, without being trapped in the local optimum during the process of search. Here are the Objectives of this study in order to accomplish its aim:

- To propose modified Particle Swarm Optimization for Graph Coloring Problem
- To enhance Particle Swarm Optimization with Fuzzy Inference System
- To compare the results with Particle Swarm Optimization method in terms of complexity, coloring failure and CPU time.

## 1.6 Scope

The scope of this research is mentioned below:

- Solving the four-color graph coloring problem for planar graphs
- Software used in this study is Matlab
- Data used in this study is random graphs with different vertex size

## **1.7 Significant of Study**

As mentioned in the problem background, the importance of the graph coloring has been considered for several issues and for this purpose different methods have been proposed. The proposed methods have advantages and disadvantages and each one of them is used in different areas based on these characteristic; for instance, fast convergence of algorithm is useful for some problems and its accuracy is beneficial for some other kind of problems. Hence in this study the proposed method utilized the benefits of Particle Swarm Optimization and Fuzzy Inference System for better result and less complexity of time and storage.